Resonant Cavity Photon Creation via the Dynamical Casimir Effect

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Motivated by a recent proposal for an experimental verification of the dynamical Casimir effect, the macroscopic electromagnetic field within a perfect cavity containing a thin slab with a time-dependent dielectric permittivity is quantized in terms of the dual potentials. For the resonance case, the number of photons created out of the vacuum due to the dynamical Casimir effect is calculated for both polarizations. It turns out that only TM modes can be excited efficiently.

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One of the most impressive manifestations of the nontrivial structure of the vacuum is the static Casimir effect, i.e., the attraction of two perfectly conducting plates, for example, generated by the corresponding distortion of the electromagnetic vacuum state [1]. The noninertial motion of a mirror can even create particles (i.e., photons) out of the vacuum [2] due to the time-dependent disturbance—which is called (in analogy) dynamical Casimir effect (see, e.g., [3] for review). Unfortunately, in contrast to the former (static) effect, the latter (dynamical) prediction has not been experimentally verified yet. To this end, it is probably advantageous to exploit the drastic enhancement of the number of created photons within a cavity occurring if the frequency of the wall vibration is in resonance with one of the (discrete) cavity modes. The difficulty of accomplishing mechanical vibrations of the wall with high frequencies (and appropriate amplitudes) experimentally has led to the idea of simulating the wall motion by manipulating the dielectric permittivity (or magnetic permeability) of some medium within the cavity (which can be done much faster). For example, filling the whole cavity with a homogeneous medium described by a time-dependent permittivity $\varepsilon(t)$ is analogous to introducing an effective length of the cavity via $L_{\mathrm{eff}}(t) = \sqrt{\varepsilon(t)}L$. However, since it is rather difficult to influence a medium filling the complete cavity, a new proposal [4] (see also [5]) for an experimental verification of the dynamical Casimir effect envisions a small slab with a fixed thickness a and a time-dependent permittivity $\varepsilon(t)$ located at one of the walls of the cavity cf. Figure 1. The question of whether and how the motion of the cavity wall can be simulated by such a small dielectric slab—especially in view of the number of created photons—will be the subject of the subsequent considerations (see also [6] for a $1 + 1$ dimensional scalar field model).

In this Letter, we present an *ab initio* derivation of the dynamical Casimir effect based on the quantization of the full macroscopic electromagnetic field within a (perfect) cavity with space-time-dependent dielectric properties—superseding previous effectively $1 + 1$ dimensional calculations (scalar field model, see, e.g., [6,7]) and approaches based on special factorization assumptions (see, e.g., [8]). For $3 + 1$ dimensional cavities with moving walls, there exist various calculations for scalar fields, but very few take into account the full electromagnetic field. For example, in [9], the electromagnetic field is effectively split up into two independent scalar fields obeying different boundary conditions via introducing different potentials for the TE and the TM modes—which leads to a decoupling of the polarizations (TE and TM) per construction. However, in the most general situation, TE and TM modes can mix—hence their coupling should not be excluded *a priori* but investigated for each special case.

Since we are considering low-frequency (e.g., microwave) photons only, we start from the macroscopic source-free Maxwell equations ($\varepsilon_0 = \mu_0 = \hbar = 1$)

$$\nabla \cdot \boldsymbol{B} = \nabla \cdot \boldsymbol{D} = 0, \quad \dot{\boldsymbol{D}} = \nabla \times \boldsymbol{H}, \quad \dot{\boldsymbol{B}} = -\nabla \times \boldsymbol{E}, \qquad (1)$$

with $\boldsymbol{H}(t,\boldsymbol{r}) = \boldsymbol{B}(t,\boldsymbol{r})$ and $\boldsymbol{D}(t,\mathbf{r}) = \varepsilon(t,\boldsymbol{r})\boldsymbol{E}(t,\boldsymbol{r})$. If we were to use the usual vector potential $\boldsymbol{A}$ in temporal gauge $\Phi = 0$, i.e., $\boldsymbol{E} = \dot{\boldsymbol{A}}$ and $\boldsymbol{B} = -\nabla \times \mathbf{A}$, the constraint $\nabla \cdot [\varepsilon(t,\boldsymbol{r})\dot{\boldsymbol{A}}(t,\boldsymbol{r})] = 0$ would render the usual canonical quantization $[\hat{A}_i(t,\boldsymbol{r}), \hat{D}_j(t,\boldsymbol{r}')] = C_{ij}(t,\boldsymbol{r},\boldsymbol{r}')$ in connection with eliminating the longitudinal degree of freedom rather tedious (cf. also [10]) because, in this case, $\nabla \cdot \boldsymbol{D} = 0$ implies $\partial_j' C_{ij} = 0$ but $\partial_i C_{ij} \neq 0$ in general.

Therefore, we avoid these difficulties with the well-known trick (see, e.g., [9]) of introducing the dual vector

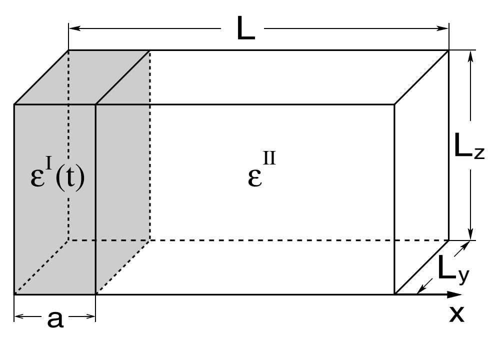

FIG. 1. Sketch of the (lossless) cavity containing a thin slab with a time-dependent dielectric permittivity $\varepsilon^I(t)$.

potential $\boldsymbol{\Lambda}$

$$\boldsymbol{H} = \dot{\boldsymbol{\Lambda}}, \qquad \boldsymbol{D} = \nabla \times \boldsymbol{\Lambda}, \tag{2}$$

which applies in this form to the source-free Maxwell equations (1) only. In terms of the dual vector potential, the constraint simply reads $\nabla \cdot \boldsymbol{\Lambda} = 0$. After the duality transformation [11], the Lagrangian $\mathfrak{L}$ is still the usual Larmor invariant—but with the opposite sign

$$\mathfrak{L} = \frac{1}{2} \int d^3r [\boldsymbol{B} \cdot \boldsymbol{H} - \boldsymbol{E} \cdot \boldsymbol{D}], \tag{3}$$

and the Hamiltonian $\mathfrak{H}$ is again the total energy

$$\mathfrak{H} = \frac{1}{2} \int d^3r [\boldsymbol{B} \cdot \boldsymbol{H} + \boldsymbol{E} \cdot \boldsymbol{D}] = \frac{1}{2} \int d^3r \Big[\dot{\boldsymbol{\Lambda}}^2 + \frac{1}{\varepsilon}(\nabla \times \boldsymbol{\Lambda})^2\Big]. \tag{4}$$

The continuity conditions ($\Delta\boldsymbol{\Lambda} = \boldsymbol{\Lambda}^I - \boldsymbol{\Lambda}^{II}$) for the dual vector potential at the interface between the regions I and II of the cavity can be derived from the Maxwell equations (1)

$$\Delta\boldsymbol{\Lambda} = 0, \quad \Delta(\nabla \times \boldsymbol{\Lambda})_\perp = 0, \quad \Delta\left(\frac{1}{\varepsilon}\nabla \times \boldsymbol{\Lambda}\right)_\parallel = 0. \tag{5}$$

Assuming that the walls of the cavity are perfectly conducting, for example, the boundary conditions read

$$\mathbf{E}_\parallel = 0 \frown (\nabla \times \boldsymbol{\Lambda})_\parallel = 0 \frown (\boldsymbol{\Lambda} \times [\nabla \times \boldsymbol{\Lambda}])_\perp = 0. \tag{6}$$

Consequently, the boundary term arising from the integration by parts (as in the Poynting theorem) of the term $(\nabla \times \boldsymbol{\Lambda})^2$ in Eq. (4) vanishes. Hence we can introduce a non-negative and self-adjoint operator $\mathcal{K}$ via

$$\mathcal{K}\boldsymbol{f}_\alpha = \nabla \times \left(\frac{1}{\varepsilon}\nabla \times \boldsymbol{f}_\alpha\right) = \Omega_\alpha^2 \boldsymbol{f}_\alpha, \tag{7}$$

with eigenfunctions $\boldsymbol{f}_\alpha$ and eigenvalues Ω_α^2. Note that we consider a lossless (ideal) dielectric medium resulting in a real permittivity $\varepsilon \in \mathbb{R}$ (and hence a self-adjoint operator $\mathcal{K}$). Owing to the time dependence of the dielectric permittivity $\varepsilon(t, \boldsymbol{r})$, the operator $\mathcal{K}(t)$ and consequently its eigenfunctions $\boldsymbol{f}_\alpha(t, \boldsymbol{r})$ as well as eigenvalues $\Omega_\alpha^2(t)$ are also explicitely time dependent in general.

The longitudinal modes $\boldsymbol{f}_\alpha^\parallel$ form the (orthogonal) eigenspace with zero eigenvalue $\nabla \times \boldsymbol{f}_\alpha^\parallel = 0$ and hence we can restrict the operator $\mathcal{K}$ to the constraint subspace $\nabla \cdot \boldsymbol{f}_\alpha = 0$. Since $\mathcal{K}$ is a real and self-adjoint operator, we can choose its eigenfunctions to be real as well $\boldsymbol{f}_\alpha = \boldsymbol{f}_\alpha^*$; and they are orthonormal (for equal times) $\int d^3r \boldsymbol{f}_\alpha(t) \cdot \boldsymbol{f}_\beta(t) = \delta_{\alpha\beta}$ and complete

$$\sum_\alpha f_\alpha^i(t, \boldsymbol{r}) f_\alpha^j(t, \boldsymbol{r}') = \delta_\perp^{ij}(\boldsymbol{r} - \boldsymbol{r}'), \tag{8}$$

with $\delta_\perp^{ij}(\boldsymbol{r} - \boldsymbol{r}')$ denoting the transversal Dirac δ distribution $\partial_i \delta_\perp^{ij}(\boldsymbol{r} - \boldsymbol{r}') = 0$. Hence a corresponding normal mode expansion of the Lagrangian and the Hamiltonian in terms of the dual potentials into the instantaneous basis

$$\boldsymbol{\Lambda}(t, \boldsymbol{r}) = \sum_\alpha Q_\alpha(t) \boldsymbol{f}_\alpha(t, \boldsymbol{r}), \tag{9}$$

leads to (see also [12])

$$\mathfrak{H}(t) = \frac{1}{2}\sum_\alpha (P_\alpha^2 + \Omega_\alpha^2(t) Q_\alpha^2) + \sum_{\alpha\beta} P_\alpha Q_\beta \mathcal{M}_{\alpha\beta}(t). \tag{10}$$

From now on, we shall drop the summation signs for convenience by declaring a corresponding (Einstein-like) sum convention. The canonical conjugated momenta P_α are given by $P_\alpha = \dot{Q}_\alpha - \mathcal{M}_{\alpha\beta}(t) Q_\beta$ and the antisymmetric intermode coupling matrix $\mathcal{M}_{\alpha\beta}$ reads

$$\mathcal{M}_{\alpha\beta}(t) = \int d^3r \dot{\boldsymbol{f}}_\alpha(t) \cdot \boldsymbol{f}_\beta(t). \tag{11}$$

The usual equal-time canonical commutation relations, e.g., $[\hat{Q}_\alpha, \hat{P}_\beta] = i\delta_{\alpha\beta}$, are equivalent to the commutators for the fields, such as $[\hat{\Lambda}^j(t, \boldsymbol{r}), \quad \hat{\Pi}^k(t, \boldsymbol{r}')] = i\delta_\perp^{jk}(\boldsymbol{r} - \boldsymbol{r}')$.

It will be convenient to classify the eigenmodes with respect to their polarization at the interface into TE (transversal electric) and TM (transversal magnetic) modes

$$E_\perp^{TE} = E_x^{TE} = 0, \qquad B_\perp^{TM} = B_x^{TM} = 0. \tag{12}$$

In terms of the dual potentials, these conditions read $(\nabla \times \boldsymbol{\Lambda})_\perp^{TE} = 0$ and $\Lambda_\perp^{TM} = 0$. Assuming the absence of any static fields and fields outside the cavity (the walls are supposed to be perfectly reflecting), the boundary condition $B_\perp = 0$ implies $\Lambda_\perp = 0$ and hence Eq. (6) imposes the restriction $\nabla_\perp \boldsymbol{\Lambda}_\parallel = 0$ at the walls. As a result, we can make the following separation *ansatz* in the homogeneous region I

$$\boldsymbol{f}_{\boldsymbol{k}\sigma}^I = \begin{pmatrix} \sin(k_x^I x)\cos(k_y y)\cos(k_z z)\epsilon_{\boldsymbol{k}\sigma x}^I \\ \cos(k_x^I x)\sin(k_y y)\cos(k_z z)\epsilon_{\boldsymbol{k}\sigma y}^I \\ \cos(k_x^I x)\cos(k_y y)\sin(k_z z)\epsilon_{\boldsymbol{k}\sigma z}^I \end{pmatrix}, \tag{13}$$

and analogously for region II with x being replaced by $x - L$. (Here σ labels the polarization.)

The wave numbers k_y and k_z are simply determined by the perpendicular cavity dimensions L_y, L_z via $k_y = n_y\pi/L_y$ and $k_z = n_z\pi/L_z$, respectively, with integers n_y and n_z. The remaining polarization factors $\epsilon_{\boldsymbol{k}\sigma}^{I/II}$ as well as the $k_x^{I/II}$-values have to be determined according to the continuity conditions in Eq. (5), the polarization condition (TE or TM) in Eq. (12), the transversality condition $\nabla \cdot \boldsymbol{\Lambda} = 0$, the overall normalization, and, finally, the eigenvalue equation (for a fixed time)

$$\Omega^2 = \frac{(k_x^I)^2 + k_y^2 + k_z^2}{\varepsilon^I} = \frac{(k_x^{II})^2 + k_y^2 + k_z^2}{\varepsilon^{II}}, \tag{14}$$

which provides a relation between k_x^I and k_x^{II}. Using the conditions mentioned above, we arrive at the transcendental equations

$$\text{TE:}\ \frac{\tan(ak_x^I)}{k_x^I} = \frac{\tan(k_x^{II}[a-L])}{k_x^{II}},$$
$$\text{TM:}\ \frac{k_x^I\tan(ak_x^I)}{\varepsilon^I} = \frac{k_x^{II}\tan(k_x^{II}[a-L])}{\varepsilon^{II}}, \tag{15}$$

which have to be satisfied simultaneously to the eigenvalue Eq. (14).

Assuming the slab to be sufficiently small $a \ll L$, we can find approximate solutions for the TE modes

$$k_x^{II} = \frac{n_x\pi}{L} + \mathcal{O}\left[\frac{a^3}{L^3}\right], \tag{16}$$

and for the TM modes (for $n_x > 0$)

$$k_x^{II} = \frac{n_x\pi}{L}\left(1 + \frac{a}{L}\left[\frac{\varepsilon^{II}}{\varepsilon^I} - 1\right]\frac{k_\parallel^2}{k_\perp^2}\right) + \mathcal{O}\left[\frac{a^2}{L^2}\right], \tag{17}$$

with $k_\parallel^2 = k_y^2 + k_z^2$ and $k_\perp = n_x\pi/L$.

We observe that the first-order (in $a/L \ll 1$) contributions to the eigenvalues Ω_α^2 of the TE modes are independent of $\varepsilon^{I/II}(t)$. [However, the higher-order terms (in $a/L \ll 1$) could become important if the variations of the permittivity are very large, see the remarks after Eq. (22) below.] Only for the TM modes, a variation of the permittivities $\varepsilon^{I/II}(t)$ induces (to first order) a change of the eigenvalues (with the label $\alpha = \{\boldsymbol{n}, \text{TM}\}$)

$$\Delta\Omega_{\boldsymbol{n},\text{TM}}^2(t) = \frac{2k_\parallel^2}{\varepsilon^{II}}\frac{a}{L}\left[\frac{\varepsilon^{II}}{\varepsilon^I(t)} - 1\right] + \mathcal{O}\left[\frac{a^2}{L^2}\right]. \tag{18}$$

The first-order term of the coupling matrix can be derived in complete analogy, it is only nonvanishing for two TM modes with $\boldsymbol{k}_{\boldsymbol{n}}^\parallel = \boldsymbol{k}_{\boldsymbol{m}}^\parallel$ but $k_{\boldsymbol{n}}^\perp \neq k_{\boldsymbol{m}}^\perp$

$$\mathcal{M}_{\boldsymbol{n},\boldsymbol{m}}^{\text{TM}}(t) = 2\frac{a}{L}\frac{k_\parallel^2}{\boldsymbol{k}_{\boldsymbol{n}}^2 - \boldsymbol{k}_{\boldsymbol{m}}^2}\frac{\partial}{\partial t}\left(\frac{\varepsilon^{II}}{\varepsilon^I}\right) + \mathcal{O}\left[\frac{a^2}{L^2}\right]. \tag{19}$$

In order to simulate a harmonic oscillation of the wall, we assume a sinusoidal time dependence of the ratio

$$\frac{\varepsilon^{II}}{\varepsilon^I(t)} = \xi + \chi\sin(\omega t), \tag{20}$$

with the amplitude χ and an irrelevant additive constant ξ (which just induces a constant shift of the eigenfrequencies). A small harmonic perturbation over a relatively long time duration (i.e., many oscillations) enables us to employ the rotating wave approximation, which neglects all nonresonant terms. According to Eq. (10) with $\Omega_\alpha^2(t) = (\Omega_\alpha^0)^2 + \Delta\Omega_\alpha^2(t)$, the perturbation Hamiltonian can be split up into two parts, the diagonal (so-called squeezing) term $\Delta\Omega_\alpha^2(t)\hat{Q}_\alpha^2/2$ and the off-diagonal (so-called velocity) contribution $\hat{P}_\alpha\hat{Q}_\beta\mathcal{M}_{\alpha\beta}(t)$ cf. [12]. The resonance condition for the former (squeezing) term reads $\omega = 2\Omega_\alpha^0$ and for the latter intermode coupling (velocity) contribution $\omega = |\Omega_\alpha^0 \pm \Omega_\beta^0|$. In the following, we shall assume a cavity with well-separated eigenfrequencies where the external oscillation frequency ω matches the diagonal resonance condition $\omega = 2\Omega_\alpha^0$ for a certain TM mode only (no resonant intermode coupling). For example, for a cavity with the aspect ratio $2\pi L/5 = L_y = L_z$, the $\text{TM}_{1,1,1}$ mode can be excited via the fundamental resonance $\omega = 2\Omega_{1,1,1}^{\text{TM}}$ without inducing resonant intermode coupling [13]. In this case, the effective Hamiltonian reads

$$\hat{\mathfrak{H}}_{\text{eff}}^{\text{TM}} = \frac{i}{2}\frac{k_\parallel^2}{\omega}\frac{\chi}{\varepsilon^{II}}([\hat{a}_\alpha^\dagger]^2 - \hat{a}_\alpha^2)\frac{a}{L} + \mathcal{O}\left[\frac{a^2}{L^2}\right], \tag{21}$$

for the resonant TM mode α, with $\hat{a}_\alpha^\dagger, \hat{a}_\alpha$ denoting the corresponding creation and annihilation operators. Via the well-known mechanism (see, e.g., [3]) of parametric resonance, the time dependence of the dielectric permittivity of the thin slab induces the squeezing of the vacuum state $|0\rangle$ and thus the creation of an exponentially increasing number $\langle\hat{N}\rangle$ of particles (photons) out of the vacuum (dynamical Casimir effect)

$$\langle\hat{N}\rangle(t) = \langle 0|\exp\{+i\hat{\mathfrak{H}}_{\text{eff}}^{\text{TM}}t\}\hat{a}^\dagger\hat{a}\exp\{-i\hat{\mathfrak{H}}_{\text{eff}}^{\text{TM}}t\}|0\rangle$$
$$= \sinh^2\left(\frac{k_\parallel^2}{\omega}\frac{\chi}{\varepsilon^{II}}\frac{a}{L}t\right). \tag{22}$$

Let us state the physical assumptions entering the above derivation. Firstly, by starting from the source-free macroscopic Maxwell equations with perfectly conducting boundary conditions we assumed an ideal cavity and omitted losses and decoherence, etc. Of course, the applicability of this assumption has to be checked (e.g., whether the Q factor of the cavity is large enough) before conducting a corresponding experiment. Secondly, the external oscillation was assumed to be harmonic with the frequency matching the resonance condition exactly. Other periodic time dependences could lead to the contribution of higher harmonics (2ω, etc.) and one has to make sure that the possibly resulting intermode coupling does not spoil the main contribution in Eq. (22). A deviation (detuning) from the exact resonance $\omega = 2\Omega_\alpha^0(1+\delta)$ is also not critical as long as the relative detuning δ is smaller than the relative perturbation amplitude cf. [14]. Thirdly, the neglect of the higher-order terms in the Taylor expansion of the transcendental matching Eqs. (15) leading to Eqs. (16) and (17) assumes a small slab $a \ll L$ is only justified if all other involved quantities are not too large. If the ratio $\varepsilon^{II}/\varepsilon^I(t)$ changes drastically, this approximation breaks down as soon as the smallness of the expansion parameter a/L is compensated by a huge variation in $\varepsilon^{II}/\varepsilon^I(t)$.

Let us study the two limiting cases: For $\varepsilon^I \gg \varepsilon^{II}$, the wave numbers behave as $k_x^I \gg k_x^{II}$ according to Eq. (14). Hence the poles of the functions $\tan(ak_x^I)$ in Eq. (15) induce drastic changes of k_x^{II} and thus Ω—for both TE and TM modes. In this case, the eigenmodes are "pulled" into the small slab (which is not desirable since the time-varying material properties will entail the danger of dissipation and decoherence). In the opposite case $\varepsilon^I \ll \varepsilon^{II}$, the wave number in the small slab becomes imaginary for $k_\parallel \neq 0$ cf. Eq. (14). In some sense, the modes are "pushed" out of the small slab similar to the phenomenon of total reflection (as in an optical fiber, for example) in this situation. With exactly the same argument as before, there is no effect to lowest order in a/L for the TE modes. However, the values k_x^{II} of the TM modes approach the pole of the tangent, i.e., $k_x^{II} \to n_x\pi/(L-a)$ owing to the occurrence of the term $\varepsilon^{II}/\varepsilon^I$ cf. Eqs. (17) and (18).

In summary, a (small and smooth) motion of the cavity wall (more precisely, its resonant features) can only be simulated for TM modes as long as $\varepsilon^I(t)$ does not become too large. Even though the set-up with $\varepsilon^I \gg \varepsilon^{II}$ may appear similar to a cavity with an effective length $L - a$, the dynamics from, say, $\varepsilon^I = \varepsilon^{II}$ to $\varepsilon^I \gg \varepsilon^{II}$ is not analogous to the motion of a wall from L to $L - a$. Instead the latter evolution (from $\varepsilon^I = \varepsilon^{II}$ to $\varepsilon^I \gg \varepsilon^{II}$) displays more similarities to the insertion of an additional wall (see, e.g., [15]) at $x = L - a$ into the cavity—which would imply a completely different dynamics (e.g., depending on what happens in the cutoff part of the cavity).

With the aid of the dual vector potential $\boldsymbol{\Lambda}$, we were able to quantize the (macroscopic) electromagnetic field within a cavity with space-time-dependent dielectric properties $\varepsilon(t, \boldsymbol{r})$ and $\mu = \text{const}$ facilitating the investigation of the influence of the polarization (TE and TM modes), etc. In the opposite case $\mu(t, \boldsymbol{r})$ and $\varepsilon = \text{const}$, an analogous derivation can be accomplished using the ordinary vector potential $\boldsymbol{A}$. In contrast to the former case, where only the TM modes feel (to first-order) the presence of the dielectric slab, both polarizations (TE and TM) are effected by the changing magnetic properties $\mu(t, \boldsymbol{r})$ in the latter situation [13]. The physical difference between the two cases can be explained by the distinct boundary conditions, which involve real macroscopic charges and currents and are therefore not invariant under the duality transformation [11].

In conclusion, there are three major differences between the scenario under investigation and a cavity with a moving wall: firstly, the dependence on the polarization (only TM modes), secondly, the dependence on the perpendicular wave number $\propto k_\parallel^2$, and thirdly, that fact that the effect does not vanish for $n_x = 0$—in this case, one obtains just half the value given in Eq. (18).

Let us insert some explicit numbers: If we assume a switching time of about 100 ps [4], the term $k_\parallel^2/\omega$ in Eq. (22) is of order GHz. According to the resonance condition, the dimensions of the cavity and hence the wavelength of the created photons should be several centimeters (i.e., microwaves). The second term χ/ε^{II} cannot exceed $1/2$ and is assumed to be of order one. The remaining parameter a/L should be small, say $1/100$—where the explicite value of a depends on the way of changing the dielectric properties, e.g., laser illumination depth [4]. Note that $a/L = 1/100$ is still by far larger than the relative amplitudes that can be achieved by mechanical vibrations (at these frequencies). Provided that the assumption of a perfect cavity (e.g., Q factor) is appropriate during such time-scales, one would create a significant amount of photons after a few microseconds.

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