Beat-Wave Excitation of Plasma Waves Based on Relativistic Bistability

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A nonlinear beat-wave regime of plasma wave excitation is considered. Two beat-wave drivers are considered: intensity-modulated laser pulse and density-modulated (microbunched) electron beam. It is shown that a long beat-wave pulse can excite strong plasma waves in its wake even when the beat-wave frequency is detuned from the electron plasma frequency. The wake is caused by the dynamic bistability of the nonlinear plasma wave if the beat-wave amplitude exceeds the analytically calculated threshold. In the context of a microbunched beam driven plasma wakefield accelerator, this excitation regime can be applied to developing a femtosecond electron injector.

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Beat-wave excitation of electron plasma waves continues attracting significant attention as a basic nonlinear plasma phenomenon, and as a viable approach to plasma-based particle acceleration [1–4]. Beat-wave excitation mechanism is realized when the driver intensity (laser or particle beam) is modulated with the temporal periodicity of the plasma wave. The linear one-dimensional theory of the beat-wave-driven plasma wave generation is well understood [1,5], and its most important predictions are as follows. First, the effectiveness of plasma wave excitation is strongly dependent on the difference between the beat-wave frequency ω_B and plasma wave frequency $\omega_p = \sqrt{4\pi e^2 n_0/m}$ (where $-e$ and m are the electron charge and mass, and n_0 is the plasma density): the smaller is the frequency detuning $\Delta\omega \equiv \omega_B - \omega_p$, the larger is the resulting plasma wave inside the beat wave. Second, only if the beat-wave pulse is short enough for its bandwidth to be comparable to $\Delta\omega$, an appreciable plasma wave is left in its wake.

In this Letter I demonstrate that these conclusions are no longer valid when the relativistic nonlinearity of a plasma wave is accounted for. In particular, a strong plasma wave can be excited in the wake of a relatively long beat-wave pulse of duration $t_L \gg 1/\Delta\omega$ due to the nonlinear phenomenon of dynamic relativistic bistability (RB) [6]. Linear estimates of the plasma wave amplitude fail when the beat-wave amplitude exceeds the detuning-dependent critical strength. As the time-dependent beat-wave strength increases and exceeds the critical value, significant pulsations of the plasma wave amplitude occur. These pulsations indicate that significant energy exchange takes place between the plasma wave and the driver. This effect can be exploited in a plasma wakefield accelerator driven by a microbunched electron beam [7]: bunches in the head of the beam excite while those in the back deplete plasma waves, thereby gaining energy.

Relativistic bistability was originally described [6] for a magnetized electron subjected to cyclotron heating. Applications of RB to electron cyclotron heating of fusion plasmas [8,9] have been later suggested. Although the nonlinear nature of electron plasma waves has been noted before [10–13], the RB of plasma waves has not been explored, either as a basic phenomenon or in the context of plasma-based accelerators.

The one-dimensional relativistic dynamics of the cold plasma driven by a beat wave can be described using a Lagrangian displacement of the plasma element originally located at z_0: $z(t) = z_0 + \zeta(t, z_0)$. It is assumed that the beat wave generated by either a pair of frequency-detuned laser beams or a modulated electron beam is moving with the speed close to the speed of light c, and therefore, all beat-wave quantities are functions of the comoving coordinate $\tau' = \omega_p(t - z/c) \equiv \tau - \omega_p\zeta/c$. Introducing the normalized displacement $\tilde{\zeta} = \omega_p\zeta/c$ and longitudinal relativistic momentum $\tilde{p} = \gamma d\tilde{\zeta}/d\tau$, where $\gamma = \sqrt{1 - \vec{v}^2/c^2}$, equations of motion take on the form

$$\frac{d\tilde{\zeta}}{d\tau} = \frac{\tilde{p}}{\sqrt{1 + \tilde{p}^2}}, \qquad \frac{d\tilde{p}}{d\tau} = -\tilde{\zeta} + a(\tau')\cos\omega\tau'. \qquad (1)$$

Assuming that $|\Delta\omega| \ll \omega_p$ (near-resonance excitation), transverse momentum of the plasma has been neglected and the relativistic γ factor simplified to $\gamma = \sqrt{1 + \tilde{p}^2}$. The first term in the force equation is the restoring force of the ions, and the second term signifies the beat wave with the frequency $\omega_B \equiv \omega\omega_p$. The nonlinear in ζ modification of the beat wave in the right-hand side of Eqs. (1) is neglected in what follows. For a pair of linearly polarized laser pulses with electric field amplitudes E_1 and E_2 and the corresponding frequencies ω_1 and $\omega_2 = \omega_1 - \omega_B$, the normalized beat-wave amplitude $a = (e/mc)^2 E_1 E_2/2\omega_1\omega_2$ [10]. For a driving electron bunch with the density profile $n_b = n_{b0} + \delta n_b \sin\omega\tau$, it can be shown that $a = \delta n_b/n_0$. Although arbitrary profiles of $a(\tau)$ are allowed, it is assumed that $|da/d\tau| \ll |a|$. The total energy density of the plasma wave $U_p/n_0mc^2 = \sqrt{1 + \tilde{p}^2} + \tilde{\zeta}^2/2$ is changed via the interaction with the beat wave. The effect of the plasma wave on the beat wave is neglected for the moment and addressed towards the end of the Letter.

Although Eqs. (1) can be solved numerically at this point, further simplification is made by assuming $\tilde{p} = u\cos(\omega\tau + \phi)$, where u and ϕ are slowly varying functions of τ. In the weakly relativistic approximation $\tilde{p}^2 \ll 1$ obtained by combining the terms proportional to $\cos\omega\tau$ and $\sin\omega\tau$:

$$\frac{du}{d\tau} = \frac{a}{2}\cos\phi \tag{2}$$

$$u\frac{d\phi}{d\tau} = -\frac{a}{2}\sin\phi - \frac{u}{2\omega}(\omega^2 - 1 + 3u^2/8). \tag{3}$$

Depending on the beat-wave frequency ω and the amplitude a, equilibrium solutions $du/d\tau = 0$ (steady amplitude) and $d\phi/d\tau = 0$ (phase-locking to the beat wave) of Eqs. (2) and (3) can have one or three real roots. For any ω, there is a stable equilibrium point: $\phi_0 = -\pi/2$ and $u_0 > 0$ found as the root of the third-order polynomial equation $\mathcal{P}(u_0) = u_0(\omega^2 - 1 + 3/8u_0^2) = \omega a$. For the most interesting $\omega < 1$ regime, additional solutions $\phi_0 = \pi/2$ and $u_0 > 0$, where u_0 is the positive root of $\mathcal{P}(u_0) = -\omega a$, may be found, depending on the beat-wave amplitude. Specifically, there are no additional positive roots for $a > a_{\mathrm{crit}}$, where $a_{\mathrm{crit}} = 4\sqrt{2}(1 - \omega^2)^{3/2}/9\omega$, and two positive roots $u_{1,2}$ for $a < a_{\mathrm{crit}}$ (one of them unstable). Stable equilibrium amplitudes u_0 with $\phi_0 = \pi/2$ (branch 1) and $\phi_0 = -\pi/2$ (branch 3), as well as the unstable one (branch 2) are plotted in Fig. 1 as a function of the beat-wave strength a for $\omega = 0.95$ ($a_{\mathrm{crit}} = 0.02$). Equilibrium bistability corresponding to branches 1 and 3 is universal for any nonlinear pendulum [6,14], including a weakly-damped one. Equilibrium solutions are meaningful only if the plasma wave is phase-locked to the beat wave: $d\phi/d\tau \approx 0$. As shown below, this is not the case when the peak beat-wave amplitude exceeds a_{crit}. Nonetheless, a *dynamic* RB described below occurs even in the absence of phase locking.

Plasma response to a time-dependent (Gaussian) beat-wave pulse $a(\tau) = a_0\exp(-\tau^2/\tau_L^2)$ (with $\tau_L \gg 1/|1 - \omega|$) strongly depends on its peak amplitude a_0. For $a_0 < a_{\mathrm{crit}}$, the wave amplitude u adiabatically follows $a(\tau)$ along the branch 1 as indicated by arrows in Fig. 1. The adiabaticity condition is $\Omega_B\tau_L \gg 1$, where Ω_B is the bounce frequency around the equilibrium point u_0 such that $\mathcal{P}(u_0) = -\omega a(\tau)$. Linearizing Eqs. (2) and (3) around $\phi = \pi/2$ and $u = u_0$ yields $\Omega_B^2 = a(u_{\mathrm{crit}}^2 - u_0^2)/4\omega u_0$, where $u_{\mathrm{crit}} = 2\sqrt{2(1 - \omega^2)}/3$ is the critical plasma wave amplitude corresponding to the merging point between branches 1 and 2 in Fig. 1. In the adiabatic regime, the action $I = (1/4\pi)\oint u^2 d\phi$ is conserved, and the system is trapped with $I \equiv I_1 = 0$. Plasma wave amplitude returns to a very small value in the wake of the beat wave, as shown by a dot-dashed line in Fig. 2. The longer the beat-wave pulse duration τ_L, the smaller the wake, because its nonvanishing amplitude is due to the adiabaticity violation for finite τ_L.

For $a_0 > a_{\mathrm{crit}}$, the adiabatic condition is violated as $a(\tau)$ approaches a_{crit} (noted in the context of electron cyclotron heating [8,9]), and phase locking at $\phi_0 = \pi/2$ is no longer possible. Thus, the transfer to branch 3 schematically shown by a vertical arrow in Fig. 1 becomes feasible, and the plasma wave amplitude can dramatically increase. In the presence of a finite plasma wave damping, this indeed happens: the subsequent decrease of the beat-wave amplitude results in phase locking at $\phi_0 = -\pi/2$, with u following along the branch 3. Without damping, there is no mechanism for the plasma wave to reach the equilibrium amplitude given by the upper branch 3.

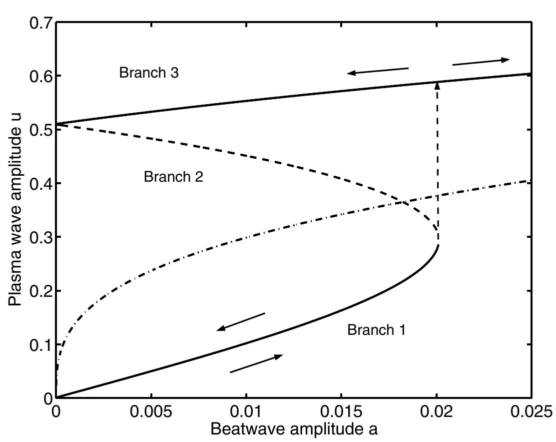

FIG. 1. Steady-state solutions of a driven plasma wave as a function of the beat-wave amplitude a. Solid lines 1, 2: stable equilibria for $\omega = 0.95$; dashed line: unstable equilibrium for $\omega = 0.95$; dot-dashed line: resonant excitation with $\omega = 1$.

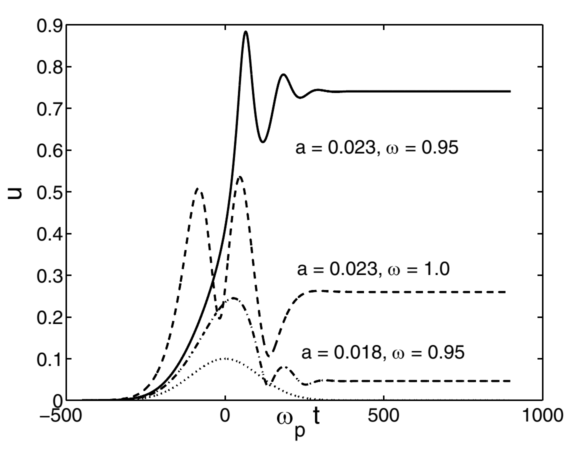

FIG. 2. Excitation of a plasma wave by a Gaussian beat-wave pulse (dotted line), $a(\tau) = a_0\exp[-\tau^2/\tau_L^2]$, $\tau_L = 150$. Solid line: $\omega = 0.95$, above-threshold excitation with $a_0 = 0.023 > a_{\mathrm{crit}} = 0.02$; dashed line: resonant excitation with $\omega = 1$ and $a_0 = 0.023$; dot-dashed line: $\omega = 0.95$, below-threshold excitation with $a_0 = 0.018 < a_{\mathrm{crit}}$.

Nevertheless, even without damping, a significant plasma wave is left behind the finite-duration beat-wave pulse (Fig. 2, solid line). This finite-amplitude solution is reached due to the effect of the dynamic RB, which is best understood through the conservation of the effective Hamiltonian of the driven plasma wave. The effective Hamiltonian

$$H = \frac{(1-\omega^2)u^2}{4\omega} - \frac{3u^4}{64\omega} - \frac{1}{2}au\sin\phi \qquad (4)$$

can be used to express Eqs. (2) and (3) in the form of $\dot{u} = -(1/u)dH/d\phi$, $\dot{\phi} = (1/u)dH/du$. For $a < a_{\rm crit}$, the Hamiltonian adiabatically responds to $a(\tau)$ according $dH/d\tau = -0.5u\sin\phi da/d\tau$, thereby assuming a parametrically-dependent on a value $H(I_1, a)$. For the initially quiescent plasma we find that $H(I_1, a = 0) = 0$. At $a = a_{\rm crit}$, the system becomes detrapped, its action changes to $I = I_2$, but the Hamiltonian does not: $H(I_2, a_{\rm crit}) = H(I_1, a_{\rm crit})$. Moreover, it can be shown that $\partial H(I_1, a_{\rm crit})/\partial a_{\rm crit} = \partial H(I_2, a_{\rm crit})/\partial a_{\rm crit}$, and, therefore, concluded that $H(I_1, a) \approx H(I_2, a)$ for all a, including $a = 0$. Although this argument is not entirely rigorous, numerical simulations confirm that for a wide range of amplitudes close to $a_{\rm crit}$ and pulse durations $150 < \tau_L$, the Hamiltonian of the system is the same before and after the beat-wave pulse: $H(\tau = +\infty) = H(\tau = -\infty) = 0$. Note that the Hamiltonian does not stay constant inside the pulse, but does return to its original value after the pulse.

Remarkably, in addition to the trivial quiescent plasma solution $u = 0$, there is a second $u_\infty = 4\sqrt{(1-\omega^2)/3}$ solution satisfying $H(u_\infty) = 0$. Thus, a plasma wave with $H = 0$ is dynamically bistable: after the passage of the beat wave, it can be either quiescent, or have the finite-amplitude u_∞. It is conjectured that, by using a beat-wave pulse with $a_0 > a_{\rm crit}$, the latter solution can be accessed, thereby leaving a wake of a substantial plasma wave with amplitude u_∞.

This conjecture is verified by numerically integrating Eqs. (2) and (3) for two different detunings (resonant, with $\omega = 1$, and nonresonant, with $\omega = 0.95$) and two beat-wave amplitudes (subthreshold, with $a_0 = 0.018$, and above threshold, with $a_0 = 0.023$). In all cases, the Gaussian pulse duration was chosen $\tau_L = 150$. In physical units, for the plasma density of $n_0 = 10^{19}$ cm^{-3}, the corresponding pulse duration is $t_L \equiv \tau_L/\omega_p \approx 750$ fs. Simulation results are shown in Fig. 2, where the solid line corresponds to the most interesting of the three cases: $\omega = 0.95$ and $a_0 = 0.023$. The plasma wave amplitude of $u \approx 0.75$ in the wake of the laser pulse is in a good agreement with $u_\infty = 0.72$. This wake owes its existence to the dynamic RB: upon interacting with the above-threshold laser beat wave, plasma wave is transferred from the quiescent state of $u = 0$ to the excited state of $u = u_\infty$. Extensive numerical simulations for a broad range of pulse durations $\tau_L > 150$ and amplitudes $a_{\rm crit} < a_0 < 1.3a_{\rm crit}$ confirmed that the finite-amplitude u_∞ wake is indeed excited. The subthreshold excitation (dot-dashed line) with the same detuning fails to transfer the plasma into the excited state, yielding a negligible wake that is an order of magnitude smaller than in the above-threshold regime. Resonant excitation (dashed line) also yields a much smaller wave. Moreover, the resonantly and the subthreshold excited plasma waves would have been even smaller had the adiabatic assumption been fully satisfied. Indeed, it is numerically confirmed that the wake amplitudes for the resonant and the subthreshold excitations rapidly decline for longer pulses, whereas the amplitude of the nonresonant above-threshold excitation is insensitive to the beat-wave pulse length τ_L.

To demonstrate the bistable nature of the relativistic plasma wake, excitation by a *pair* of identical beat-wave pulses is considered. By varying the time delay $\tau_d \equiv \omega_p t_d$ between the pulses, plasma wave can be either returned to the original quiescent state $u = 0$ [Fig. 3(a), solid line, delay time $\tau_d = 920$) or brought into the excited state $u = u_\infty$ [Fig. 3(a), dashed line, delay time

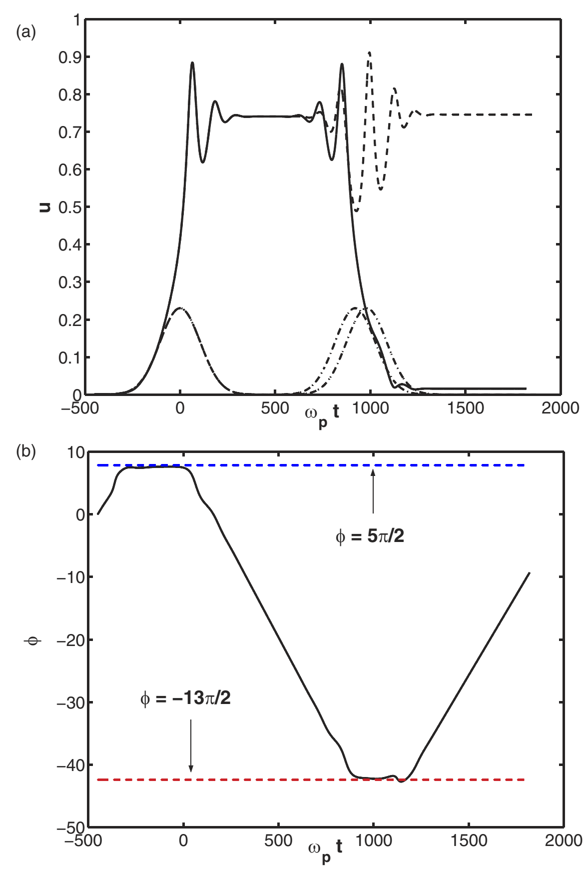

FIG. 3 (color online). (a) Excitation of a plasma wave by a pair of identical Gaussian beat-wave pulses (dot-dashed line) separated by the delay times $\tau_d = 920$ (solid line: wake depleted by the second pulse) and $\tau_d = 980$ (dashed line: wake unperturbed by the second pulse). Pulse parameters: same as in Fig. 2: $a_0 = 0.023$, $\omega = 0.95$, and $\tau_L = 150$. (b) Sequence of phase lockings and phase releases for $\tau_d = 920$.

$\tau_d = 980$). Depending on the time delay τ_d, there are, essentially, only two outcomes for plasma wave amplitude: $u \approx 0$ or $u = u_\infty$. This result is remarkably nonlinear: the linear theory predicts that the wake behind two pulses depends on their separation in a sinusoidal way: $u(t = \infty) = 2u(t_1)\cos^2[\pi\tau_d(\omega - 1)]$, where $\tau_L \ll t_1 \ll \tau_d$ is the instance well after the end of the first and before the beginning of the second pulse. Therefore, a complicated bi-Gaussian pulse shape is capable to transfer the plasma wave into either one of the allowed under the conservation of H solutions depending on the pulse separation.

Dynamical RB described in this Letter is different from the standard equilibrium bistablity of a weakly-damped nonlinear oscillator [6,14] in that the former does not require phase locking, only the constancy of the effective Hamiltonian H before and after the pulse. As Fig. 3(b) indicates, phase locking at $\phi_0 = \pi/2$ exists only during the switch-on half of the beat wave, $-2\tau_L < \tau < 0$. The plasma wave phase is released afterwards ($0 < \tau < \tau_d$) as the pulse amplitude settles into $u = u_\infty$ and the system becomes detrapped. Depending on the delay time, the second pulse can either (i) lock the phase at $\phi_0 = \pi/2$ and trap the system (as shown in Fig. 3(b)], with the consequent decay of the plasma wave to $u \approx 0$, or (ii) fail to lock the phase, resulting in $u = u_\infty$ after the pulse pair. Phase locking at $\phi_0 = -\pi/2$ indicative of a transfer to the equilibrium branch 3 and, therefore, equilibrium bistability, is never observed.

So far the effect of the plasma wave on the driver has been neglected. Of course, the energy of the plasma wave is supplied by the beat wave. Since the plasma wave energy changes nonmonotonically, different portions of the beat wave either lose or gain energy. In the weakly relativistic case, the plasma energy density $U_p \approx n_0 mc^2 u^2/2$. For concreteness, I concentrate on the above-threshold case plotted in Fig. 2 (solid line). The leading portion of the beat wave ($-\infty < \tau < 64$) contributes energy to the beat wave and is, therefore, depleted. If the beat wave is produced by a laser pulse, this depletion can be described in the language of photon deceleration, or red-shifting [15]. In the context of the laser beat wave, the red-shifting corresponds to the scattering of the photons from the higher frequency into the Stokes component. Assuming equal amplitude lasers, $E_1 = E_2$, the rate of the frequency shifting (per unit of the propagation length) can be found as $-d\omega/dz \approx (\omega_p^3/4c\omega_1 a) \times d(u^2)/d\tau$. Therefore, the laser pulse is red (blue) shifted if $du/d\tau > 0$ ($du/d\tau < 0$).

If the beat wave is produced by a microbunched electron beam, the sign of $du/d\tau$ can be related to the acceleration or deceleration gradient of the drive electron bunch E_z through

$$\frac{E_z(\tau)}{E_{\rm WB}} = \frac{\delta n_b}{n_{b0}}\left(\frac{1}{2a(\tau)}\frac{du^2}{d\tau}\right), \tag{5}$$

where $E_{\rm WB} = mc\omega_p/e$ is the nonrelativistic wave breaking electric field. Again, the sign of $du/d\tau$ determines whether the driving bunch is accelerated or decelerated. For a microbunched electron driver consisting of femtosecond bunches with duration $\delta t \ll 1/\omega_p$ [7] produced by an inverse free-electron laser, $\delta n_b \sim n_{b0}$. It is estimated that in the plasma wave decay region of the driving bunch ($64 < \tau < 112$), the beam is decelerated at a rate of $E_z \approx 30$ GeV/m for $n_0 = 10^{19}$ cm^{-3}. Therefore, the marriage of the microbunched plasma wakefield accelerator and the dynamic relativistic bistability concepts yields a new advanced acceleration technique that takes advantage of the temporal drive beam structure to produce high energy femtosecond electron beams.

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