Transfer and Storage of Vortex States in Light and Matter Waves

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We theoretically explore the transfer of vortex states between atomic Bose-Einstein condensates and optical pulses using ultraslow and stopped light techniques. We find shining a coupling laser on a rotating two-component ground state condensate with a vortex lattice generates a probe laser field with optical vortices. We also find that optical vortex states can be robustly stored in the atomic superfluids for times, in Rb-87 condensates, limited only by the ground state coherence time.

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One of the most dramatic developments in recent experiments on atomic Bose-Einstein condensates (BECs) is the creation and study of quantized vortices and vortex lattices [1,2]. In this Letter we show that light modes with orbital angular momentum can be efficiently stored in superfluid vortex configurations and later rewritten back onto light fields. This also provides a feasible mechanism to transfer BEC vortex lattice ground states to light pulses. The strong light-matter coupling can be implemented using the recent technology of ultraslow and stopped light pulses in atomic vapors [3–6], based on the method of electromagnetically induced transparency (EIT) [7]. In these experiments, an optical field's amplitude and phase pattern was written onto superpositions of atomic states, stored for several milliseconds, and then rewritten back onto the original light field and output. Slow light and EIT are now inspiring important coherent matter wave technology applications, e.g., BEC shock waves [8], probing decoherence in BECs [9], optical black holes [10], storage and production of nonclassical states [11,12], and optical processing [13].

One of the unique characteristics of slow light is the strong yet coherent action of the matter fields on the light fields. The phase coherence between two overlapping BECs provides a means of coherently generating light fields reflecting the relative phase and amplitude of the BECs [13]. Here we study the transfer and storage of coherent information in terms of vortex configurations, with experimentally feasible parameters for ^{87}Rb. We find that shining a coupling laser pulse can generate a probe pulse which contains optical vortices corresponding to vortices originally in the atomic fields. Besides providing a simultaneous probe of the phase and density patterns of two-component vortex states (making it distinct from previous proposals for ultraslow light based imaging [14]), this is a unique method of generating optical vortices from preexisting atomic vortices. Moreover, the output light field could potentially be input into a second optically connected BEC (even if it is spatially distant), allowing transfer of vortex states between BECs.

The strong coupling between the quantized optical and BEC vortices is an interesting phenomenon in its own right, as a demonstration of nonlinear superfluid-light optics, but it also points to possibilities for information processing and storage. Experiments have seen advances in the generation of Laguerre-Gaussian (LG) modes of light with orbital angular momentum and applications of these modes to quantum information [15]. It may be possible to utilize modes with several different angular momentum quantum numbers to build a quantum information architecture based on a larger alphabet than the traditional two-state systems [16]. However, optically based quantum information schemes suffer from the difficulty of trapping and storing optical fields.

Here we perform a comprehensive analysis of how LG beams can be written into BECs, stored in the atomic fields, and later rewritten onto light fields. We find one can choose parameters so the coherent BEC dynamics do not dissipate the information during the storage time, allowing storage fidelities on the order of 70% for several hundred milliseconds with typical parameters. This fidelity can be further improved by using BECs with larger optical densities. Moreover, the phase information in vortex lines can exhibit very long life times of tens of seconds or can even be energetically stable [1]. While LG beams have been proposed as a method to generate vortices in BECs [17] and in experiments vortices have been imprinted on BECs [2], our scheme here is very different: In the nonlinear light-matter coupling both amplitude and phase are robustly and simultaneously exchanged, allowing the controlled storage of information.

We first show that BEC vortex lattice ground states can be transferred to light pulses. In the second part we then show how to apply this mechanism to store light modes with orbital angular momentum in superfluid vortices and later rewrite them back onto light fields. We first consider two BECs of Rb-87 atoms, in stable internal states $|1\rangle \equiv |5S_{1/2}, F = 2, M_F = +1\rangle$, and $|2\rangle \equiv |1, -1\rangle$, with macroscopic wave functions ψ_1 and ψ_2. These are connected, by resonant, $+z$ propagating *coupling* and *probe* light fields, Ω_c and Ω_p (each of wavelength $\lambda = 2\pi/k = 780$ nm), to the excited state $|3\rangle \equiv |5P_{1/2}, F = 2, M_F = 0\rangle$ (with wave function ψ_3), which decays at $\Gamma = (2\pi)$ 6 MHz, forming a Λ three-level structure. Here the Rabi frequencies $\Omega_i(\mathbf{r}, t) \equiv -\mathbf{d}_{i3} \cdot \mathbf{E}_i(\mathbf{r}, t)$ ($p = 1, c = 2$) are defined in terms of the atomic dipole matrix elements $\mathbf{d}_{i3}$ and the slowly varying envelope (SVE) of the electric fields $\mathbf{E}_i$ (with the rapid phase rotation at the optical frequencies and optical wave numbers factored out). The light propagation then follows from SVE approximation to the Maxwell's equations [18]

$$\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_i = -\frac{ik}{\hbar\epsilon_0}\mathbf{d}_{i3} \cdot \mathbf{P}_{i3}. \quad (1)$$

Here the polarization is $\mathbf{d}_{i3} \cdot \mathbf{P}_{i3} \simeq f_{i3}\mathcal{D}^2 N \psi_i^* \psi_3$, N is the initial total number of BEC atoms, $f_{13} = 1/4$ and $f_{23} = 1/12$ are dimensionless oscillator strengths, $\mathcal{D} = \sqrt{\hbar\epsilon_0\sigma\Gamma/k}$ is the reduced dipole matrix element, and $\sigma \equiv 3\lambda^2/2\pi$ is the resonant cross section. The BEC wave functions ψ_1, ψ_2 evolve according to generalized Gross-Pitaevskii equations (GPEs):

$$i\hbar\dot{\psi}_i = \left(H_0 + \sum_{j=1,2} U_{ij}|\psi_j|^2\right)\psi_i + \hbar\Omega_i^*\psi_3, \quad (2)$$

where $H_0 \equiv -\hbar^2\nabla^2/2m + V(\mathbf{r})$, with identical trapping potential for both states $V(\mathbf{r}) = \frac{m}{2}[\omega_r^2(x^2 + y^2) + \omega_z^2 z^2]$. In the interaction coefficients $U_{ij} = 4\pi N\hbar^2 a_{ij}/m$, the a_{ij} are the scattering lengths between atoms in states $|i\rangle$ and $|j\rangle$. For ^{87}Rb, $a_{12} = 5.5$ nm, and a_{11}:a_{12}:a_{22}::0.97:1.0:1.03 [19]. Often in practice, and throughout this Letter, we will be working in a regime where, whenever the light fields are present, the atomic internal state dynamics and light field couplings are on a much faster time scale than the external dynamics. In the context of slow light experiments we can adiabatically eliminate $\psi_3 \simeq -i(\Omega_p\psi_1 + \Omega_c\psi_2)/\Gamma$ in Eqs. (1) and (2) [13]. Then the last term in Eq. (2) results in both coherent exchange between $|1\rangle, |2\rangle$ as well as absorption into $|3\rangle$. In our model, atoms which populate $|3\rangle$ and then spontaneously emit are assumed to be lost from the BECs.

Vortex lattices can be produced by imparting angular momentum to the system with a time-dependent rotating potential. We numerically find the ground state of a two-species BEC rotating about the z axis at the frequency $\bar{\omega} = 0.3\omega_r$ by evolving the GPEs (2) in imaginary time in the absence of the light fields in the rotating frame, obtained by replacing H_0 by $H_0 - \bar{\omega}\hat{L}_z$, with $\hat{L}_z = i\hbar(y\partial_x - x\partial_y)$. The full 3D integration, without imposing any symmetry on the solution, is performed in a "pancake-shaped" trap $\omega_r = 0.1\omega_z = (2\pi)10$ Hz for $N_1 = N_2 = N/2 = 1.38 \times 10^5$.

In Fig. 1 we present a calculation of such a two-species lattice. The phases of the wave functions ϕ_i indicate a rectangular lattice of singly quantized vortices in each component (at singular points where the entire spectrum

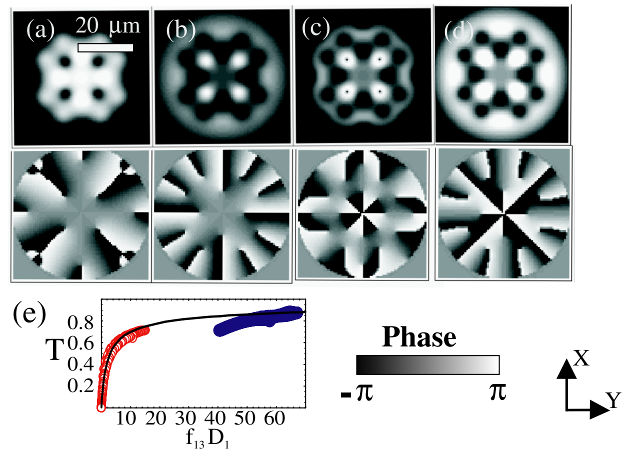

FIG. 1 (color online). Writing an atomic vortex lattice onto a light field. The optical density (top row) $D_i = N\sigma\int dz|\psi_i|^2$ (black = 0, white = 55) and the phase ϕ_i in the $z = 0$ plane (bottom row, and according to the gray-scale bar) for a vortex lattice (a) in $|1\rangle$ and (b) in $|2\rangle$. The total initial atom density in the center is 6.9×10^{13} cm^{-3}. (c) n_p (the number of probe photons output per cross sectional area σ) upon a switch-on of the coupling field (black = 0, white = 25) and the phase profile of ϕ_p, which contains vortices of both circulation. The temporal width of the output probe pulse was about 0.8 μs, determined by a combination of the coupling switch-on time and slow light pulse delay (which are of similar magnitude here). (d) Same quantities as (c) for a nonrotating ground state BEC in $|1\rangle$, resulting in vortices of only one circulation. (e) A scatter plot of the transfer efficiency in each column, $T = n_p/n_2$, where n_2 is the number of $|2\rangle$ atoms per σ before the switch-on, versus $f_{13}D_1$ for the case in (c) (open circles, red online) and for (d) (closed circles, blue online). The solid curve shows the estimate $1 - (1 + f_{13}D_1)^{-1/2}$.

of phase values converge), with the positions of the vortex cores in the two-species offset from each other. The filling of the vortex cores by the other BEC significantly increases the core size, as compared to vortices in a single-component BEC, and makes them observable even without a ballistic expansion. The 3D results are qualitatively similar to the previous 2D calculations [20]. The vortex core positions (and ϕ_1, ϕ_2) change very little along z.

To model the writing of this vortex lattice to the probe field Ω_p we numerically solved Eqs. (1) and (2) when a coupling field with a peak input value of $\Omega_c^{(\mathrm{in})} = (2\pi)$ 4 MHz was switched on suddenly (in a time 0.8μs). When this happens the coherence $\psi_1^*\psi_2$ acts to generate a probe field Ω_p in such a way that the contributions to the polarization in Eq. (1) cancel out, and a dark state is formed. The resulting output intensity and phase pattern of Ω_p is shown in Fig. 1(c).

To understand the results, we note that solving Eq. (1), ignoring the negligible z variation of ψ_1/ψ_2, shows the fields eventually reach the asymptotic values $\Omega_p = -\Omega_c^{(\mathrm{in})}(f_{13}\psi_1^*\psi_2)/(f_{13}|\psi_1|^2 + f_{23}|\psi_2|^2)$ and $\Omega_c = \Omega_p\psi_1/\psi_2$. Note that $\Omega_p \to 0$ whenever *either* $\psi_1, \psi_2 \to 0$, giving vanishing intensity at all the phase singularities. The phase ϕ_p is determined by the relative BEC phase $\phi_p = \phi_2 - \phi_1 + \pi$ (the coupling field acquires no phase shift here). Thus ϕ_p contains singularities with one circulation at the location of vortices in ψ_2 and of the opposite circulation at vortices in ψ_1.

In the columns where $|\psi_2/\psi_1| \ll 1$, the number of generated probe photons output (per unit area) is nearly equal to the original $|2\rangle$ column density $\int dz|\psi_2|^2$ (there is small absorption loss in a "preparation" region of about one optical depth before the dark state is established). However, near the vortex cores of $|1\rangle$, where $\psi_1 \to 0$, there is not enough coherence to generate intense probe field and the dark state is established primarily via absorption of Ω_c. We found that the transfer efficiency for each column agrees well with the prediction $1 - 1/\sqrt{1 + f_{13}D_1}$; see Fig. 1(e). The output efficiency, integrated over x, y, is 35%.

Motivated by the fact that the fidelity improves with D_1 we also considered coupling the same ψ_2 vortex lattice of Fig. 1(b) to a vortex-free, nonrotating ground state BEC in $|1\rangle$, with $N_1 = 3 \times 10^6$ [Fig. 1(d)]. There the peak optical density $f_{13}D_1$ is 70 and is >35 throughout the region occupied by $|2\rangle$. To a good approximation, the probe intensity simply reflects ψ_2 [see Fig. 1(b)] and only contains vortices of one helicity. In this case the fidelity is now higher [Fig. 1(e)] with the overall efficiency 82%.

We have determined that vortex states can indeed be written from atomic to light fields. Turning now to information storage applications, we calculate whether vortex states, originally in light fields, could be robustly stored in a BEC using stopped light techniques [4,5,13]. For this study, we switch the roles of the two stable ^{87}Rb states so $|3\rangle$ is coupled to $|2\rangle \equiv |2, +1\rangle$ by Ω_c and to $|1\rangle \equiv |1, -1\rangle$ by Ω_p. A BEC with $N = 4 \times 10^6$ atoms is initially in the (nonrotating) ground state of $|1\rangle$, labeled $\psi_1^{(G)}(\mathbf{r})$, in an isotropic trap $\omega_z = \omega_r = (2\pi)21$ Hz, giving a BEC with a peak optical density of $f_{13}D_1 = 213$. Initially, the coupling field is on at $\Omega_c^{(\mathrm{in})} = (2\pi)$ 8 MHz [see Fig. 2(a)], while the LG probe input reads (in cylindrical coordinates [21]):

$$\Omega_p^{(\mathrm{in})}(r, \theta, t) = \Omega_p^{(0)}\left(\frac{r\sqrt{2}}{w}\right)^{|m|} e^{-r^2/w^2} e^{im\theta} e^{-t^2/2\tau_0^2}, \qquad (3)$$

where w is the beam waist. Each photon in the LG mode with $|m| \geq 1$ contains a vortex at $r = 0$ with a vanishing intensity and m units of orbital angular momentum.

We consider a light pulse in an $m = 1$ mode with $\tau_0 = 0.25$ μs, $w = 6.3$ μm, $\Omega_p^{(0)} = (2\pi)4$ MHz, and $N_p^{(\mathrm{in})} = 3.83 \times 10^4$ photons, propagating slowly, with little attenuation or distortion, through the BEC, with the group velocity $v_g \propto \Omega_c^{(\mathrm{in})2}/|\psi_1|^2$. As it is input, the pulse induces coherent transfer of atoms from $|1\rangle$ to $|2\rangle$. Once the pulse is completely input (0.6 μs), N_2 nearly reaches $N_p^{(\mathrm{in})}$; see Fig. 2(c). Because of the finite bandwidth of the EIT transmission window there is some nonadiabatic loss from the BEC during the propagation.

This pulse is in the weak-probe limit $|\Omega_p| \ll |\Omega_c^{(\mathrm{in})}|$ and so, during the probe input, ψ_2 is coherently driven into the dark state $\psi_2 = -\psi_1^{(G)}(\Omega_p/\Omega_c^{(\mathrm{in})})$ and acquires the amplitude and vortex phase pattern of the input LG mode. At $t = 0.6$ μs, Ω_c is then switched off in about 0.2 μs, and N_2 and N_{loss} are unaffected by this rapid switch-off; see Fig. 2(c). Both Ω_c and Ω_p smoothly ramp to zero intensity and the LG mode is then stored in ψ_2. The first panel of Fig. 2(d) shows the relative density profile $|\psi_2|^2/\rho$, where $\rho \equiv |\psi_1|^2 + |\psi_2|^2$. Note the density is cylindrically symmetric and the full 3D

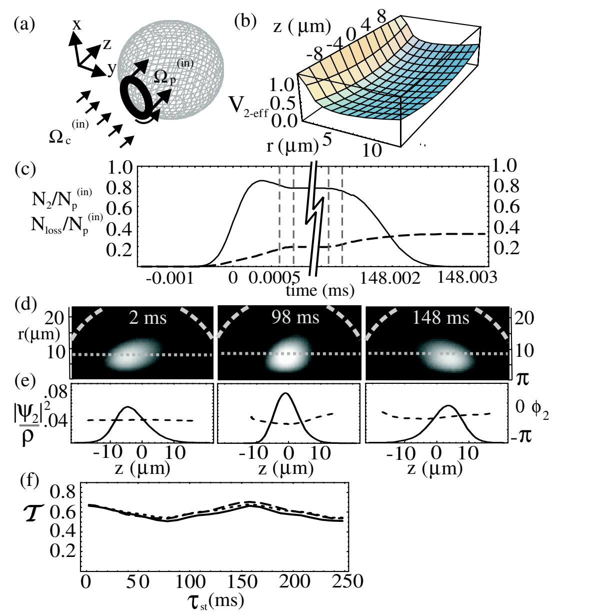

FIG. 2 (color online). Inputting, storing, and outputting an optical vortex in a BEC. (a) A spherical BEC (gray sphere) is originally illuminated by a c.w. coupling field Ω_c. A probe field pulse in an LG ("doughnut") mode (black torus), is then input and stopped when it has propagated to the center. (b) The effective potential seen by ψ_2, $V_{2\text{-eff}}(\mathbf{r})$ in units of $\hbar\omega_z$, during the storage for $m = 1$, with a minimum near $r = 6$ μm, $z = 0$. (c) The number of atoms in $|2\rangle$, N_2, relative to the number of input photons $N_p^{(\mathrm{in})}$ (solid curve) and the number of atoms lost due to spontaneous emission N_{loss} (dashed curve) during the probe pulse input and coupling switch-off (the vertical dotted lines shown the points at which the coupling field is 90% and 10% of its full value). These are constant during the storage time (the time break). After the time break we then plot the same quantities during the coupling switch-on and probe pulse output. (d) The relative density profile $|\psi_2|^2/\rho$ during the storage at the times indicated. The dashed curves indicates the BEC Thomas-Fermi boundary. The dotted lines shows the cuts plotted in (e). (e) $|\psi_2|^2/\rho$ and the phase ϕ_2 along the cuts indicated in (d). (f) The corresponding output fidelity $\mathcal{T}$ upon switch-ons after various storage times τ_{st}, for $m = 1, 2, 0$ (solid, dashed, and dotted lines, respectively).

structure of the vortex can be visualized by rotating the plots in Fig. 2(d) about $r = 0$. The first panel of Fig. 2(e) plots a cut of the relative density and phase ϕ_2. The slow light propagation introduces virtually no phase gradient in z or r.

For times short compared to the BEC dynamics, determined by the GPEs (2) with $\Omega_i = 0$ (typically milliseconds), any spatial mode can then be robustly stored. However, much longer storage can be achieved if the pulse parameters are chosen such that ψ_1 and ψ_2 remain nearly stationary. In the weak-probe limit ψ_1 remains $\simeq \psi_1^{(G)}$, while ψ_2 will evolve according to an effective potential $V_{2\text{-eff}}(\mathbf{r})$, determined by the sum of the centrifugal potential m^2/r^2, the trap $V(\mathbf{r})$, and the mean field potential $U_{12}|\psi_1|^2$; see Fig. 2(b). Our choice of pulse parameters is such that ψ_2 nearly matches the ground state of the potential $V_{2\text{-eff}}(\mathbf{r})$, which is trapping and approximately harmonic for $a_{11} > a_{12}$. This results in the dynamics being suppressed, as seen in the second and third panels of Figs. 2(d)–2(f). The dominant motion is a slight dipole sloshing in z [at the frequency of the potential $V_{2\text{-eff}}(\mathbf{r})$], an unavoidable artifact of the group velocity having a small dependence on r.

After an arbitrary time, one can then switch the coupling field back on, regenerating a probe pulse according to $\Omega_p = -\Omega_c^{(\text{in})}(\psi_2/\psi_1^{(G)})$ [13] which is then output. Figure 2(c) shows N_2 and N_{loss} upon a switch-on at 148 ms. The total accumulated loss after the pulse has been output is 33% and nearly identical to the 37% obtained when we propagate the LG mode through the BEC without stopping and storing it. Furthermore, the vortex phase profile has been preserved.

We performed a series of numerical calculations of output pulses for the case in Fig. 2, but with various switch-on times and winding numbers m. The dynamics are sufficiently suppressed that the variation of the storage efficiency $\mathcal{T} = N_p^{(\text{out})}/N_p^{(\text{in})}$ with storage time is not significant; see Fig. 2(f). The variations of $\mathcal{T}$ can be accurately estimated by the model of [13]. They are due to spatial features generated in the BECs during the dynamics which get written into small temporal features on the revived probe fields, resulting in additional EIT bandwidth loss.

In assuming cylindrical symmetry in our calculations [21], we have not considered the issue of stability of the vortex states against drift from the trap center or splitting of $|m| > 1$ vortices into several singly quantized vortices. The addition of a blue-detuned laser potential [22] in the center of the trap would form a ‘‘Mexican-hat’’ potential, which would pin the vortices to the trap center and make the winding number m very robust against the splitting [23]. This would not essentially effect our conclusions about the storage dynamics.

We have shown how vortices can be written from atomic to optical fields and vice versa, providing a unique nonlinear technique to create optical vortices as well as directly image phase singularities in BECs. This method should also allow the transfer of vortex configurations between BECs which are optically connected. Furthermore, we considered the ability of a BEC to robustly store a vortex state input from light fields. While the case studied here allowed 70% efficiency, this number can be further improved by using larger BECs (with the loss $\propto 1/\sqrt{D_1}$). We have found that an appropriate choice of parameters can suppress the BEC dynamics. The ultimate storage times should then only be limited by decoherence due to the small inelastic collision rates and thermal and quantum fluctuations (>100 ms) [19].

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