Spin Asymmetry and Gerasimov-Drell-Hearn Sum Rule for the Deuteron

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An explicit evaluation of the spin asymmetry of the deuteron and the associated Gerasimov-Drell-Hearn sum rule is presented which includes photodisintegration, and single and double pion and eta production as well. Photodisintegration is treated with a realistic retarded potential and a corresponding meson exchange current. For single pion and eta production the elementary operator from MAID is employed, whereas for double pion production, an effective Lagrangian approach is used. A large cancellation between the disintegration and the meson production channels yields for the explicit GDH integral a value of 27.31 μb to be compared to the sum rule value 0.65 μb.

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Introduction.—In recent years the interest in the Gerasimov-Drell-Hearn (GDH) sum rule [1,2] has been revived and has become subject to quite intensive research [3–5]. This sum rule links the anomalous magnetic moment κ of a particle *of spin S* and mass M to the integral over the energy weighted spin asymmetry of the absorption cross section with respect to circularly polarized photons and a polarized target:

$$I^{\rm GDH} = \int_0^\infty \frac{d\omega}{\omega}[\sigma^P(\omega) - \sigma^A(\omega)] = 4\pi^2\kappa^2\frac{e^2}{M^2}S,$$

where σ^P (σ^A) denotes the cross sections for parallel (antiparelle l) orientation of photon and deuteron spins.

Obviously, for $\kappa \neq 0$ the particle possesses an internal structure. However, the opposite is not in general true. A particle having a vanishing or very small κ need not be pointlike or nearly pointlike. In this respect, the deuteron is a particularly instructive example because it has a very small anomalous magnetic moment $\kappa_d = -0.143$ nm and, thus, a very small sum rule value $I_d^{\rm GDH} = 0.65$ μb. On the other hand, it is well known that the deuteron has quite an extended spatial structure due to its small binding energy. The smallness of κ_d arises from an almost complete cancellation of the anomalous magnetic moments of neutron and proton because their spins are parallel and predominantly aligned along the deuteron spin.

Therefore, it is expected that, also for the sum rule integral, such cancellation of different contributions occurs. Indeed, it has been shown in previous work [6,7] that a large negative contribution to the sum rule of -413 μb arises from the photodisintegration channel, which has its origin in a large negative spin asymmetry right above breakup threshold. It is almost equal in absolute size to the sum of neutron and proton GDH values $I_p^{\rm GDH} + I_n^{\rm GDH} = 438$ μb, the latter value taken as a rough estimate of the meson production contributions.

This cancellation of contributions from quite different energy regions might be surprising on first sight. But a closer look reveals indeed an intimate connection via the underlying strong interaction dynamics because the latter is dominated in both energy regions by pion degrees of freedom, namely, at low energy via the dominance of the one-pion-exchange potential and at high energy via pion production as the primary absorptive mechanism on the nucleon.

However, the previous explicit evaluation in Ref. [6] suffered from several shortcomings. First of all, only single pion production was considered. The production operator was very simple and included, besides Born terms, only the Δ resonance, and final state interaction (FSI) was neglected. Thus, the GDH integral was evaluated up to 550 MeVonly where convergence had not been reached. Furthermore, for the photodisintegration channel the Δ excitation was treated in the impulse approximation (IA) neglecting the influence of the $N\Delta$ interaction. As a result, a value of -187 μb for the total GDH integral up to 550 MeV was obtained to be compared to the sum rule value $I_d^{\rm GDH} = 0.65$ μb. In subsequent work [8], the treatment of single pion production has been improved by including complete FSI in the πN and NN subsystems. However, the simple operator form was kept and, thus, the limited integration range. The major FSI effect was a reduction of the spin asymmetry of the incoherent π^0 contribution which increased the discrepancy of the explicit calculation to the sum rule value even further.

The scope of the present Letter is to present a considerably improved calculation with respect to photodisintegration and single pion production, with additional inclusion of two-pion and eta production.

Spin asymmetries for the deuteron.—As elaborated in detail in Ref. [6], the asymmetry of the photodisintegration channel is governed in the near threshold region by the dominant isovector $M1$ transition to the $^1S_0$ state which can only be reached for antiparallel photon and deuteron spins. Thus, a huge negative spin asymmetry arises as is shown in Fig. 1. In addition to the earlier evaluation [6], we show the result of a recent considerably

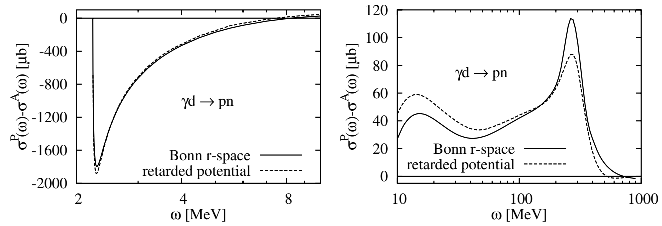

FIG. 1. Spin asymmetry of deuteron photodisintegration using (a) Bonn r-space potential [14]+MEC + IC + RC and (b) retarded potential + retarded π MEC, Δ degrees in coupled channel, πd channel +RC [9]. Left panel: low energy region; right panel: high energy region.

improved calculation [9] which is based on a retarded potential with retarded π exchange currents (MEC), incorporation of the $N\Delta$ dynamics into a coupled channel approach with retarded interactions, and inclusion of the πd channel and relativistic contributions. In comparison to Ref. [9], one finds, besides a small increase of the asymmetry right at threshold, a significant change at higher energies, in particular, in the Δ region.

For incoherent single pion and eta production on the deuteron an improved evaluation of the spin asymmetries is now available in which, for the elementary production operator, the more realistic MAID model [10] is used allowing one to extend the calculation to considerably higher energies. The results for π production in IA and with inclusion of complete rescattering in the final NN and πN subsystems (like in Ref. [8]) are shown in Fig. 2

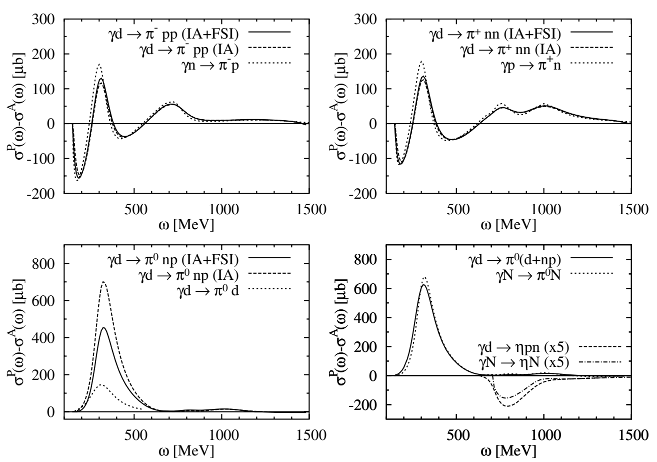

FIG. 2. Spin asymmetries of single pion and eta production on nucleon and deuteron for various charge channels. The results for the deuteron are in IA and with inclusion of FSI in final NN and πN subsystems. Upper panels: charged single pion production; lower left panel: coherent and incoherent π^0 production on deuteron; lower right panel: total π^0 and η production on nucleon and deuteron. For π^0 and η production on the nucleon the sum of spin asymmetries of neutron and proton is shown. The result for coherent π^0 production is taken from [6].

together with the corresponding asymmetries for the elementary reactions. Furthermore, in the lower right panel of Fig. 2 we show η production on nucleon and deuteron, enlarged by a factor 5. For the deuteron only incoherent production with NN-FSI is shown while coherent production is negligible. For π production one notes, besides a positive contribution from the $\Delta(1232)$ excitation, another one above a photon energy of 600 MeV from $D_{13}(1520)$ and $F_{15}(1680)$. For charged pion production FSI effects are in general quite small, except near threshold and in the maximum. The same is true for eta production. But FSI is quite sizable for incoherent neutral pion production due to the nonorthogonality of the final state plane wave in IA to the deuteron bound state wave function. Comparing the deuteron spin asymmetries in Fig. 2 to the corresponding nucleon spin asymmetries, one finds a significant difference.

For two-pion production our evaluation is based on a traditional effective Lagrangian approach similar to the one in Ref. [11]. A detailed description of the model will be published elsewhere [12]. The model gives a satisfactory description of the available total cross section data on the proton as is shown in Fig. 3 for the two dominant channels.

The resulting spin asymmetries for double pion production on the deuteron are shown in Fig. 4, both in IA and with inclusion of NN rescattering (NN-FSI). The latter is small. The largest contribution comes from the $\pi^-\pi^+$ channel which has not reached convergence at 2.2 GeV in contrast to the other three channels. Furthermore, one notes again significant differences to the elementary reaction.

Explicit evaluation of the GDH sum rule for the deuteron.—Now we will turn to the explicit evaluation of the finite GDH integral as defined by

$$I^{\mathrm{GDH}}(\omega) = \int_0^{\omega} \frac{d\omega'}{\omega'}[\sigma^P(\omega') - \sigma^A(\omega')].$$

The results for photodisintegration, and single and double pion and eta production, are exhibited in Fig. 5. With respect to the photodisintegration channel, one notes a

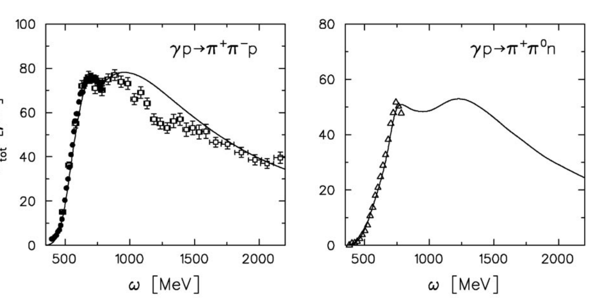

FIG. 3. Total cross section for the dominant two-pion production channels on the proton. Experimental data from [15,16].

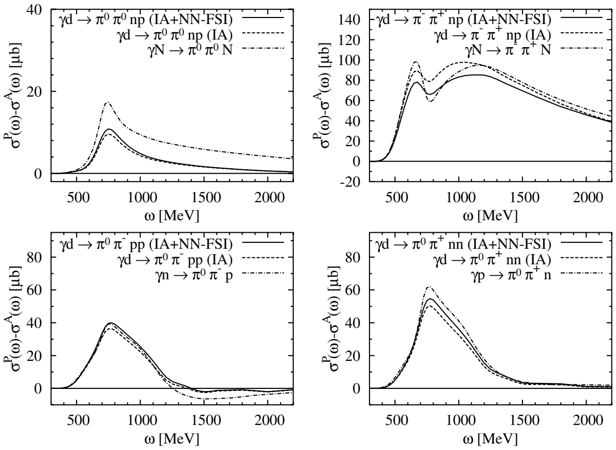

FIG. 4. Spin asymmetries of double pion production on nucleon and deuteron for various charge channels. For $\pi^0\pi^0$ and $\pi^-\pi^+$ production on the nucleon, the sum of spin asymmetries of neutron and proton is shown. The deuteron results are in IA and with inclusion of FSI in the final NN subsystem.

significant reduction of the absolute size from retardation in potential and π MEC at lower energies and at higher energies from an improved treatment of the Δ excitation in a coupled channel approach. For single and double pion production on the deuteron we show, in addition to the IA, the influence of NN-FSI which gives the most important FSI contribution. It is obvious that convergence of the GDH integral is reached for $\gamma d \to np$ already at about 0.8 GeV and for neutral pion and eta production at 1.5 GeV, however, not completely for charged π production and also not for double pion production, in particular, for the channel $\pi^-\pi^+$.

The comparison of the finite GDH integral for single and double pion production on nucleon and deuteron is also shown in Fig. 5. One sees quite clearly significant differences between single charged pion production on nucleon and deuteron because of (i) Fermi motion and to a lesser extent (ii) final state interaction. Also for double pion production the differences are sizable and FSI effects are comparable for some channels. The difference between pion production on nucleon and deuteron is also seen in Table I, where the separate contributions from the various charge channels of pion production to the GDH integral are listed.

The contributions of various channels to the finite GDH integral for nucleon and deuteron are listed in Table II. While for the neutron the total sum is about 8% lower than the sum rule value, it is too large by about 28% for the proton. In contrast to our earlier evaluation one now indeed finds for the deuteron a large cancellation between the contributions of photodisintegration and meson production to the GDH sum rule. The sum of all contributions of 27.31 μb now is positive, though somewhat too large. However, one should keep in mind that the present theoretical evaluation still contains several uncer-

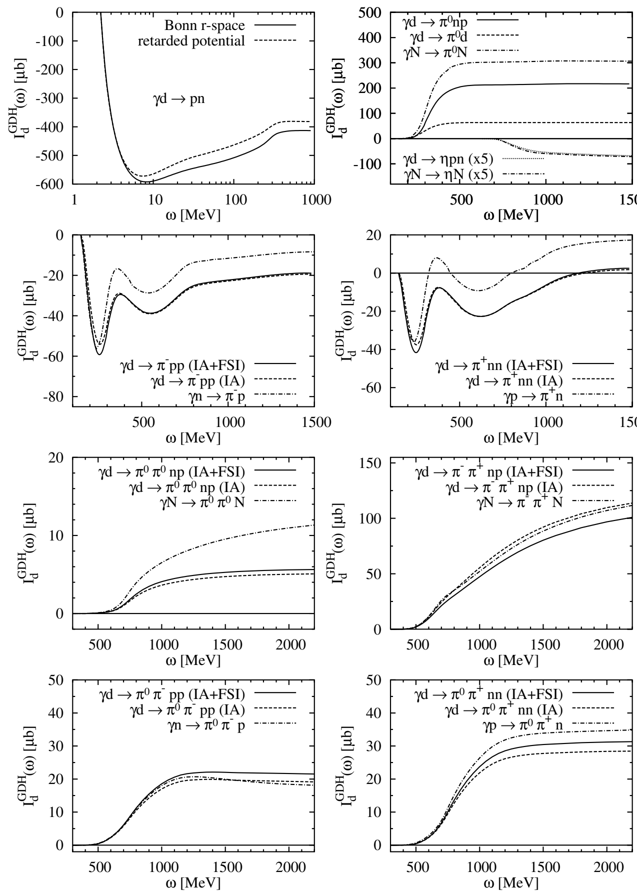

FIG. 5. Contribution of various channels to the finite GDH integral as function of the upper integration limit for deuteron disintegration, and single and double pion and eta production on nucleon and deuteron. For the neutral charge channels π^0, η, $\pi^0\pi^0$, and $\pi^-\pi^+$, the nucleon integrals are the sum of neutron and proton integrals.

tainties arising from shortcomings of the theoretical model and the neglect of contributions at higher energies. Probably the largest uncertainty arises from two-pion production. As a rough estimate of this uncertainty we take from Table II the difference of about 40 μb between the explicit evaluation for neutron plus proton and the corresponding sum rule value.

Conclusions and outlook.—The GDH sum rule of the deuteron is a very interesting observable on its own value

TABLE I. Contributions of various charge states of single and double π production on neutron, proton, and deuteron to the finite GDH integral (in μb), integrated up to 1.5 GeV for single pion and up to 2.2 GeV for double pion production.

	π^0	π^-	π^+	$\pi^0\pi^0$	$\pi^+\pi^-$	$\pi^0\pi^-$	$\pi^0\pi^+$
Neutron	147.34	−8.39		6.42	57.49	18.11	
Proton	159.10		17.28	4.91	54.18		34.84
Deuteron	279.87	−18.94	2.51	5.64	100.87	21.52	31.31

TABLE II. Contributions of various channels to the finite GDH integral (in μb), integrated up to 0.8 GeV for photodisintegration, 1.5 GeV for single pion and eta production, and 2.2 GeV for double pion production on nucleon and deuteron.

	np	π	$\pi\pi$	η	Σ	Sum rule
Neutron		138.95	82.02	−5.77	215.20	233.16
Proton		176.38	93.93	−8.77	261.54	204.78
Deuteron	−381.52	263.44	159.34	−13.95	27.31	0.65

because of a strong anticorrelation between the spin asymmetries of low energy photodisintegration and the one of meson production channels at higher energies. For the photodisintegration channel the spin asymmetry is sensitive to relativistic contributions, in particular, to the spin-orbit current, to meson retardation in potential and associated two-body currents, and to a dynamical treatment of the Δ excitation. Retardation and $N\Delta$ dynamics result in a sizable reduction by about 8% of the GDH contribution compared to previous evaluations without such refinements. An experimental test of these features would be highly desirable. For this channel, the integral is well converged. The uncertainty introduced by the neglect of higher isobar configurations beyond the most important $N\Delta(1232)$ one is estimated in IA [13] to be of the order of 5 μb. The second improvement concerns the extension of integration up to 1.5 GeV in single pion production by using the MAID model resulting in an increase of its GDH contribution by 113 μb compared to Ref. [8]. However, most important is the inclusion of double pion production for the first time, yielding 153 μb to I^{GDH} integrated up to 2.2 GeV. As a result, we found from meson production a positive contribution about equal in absolute size than the one from photodisintegration, confirming thus a large cancellation between photodisintegration and meson production. However, the resulting total value of the finite integral of 27.31 μb overshoots the sum rule value. But we do not consider this as serious, because of the above mentioned model uncertainties. Moreover, not all channels of single and double pion production had reached complete convergence. In addition, one should keep in mind that the interaction effects of meson production are taken into account only in an approximate manner. Another uncertainty, though probably quite small, arises from the neglect of three-pion, kaon, etc., production. Thus, there is room for improvements in the theoretical framework which hopefully will allow one to close the gap between the explicit evaluation and the sum rule value.

The strong cancellation between the regions at low and high energies is a fascinating feature clearly demonstrating the decisive role of the pion as a manifestation of chiral symmetry governing strong interaction dynamics in these two different energy regions. The cancellation constitutes also a challenge for any theoretical framework since it requires a unified consistent treatment of hadron and electromagnetic properties for both energy regions.

With respect to meson production channels on nucleon and deuteron, the corresponding spin asymmetries show a significantly different behavior. This means that a direct experimental access to the neutron spin asymmetry from a measurement of the spin asymmetry of the deuteron by subtracting the one of the free proton is not possible. On the other hand, polarization data of meson production on the deuteron certainly will provide a more detailed test of meson production on the neutron and thus, in an indirect manner, on its spin asymmetry. However, for this endeavour a reliable theoretical model is needed. The first significant steps in this direction have been presented in this Letter.

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