Vortex Arrays in a Rotating Superfluid Fermi Gas

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The behavior of a dilute two-component neutral superfluid Fermi gas subjected to rotation is investigated within the context of a weak-coupling BCS theory. The microscopic properties at finite temperature are obtained by iterating the Bogoliubov–de Gennes equations to self-consistency. In the model, alkali atoms are strongly confined in quasi-two-dimensional traps produced by a deep one-dimensional optical lattice. The lattice depth significantly enhances the critical transition temperature and the critical rotation frequency at which the superfluidity ceases. As the rotation frequency increases, the triangular vortex arrays become increasingly irregular, indicating a quantum melting transition.

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Great experimental strides have been made over the past year in the goal to form a BCS-like superfluid state with ultracold neutral Fermi gases. Through the use of magnetic field-induced Feshbach resonances [1–3], strong interactions between quantum degenerate fermions in two different hyperfine states have been induced [4–7]. Molecules result when the resulting interactions are repulsive [8–12]; though Cooper pairing at high temperatures (within an order of magnitude of the degeneracy temperature T_F) is widely expected for attractive interactions near the Feshbach resonance [13,14], the state actually produced in recent experiments [15,16] remains to be fully elucidated.

Another proposed approach to high-temperature superfluidity in these systems is to confine the gases in optical lattices [17,18]. The flattening of the energy bands as the lattice depth increases ensures that a large number of atoms participate in the pairing even for relatively weak interactions. A deep one-dimensional (1D) lattice corresponds to an array of quasi-2D traps, whose large effective axial confinement can enhance the interaction strength sufficiently [19] to raise the superfluid transition temperature T_c to within 10% of T_F [20].

Rotating the superfluid around the optical lattice axis should yield vortex arrays akin to those observed recently in trapped Bose-Einstein condensates [21]. Few clear signatures of superfluidity exist for dilute Fermi gases [22]; the first evidence has been obtained only very recently [23–25]. The experimental observation of quantized vortices would be an unambiguous indicator of superfluidity in the system, though significant depletion of particles in the vortex cores is expected only in the strong-coupling limit [26]. Vortices in the array of weakly coupled two-dimensional (2D) traps serve as a clean model of the pancake vortices in strongly layered high-T_c superconductors such as $Bi_2Sr_2CaCu_2O_{8+\delta}$ [27]. Furthermore, because a small fraction of the total number of atoms reside in any individual lattice site, one might expect to readily access the quantum Hall limit at high rotation frequencies where the number of vortices becomes comparable to the number of atoms [28,29].

To my knowledge, the calculations presented below constitute the first microscopic determination of vortex arrays in inhomogeneous Fermi superfluids. The central results of the work are the following: (1) the critical rotation frequency for the cessation of superfluidity (the rotational analogue of the critical velocity v_c in superconductors) scales with T_c; (2) the vortex cores for superfluids in the BCS limit that are confined in 1D optical lattices should be directly visible experimentally at low temperatures; and (3) the vortex lattice is expected to melt at zero temperature at high rotation frequencies approaching the expected quantum Hall transition.

The calculations were inspired by experiments with attractive Fermi gases confined in optical traps [7,16]. The experiments were performed at magnetic fields above but near the Feshbach resonance at 860 G [30] at which the s-wave scattering length between atoms in the hyperfine states $|F = \frac{1}{2}, m_F = \pm\frac{1}{2}\rangle$ (labeled $\sigma = \uparrow, \downarrow$ below) is $a \sim -10^4 a_0$ ($a_0 = 0.0529$ nm). With $N = 7.5 \times 10^4$ atoms in each species [7], the Thomas-Fermi (TF) approximation $N = \frac{1}{6}\frac{\omega_\rho}{\omega_z}(\frac{\mu_{\rm TF}}{\hbar\omega_\rho})^3$ can be inverted to yield the TF chemical potential $\mu_{\rm TF} \approx 25\hbar\omega_\rho$ and the Fermi momentum at the trap center $\hbar k_F^0 \approx \sqrt{2m\mu_{\rm TF}}$; one then obtains $k_F^0|a| \gg 1$, which implies very strong coupling.

In contrast, the BCS mean-field approximation is applicable only for $k_F^0|a| \lesssim 0.6$, beyond which one needs to include pairing fluctuations [31]. The BCS transition temperature is given approximately by the uniform 3D expression [32] $T_c = (8e^{\gamma-2}/\pi)\mu_{\rm TF}\exp[-\pi/(2k_F^0|a|)]$, where $\gamma = 0.5771\cdots$ is Euler's constant; this does not take into account polarization corrections (not considered in the present work), which lower T_c by a factor of approximately 2.2 [33]. Decreasing $|a|$ such that $k_F^0|a| = 0.6$ yields $T_c \approx 1.1\hbar\omega_\rho \approx 0.06T_F$ which remains too low to easily access experimentally.

This value for T_c can be substantially increased if the laser were retroreflected to form a 1D optical lattice, with potential $V_{\rm lat} = nE_R \sin^2(2\pi z/\lambda)$ where n is the lattice depth in units of the recoil energy $E_R = \frac{\hbar^2}{2m}(\frac{2\pi}{\lambda})^2$. Near the center of each well, the potential may be approximated as quadratic in z, with the ratio of the effective axial and radial frequencies $\alpha \equiv \tilde{\omega}_z/\omega_\rho = \sqrt{n}(2\pi d_\rho/\lambda)^2 = 20$–$80$ where $d_\rho = \sqrt{\hbar/m\omega_\rho}$ and the lower and upper limits correspond to $\lambda = 1064$ nm, $n \approx 0.4E_R$ [16] and $\lambda = 10.6$ μm, $n \approx 50\,000E_R$ [7]. Neglecting interwell tunneling, all of the atoms occupy the axial oscillator ground state in the quasi-2D limit $\mu \ll \hbar\tilde{\omega}_z$. A straightforward calculation [34] gives the superfluid transition temperature for a uniform quasi-2D system $k_B T_c^{\rm 2D} = (2e^\gamma/\pi)\mu_{\rm TF}^{\rm 2D} \exp(-\sqrt{\frac{\pi}{2}}\frac{\tilde{d}_z}{|a|})$ where $\tilde{d}_z = \sqrt{\hbar/m\tilde{\omega}_z}$, not including polarization effects which reduce $T_c^{\rm 2D}$ by a factor of e [20]. In the calculations below, $\mu = 10\hbar\omega_\rho$, $d_\rho = 1$ μm, $a = -2160a_0$ (corresponding to the triplet scattering length that would be obtained for an external bias field around 2000 G far above the Feshbach resonance), and $20 \le \alpha \le 40$. Choosing $\alpha = 40$, one obtains $T_c^{\rm 2D} \approx 0.16$ μK $\approx T_F/5$. This significantly improves T_c while $k_F^{\rm 2D}|a| \approx 0.5$ remains in the weak-coupling limit.

The calculations are based on a BCS mean-field approximation to the full quasi-2D interaction Hamiltonian [35], where the particle density and gap functions are defined by the thermal averages $n_\sigma(\mathbf{x}) = \langle \psi_\sigma^\dagger(\mathbf{x})\psi_\sigma(\mathbf{x})\rangle$ and $\Delta(\mathbf{x}) = -\tilde{g}'\langle\psi_\uparrow(\mathbf{x})\psi_\downarrow(\mathbf{x})\rangle$, respectively [here and below $\mathbf{x} = (x, y)$ corresponds to a 2D vector]. The ultraviolet divergence in the definition of the superfluid gap is regularized using the pseudopotential method [36,37], giving rise to a regularized coupling constant $\tilde{g}'$. An equal population of the two hyperfine components $N_\uparrow = N_\downarrow$ is chosen to maximize T_c [32]. Diagonalizing the mean-field Hamiltonian in a frame rotating around the lattice axis at angular frequency Ω, one obtains the Bogoliubov–de Gennes (BdG) equations [35]

$$\begin{bmatrix} \mathcal{H} - \mu & \Delta(\mathbf{x}) \\ \Delta^*(\mathbf{x}) & -(\mathcal{H} - \mu)^* \end{bmatrix} \begin{bmatrix} u_n(\mathbf{x}) \\ v_n(\mathbf{x}) \end{bmatrix} = E_n \begin{bmatrix} u_n(\mathbf{x}) \\ v_n(\mathbf{x}) \end{bmatrix}. \quad (1)$$

Here $\mathcal{H} = -\frac{\hbar^2}{2m}\nabla_{\mathbf{x}}^2 + \frac{1}{2}m\omega_\rho^2(x^2 + y^2) + g' n_\sigma(\mathbf{x}) - \Omega L_z$, where $L_z = i\hbar(y\partial_x - x\partial_y)$. The density and gap functions are $n_\sigma(\mathbf{x}) = \sum_n[|u_n|^2 f(E_n) + |v_n|^2(1 - f(E_n))]$ and $\Delta(\mathbf{x}) = -\tilde{g}'\sum_n u_n v_n^*(1 - 2f(E_n))$, respectively, and must be iterated to self-consistency together with the quasiparticle amplitudes u_n and v_n appearing in Eqs. (1) to obtain equilibrium solutions; the sums run over positive E_n and $f(E_n) = (e^{E_n/k_BT} + 1)^{-1}$.

The BdG matrix was evaluated numerically using a discrete variable representation [38] based on Hermite polynomials with up to 100 functions/points each in x and y. The eigenvectors for energies up to E_n(max) were obtained using the routine PZHEEVX on a 32-processor xeon cluster. For the largest grid, each diagonalization took approximately 50 min. The procedure was deemed converged when the magnitudes of Δ and n_σ at successive iterations were smaller than some predefined tolerance; for large Ω as many as 3000 iterations were required to verify that the ground state had been obtained. The sums over E_n were found to fully converge for $E(\max) \sim 3\mu$. N_σ was on the order of 3000 for $\Omega = 0.98\omega_\rho$ for $\alpha = 40$.

Because the effective coupling strength $g' \propto \sqrt{\alpha} \propto n^{1/4}$ (where n is the lattice depth in recoils) is a parameter adjustable by varying the trapping laser intensity, it is useful to explore the (T, Ω) superfluid phase diagram for various α, subject to the constraints $\mu/\hbar\omega_\rho < \alpha$ (single axial mode approximation) and $k_F^0|a| < 1$ (weak coupling). Results for $\alpha = \{20, 30, 40\}$ are shown in Fig. 1; the superfluid transition was obtained assuming cylindrical symmetry (when $\Delta = 0$ there are no vortices) with random points checked using the full 2D code. The numerical value of T_c for $\Omega = 0$ is comparable to the TF value of $T_c^{\rm 2D}$ given above, in spite of the small total number of atoms and the inhomogeneous density. Superfluidity also ceases at zero temperature at a critical angular frequency Ω_c. This is entirely due to the breaking of Cooper pairs by the quasiparticles' rotational motion, and is the neutral analog of the critical velocity v_c in uniform superconductors subjected to a voltage drop [35]. In the latter case, $v_c \propto \Delta(T = 0) \propto T_c$; the numerical results clearly indicate that while the critical frequency is proportional to T_c, it is much smaller than ω_ρ unless T_c is enhanced by the trap anisotropy.

The immediate question is whether Ω_c can be made sufficiently large that one or more vortices can be stabilized. If Ω is too small, the vortex-free state is energetically favored, and for $\Omega \sim \Omega_c$ the magnitude and spatial extent of Δ are too small to support vortices whose characteristic size is the local healing length $\xi = \hbar^2 k_F/m\pi\Delta$.

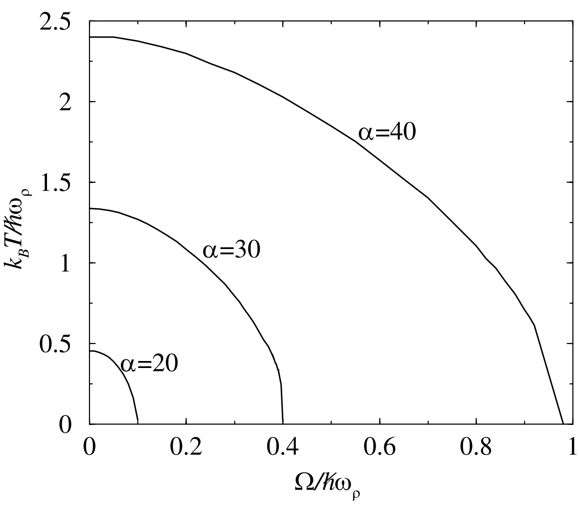

FIG. 1. (T, Ω) phase diagram for the quasi-2D system as a function of $\alpha = \tilde{\omega}_z/\omega_\rho$. Regions to the left (right) of a given curve correspond to superfluid (normal) states. Note that $\hbar\omega_\rho/k_B \approx 80$ nK, so that $T_c(\Omega = 0) \approx 200$ nK for $\alpha = 40$.

No vortices were observed for $\alpha = 20$, while $\alpha = 30$ yielded a small number, fewer than 10. As shown in Fig. 2, the $\alpha = 40$ case, with $\Omega_c \sim \omega_\rho$, is able to support large numbers. For small Ω, the vortices distribute themselves into a regular triangular pattern; however, for increasing angular frequency the arrays become disordered, so that by $\Omega = 0.7\omega_\rho$ the $T = 0$ lattice has completely melted. These results were independent of the initial guesses for Δ and n_σ and of the convergence criterion. The vortices tend to be found on circles defined by the boundary between orbitals of angular momentum m and $m + 1$, and concomitantly exhibit considerable azimuthal distortion (the effect is most noticeable in the number density).

The zero-temperature melting of a 2D vortex array was suggested some time ago in the context of the layered high-T_c superconductors without impurities [39,40]. According to the Lindemann criterion, melting occurs when the zero-point amplitude of vortex position fluctuations is some critical fraction c_L of the intervortex separation ℓ_v; for most materials, $c_L \approx 0.1$–0.2. The quasiparticle bound states in the vortex core account for the vortex fluctuations, and have a spatial extent $\sim\xi$. The vortex separation ℓ_v for a given Ω is determined by matching the angular velocity of the normal-state atoms with the average circulation $\kappa = h/2m$ of the superfluid vortices, which yields the vortex density $n_v = 2\Omega/\kappa$ or $\ell_v = 1/\sqrt{n_v} \approx 2d_\rho = 2\ \mu$m which depends only weakly on Ω. For $\Omega/\omega_\rho = \{0.4, 0.5, 0.7\}$ shown in Figs. 2(a)–2(c), the numerics yield $\xi/\ell_v \approx \{0.15, 0.17, 0.2\}$, consistent with the Lindemann criterion. Indeed, visual inspection of the lowest-energy quasiparticle amplitudes u_n and v_n reveals that they are highly localized near the vortex cores for $\Omega = 0.4$ but begin to overlap those of adjacent vortices for larger Ω. Furthermore, for systems in the lowest Landau level the relation $N/N_v \sim c_L^{-1}$ is expected [40], where N_v is the number of vortices; for the values of Ω considered above one obtains $N_\sigma/N_v = \{12.6, 8.7, 5.5\}$. An important question beyond the scope of the present work is to elucidate the relationship (if any) between the vortex array melting observed here and any impending transition into a quantum Hall state (not accessible with the current formalism). Fractions $N/N_v \lesssim 6$ have been predicted to favor a quantum Hall state in Bose gases [28].

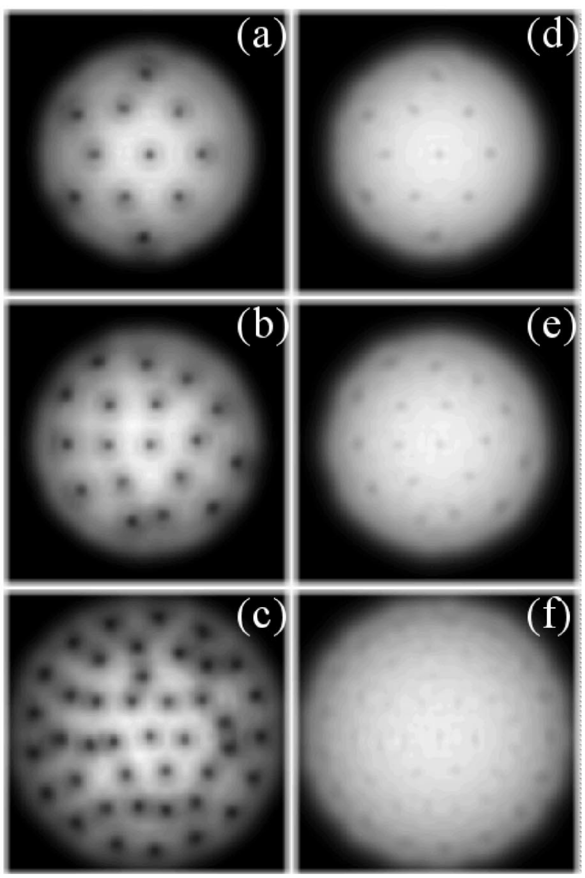

FIG. 2. Density plots for the in-trap gap function $\Delta(\mathbf{x})$ (a)–(c) and number density $n_\sigma(\mathbf{x})$ (d)–(f) for $\alpha = 40$ and $T = 0$. (a)-(d), (b)-(e), and (c)-(f) correspond to $\Omega/\omega_\rho = 0.4$, 0.5, and 0.7, respectively. Each box is 16 μm on a side.

Perhaps the most important result of this work is that the vortex cores are faint but nevertheless clearly visible as depressions in the particle density, particularly for lower Ω and T. For $\Omega/\omega_\rho = 0.4$ and 0.7, the density in the vortex core is reduced by approximately 1/2 and 1/4, respectively. Such large depressions were previously thought to occur only far from the weak-coupling BCS limit relevant to these calculations [37].

As shown in Fig. 3, the results for a given Ω are strongly affected by the temperature. As T increases, only atoms in the vicinity of the trap center participate in the pairing because the local value of T_c is lowest at the low-density surface of the cloud; alternatively, most of the pair-breaking quasiparticle excitations occur at the surface where the effective potential is lowest and the tangential particle velocities are largest. Similar results have been obtained with a local-density Ginzburg-Landau theory [41]. Because the healing length, which governs the vortex core size, also diverges as the temperature increases, fewer vortices can fit into the reduced superfluid region. The visibility of vortices in the particle density is reduced at finite temperature due to the thermal occupation of core states.

The most important outstanding issue to be addressed in future work is the vortex coherence between the wells of the optical lattice. For deep lattices such as are considered here the Josephson coupling between sites becomes small [42]. In this regime, vortex lines could break up into disconnected "pancake" vortices in each layer, rendering them unobservable experimentally. In the high-T_c superconductors, this vortex decoupling occurs when the anisotropy parameter $\gamma = \sqrt{m_z/m_\rho} \gtrsim 150$

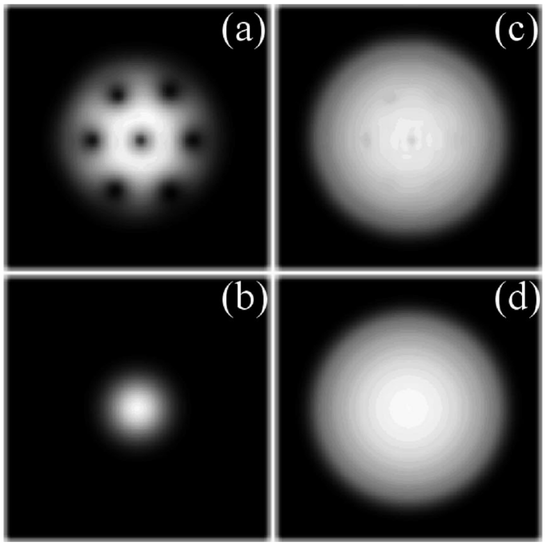

FIG. 3. Density plots for the in-trap gap function $\Delta(\mathbf{x})$ (a),(b) and number density $n_\sigma(\mathbf{x})$ (c),(d) for $\alpha = 40$ and $\Omega/\omega_\rho = 0.4$, for which $k_B T_c/\hbar\omega_\rho \approx 2.03$. (a),(c) and (b),(d) correspond to $k_B T/\hbar\omega_\rho = 1$ and 2, respectively [the $T = 0$ cases are shown in Figs. 2(a) and 2(d)]. Each box is 16 μm on a side.

[27,43] (m_z and m_ρ are the effective masses perpendicular and parallel to the layers), though a full description of such a decoupling transition is not yet complete [44]. Assuming $m_z = \hbar^2/(\partial^2\varepsilon_k/\partial k^2)$ where ε_k is the dispersion of the lowest-lying band near the trap center, the decoupling γ corresponds to an optical lattice depth approaching $70E_R$. This value is much lower and higher than those of Refs. [7,16], respectively, suggesting that the melting of well-defined vortex lines should be directly observable in experiments employing Nd:YAG (but not CO_2) lasers. A full calculation incorporating interwell tunneling remains necessary; the disappearance of vortices observed experimentally could be due not only to vortex decoupling but also to the onset of a quantum Hall phase, which is expected to restore the cylindrical symmetry broken by the vortices.

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