New Fluxon Resonant Mechanism in Annular Josephson Tunnel Structures

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(Received 20 May 2004; published 25 October 2004)

A novel dynamical state has been observed in the dynamics of a perturbed sine-Gordon system. This resonant state has been experimentally observed as a singularity in the dc current-voltage characteristic of an annular Josephson tunnel junction, excited in the presence of a magnetic field. In this respect it can be assimilated to self-resonances known as Fiske steps. Differently from these, however, we demonstrate, on the basis of numerical simulations, that its detailed dynamics involves rotating fluxon pairs, a mechanism associated, so far, to self-resonances known as zero-field steps. This occurs because the size of nonlinear excitations is comparable with that of the system.

DOI: 10.1103/PhysRevLett.93.187001 PACS numbers: 74.50.+r, 03.75.Lm, 05.45.Yv

Singularities, or resonances, in the current-voltage characteristic of a Josephson tunnel junction reflect the underlying dynamics of the phase difference between the two superconducting junction electrode order parameters. Two kinds of singularities are usually observed: resonances, known as Fiske steps (FS), that arise when a magnetic field is externally applied in the junction plane and the so-called zero-field steps (ZFS), excited even in the absence of an external magnetic field, exclusively in large junctions (a Josephson junction is said to be large or extended when one of its dimensions is wider than the Josephson penetration depth λ_J) [1]. Two different resonance mechanisms have been proposed to explain the existence of these current-voltage singularities: interaction of cavity modes with the ac Josephson effect and fluxon oscillations. For small junctions, the theory of Fiske steps developed by Kulik [2], based on the excitation of electromagnetic standing waves, accounts for the experimental observations. In large junctions, the fluxon based picture, first proposed by Fulton and Dynes [3], constitutes the convincing explanation frame of reference of all ZFS phenomenology. In the last approach, fluxons, or particlelike magnetic flux quanta (solitons), shuttle back and forth along the extended dimension of the junction. Here the relevant equation is the one-dimensional sine-Gordon equation with appropriate perturbing terms and boundary conditions [4]. There have been attempts to extend the Kulik theory to long junctions such that a single type of analysis could work for both FS and ZFS. Alternatively, the idea that fluxon propagation was responsible also for existence of FS, besides ZFS, in long junctions was put forward [5,6]. This hypothesis, sometimes referred to as "Samuelsen hypothesis," is based on the simple observation that an applied magnetic field renders the junction dynamics asymmetric through the boundary conditions; i.e., the fluxon propagation becomes unidirectional: fluxons enter one of the junction edges and annihilate on the opposite one. The situation has been partially redefined by a number of experiments performed by Cirillo *et al.* [7]. In these experiments there is evidence that separate regimes would exist in long junctions, depending on the intensity of applied magnetic field, in which the two different mechanisms are active. At small fields the fluxon picture applies; for larger fields, i.e., beyond the threshold represented by $H_1 = 2\lambda_J j_c$ [8], where j_c is the maximum critical Josephson current density, the field penetrates stably the junction which starts to behave as a short junction as far as its magnetic properties are concerned. In other words, the cavity mode mechanism described by Kulik operates beyond H_1, while the Samuelsen hypothesis acts at low field values, when the external magnetic field localizes at the edges of the junction.

In this Letter we connect for the first time an observed feature in the *I-V* characteristic of a Josephson tunnel junction with a new possible *hybrid* kind of resonance whose nature stands halfway between a FS and a ZFS. Our observations have been made on an annular Josephson junction, i.e., a junction in which the two electrodes are stacked superconducting rings coupled through a thin dielectric tunnel barrier. We have studied experimentally and numerically the phase dynamics underlying the resonances appearing in the *I-V* characteristic of a moderately extended annular sample. From our analysis a new picture of the Samuelsen mechanism appears, which permits the identification of the hybrid character of the observed resonant dynamical state. The underlying dynamics of the observed state is one in which *both* the Samuelsen unidirectional FS mechanism *and* ZFS free fluxon propagation coexist. For larger fields this hybrid dynamics disappears and we observe only the Kulik cavity mode dynamics. In annular Josephson junctions, the resonance phenomena connected to FS or ZFS have a peculiar character due to the absence of fluxon reflections at the boundaries and to the capability of a superconducting ring to stably trap flux quanta [9]. The

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detailed comprehension of the phase dynamics in these structures remains of paramount interest. These devices have been considered recently in connection with a number of fundamental physics experiments: the macroscopic quantum tunneling of a single superconducting vortex in real space [10], the quantum generation of fluxon-antifluxon (F-AF) pairs [11] in view of the potential implementation of vortex qubit circuits based on annular Josephson structures [12]. They have been proposed as advanced radiation cryodetectors [13], as devices to detect effects of the geometric Berry phase on the vortex dynamics [14], and even for testing, in laboratory, quantum field theory models of the early Universe [15]. In long annular junctions FS and ZFS appear at the asymptotic voltage positions $V_n = n\bar{c}\Phi_0/L$, where n is an integer number, L is the length of the circumference of the junction, $\bar{c}$ is the velocity of light in the junction, and Φ_0 is the flux quantum. ZFS dynamics involves free propagation of fluxons (antifluxons) around the circle. Unless fluxons are inserted into the junction [9], thus changing the so-called winding number, ZFS correspond in fact to the free propagation of F-AF pairs, with zero winding number. On the other hand, the Samuelsen hypothesis for FS in long annular junctions corresponds to the following picture: in the presence of an external magnetic field, F-AF pairs are enucleated and successively annihilated at two opposite points along a diameter normal to the magnetic field direction where the tangential component of magnetic field H_t is maximum and minimum, respectively [11,16]. These two distinct mechanisms provide, through the above formula for V_n, the even voltage positions ($n = 2, 4, \ldots,$) for ZFS (i.e., ZFS1, one pair, at V_2; ZFS2, two pairs, at V_4; etc.) and all the positions ($n = 1, 2, \ldots,$) for the case of FS. This means that on the *even* positions it is possible to observe at the same time both resonances (for example, at V_2, FS2 and ZFS1). It should not be possible, on the contrary, to observe on an *odd* position anything other than a single Fiske resonance. Our experiment was performed on a $Nb_{bottom}(150\ nm)/Al_{bottom}(25\ nm)/Al_2O_3/Al_{top}(25\ nm)/Nb_{top}(50\ nm) + Nb_{wiring}(530\ nm)$ annular Josephson tunnel junctions. As for the junction geometry, we have chosen island-type configuration with narrow ($\approx 8\ \mu m$) wiring leads. The sample photograph is shown in the inset of Fig. 1(a). Internal and external junction diameters were 61 and 91 μm, respectively, giving a mean radius of $\bar{R} = 38\ \mu m$. Physical parameters of the junction were the following: $j_c = 54\ A/cm^2$, $\lambda_J = 45\ \mu m$, $H_1 = 0.6$ Oe, $\bar{l} = 2\pi\bar{r} = 5.23$, $\bar{r} = \bar{R}/\lambda_J$. The Josephson penetration depth was almost equal to the external junction radius so that our sample can be considered as a slightly extended annular junction. We have measured the dependence of the amplitude of the resonance steps of the current-voltage characteristic as a function of parallel magnetic field H. The temperature of the sample was 4.2 K. The amplitudes of four resonances were accessible to be registered [17].

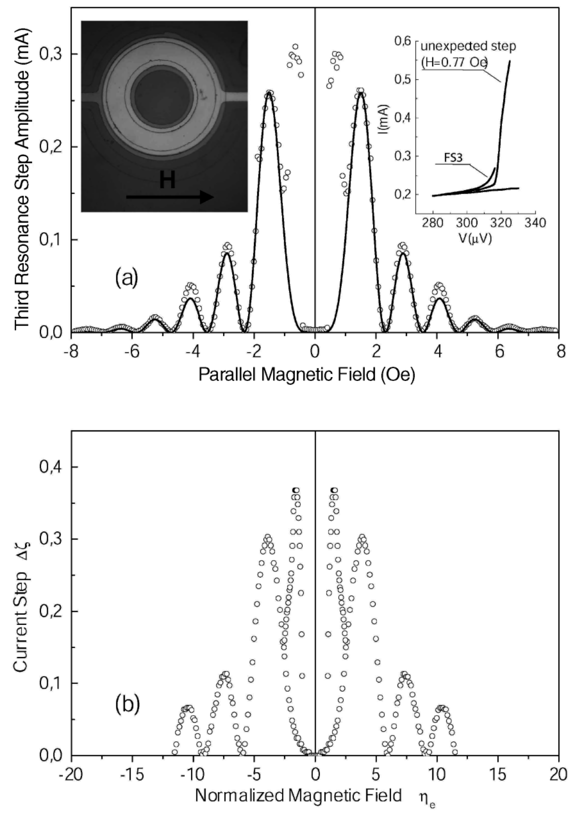

FIG. 1. (a) Experimental magnetic field dependence of the current singularity at voltage position corresponding to the third Fiske step (FS3) (open circles). The solid line has been obtained by an extension of the Kulik theory to the 2D annular junction case. Two extra lobes are visible beside the Kulik predicted dependence of FS3. Insets: the photograph of the sample and the portion of I-V characteristic showing FS3 and the new described resonant state (at different fields). (b) Simulated magnetic field dependence of the critical current of third singularity obtained by numerical solution of Eq. (1). In the simulation $\bar{l} = 5.23$, $\Delta r = 0.33$, and $\alpha = 1/Q = 0.15$.

Further measurement details are described in [18]. Figure 1(a) shows the experimental parallel magnetic field dependence of the amplitude of the third current singularity (open circles). The inset of Fig. 1(a) shows the portion of the I-V characteristic where this singularity appears. Two clearly distinct current peaks are visible: the lower in amplitude is the third Fiske step (FS3), while the higher, optimized in amplitude by a field of 0.77 Oe, is the hybrid state described in this Letter. We used our model theory, an extension of the Kulik theory to small 2D annular junctions [18], as a tool to discriminate between features which do not match with the assumption of a cavity mode mechanism. The solid curve of Fig. 1(a) is the theoretical dependence derived from this theory. The experiment shows clearly two lobes, in the field range 0.46 Oe$< | H | <$1.0 Oe, which the theory does not predict at all. These are the magnetic field modulation of the fluxon sustained voltage step, identified in the inset as "unexpected." This feature occurs in a field region of the order of H_1. According to the Samuelsen fluxon picture, three F-AF pairs should be enucleated at the point where H_t is maximum and annihilate on the opposite side. This would result in an increment of the phase of 6π in a period giving the required FS3 voltage. However, the length of the junction assumes here a critical relevance in that in order to realize the above mechanism six solitons should be accommodated in the junction. Different from the case of one-polar states, i.e., only fluxons or only antifluxons trapped into the junction [17], existence of stable F-AF pairs requires that the junction be sufficiently extended to permit the pair dynamics. In particular here, for $\bar{l} = 2\pi\bar{r} < 2\pi$, the occurrence of resonances involving more than two pairs appears problematic [19]. Therefore the above described Samuelsen mechanism requires some modifications induced by the junction having such a critical dimension.

In order to investigate this possibility and the nature of the observed singularity deeper, we solved numerically the following 2D perturbed sine-Gordon equation for the dynamics of the phase $\varphi(r, \vartheta, t)$ in an annular junction:

$$\frac{1}{r^2}\frac{\partial^2\varphi}{\partial\vartheta^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) - \frac{\partial^2\varphi}{\partial t^2} - \alpha\frac{\partial\varphi}{\partial t} = \sin\varphi, \qquad (1)$$

$$\left.\frac{\partial\varphi}{\partial r}\right|_{r_e} = \zeta(\vartheta) - \eta_e\cos(\vartheta), \quad \left.\frac{\partial\varphi}{\partial r}\right|_{r_i} = -\eta_i\cos(\vartheta), \quad (2)$$

$$\varphi(\vartheta + 2\pi) = \varphi(\vartheta). \qquad (3)$$

In the above equation lengths are normalized to the Josephson penetration depth λ_J and times to the Josephson plasma frequency $\omega_J = \bar{c}/\lambda_J$. η_e and η_i are the external and internal magnetic field normalized with respect to $\lambda_J j_c$, respectively. We assume a θ dependence of the normalized bias current which is closer to the experiment; i.e., two thin leads feed the bias current to the junction. In this case $\zeta = (j_b/j_c)\gamma$, with j_b the bias current density and $\gamma = (\pi/\Delta\theta)(1 + r_i/2r_e)\Delta r$, with $\Delta\theta$ the angular width of current lead, Δr the normalized junction width, r_e (r_i) the normalized external (internal) radius. Figure 1(b) shows the result of simulation of the dependence of third resonance current on the external magnetic field η_e. The internal magnetic field was set to $0.85\eta_e$ to optimize the agreement with the experimental data, taking into account the screening effects. The simulation reproduces fairly well the result of Kulik theory for fields higher than H_1. The lobes at small fields are also well reproduced. From Fig. 1(b) it is seen that these are coexisting with the beginning of the subsequent lobes as two separate branches. The dynamics involved in the low field lobes is illustrated by the numerical simulation results reported in Fig. 2. The phase difference evolution between the two electrodes can be followed during a full period $T = 3\Phi_0/V_3\omega_j \sim \bar{l}$. The phase distribution appearing in Fig. 2 represents only a rough approximation to single fluxon propagation, but the overall picture is sufficiently clear to draw the following description. A F-AF pair (identified with the two points of steepest phase slope, where the magnetic field maximizes) is enucleated at the point A and it propagates up the positions indicated by the blue or grey arrows as is seen at $t = 0.000$ of Fig. 2. While it moves towards the opposite edge ($t = 0.664$ in Fig. 2 blue or grey arrows) a second pair

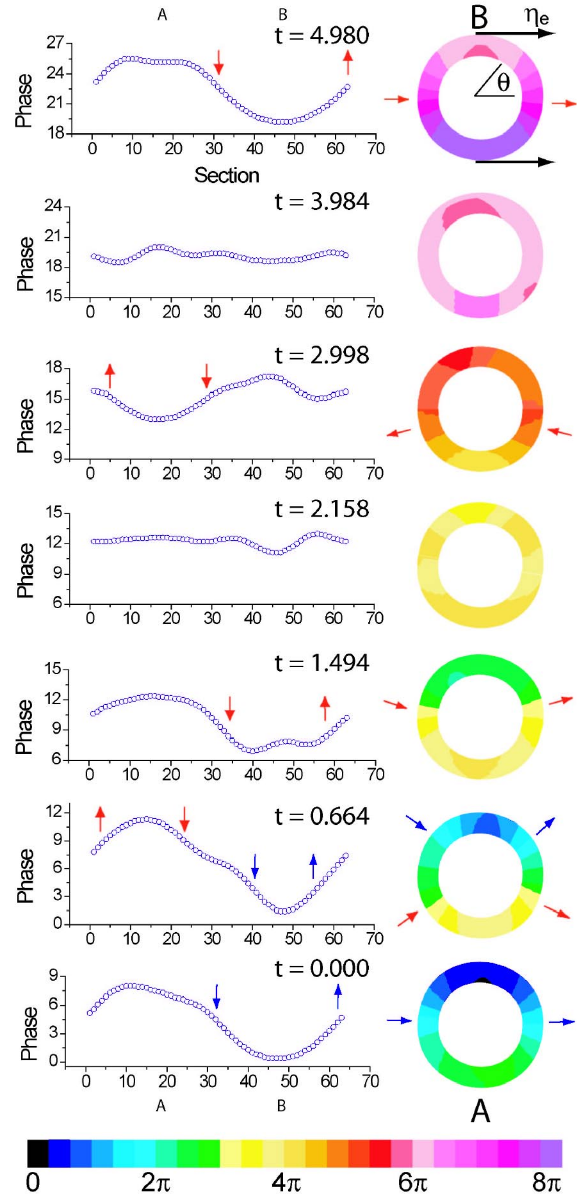

FIG. 2 (color online). Time plots of the phase at the external border of the junction with the corresponding 2D phase snapshots for the annular junctions considered in Fig. 1. $\zeta = 1.104$ and $\eta_e = 1.55$ just at the top of the lobe. Equation (1) is discretized in 63×10 spatial sections each about $0.1\lambda_J$ long. Section 63 is followed by section 1. The bias current drive is located just at sections 1–5 and 32–36. Points A and B are the maximum tangential field locations where pair nucleation (annihilation) occurs. Blue or grey arrows indicate the first F-AF pair, red or light grey arrows the second one. Directions of arrows indicate the fluxon polarity.

enucleates at point A and moves towards the former (see again $t = 0.664$ in Fig. 2, red or light grey arrows). After the first pair has reached point B ($t = 1.494$ in Fig. 2), it has an oscillation mode there because energy is subtracted and the pair prepares to annihilate. Then the second pair arrives ($t = 2.158$). Figure 2 (red or light grey arrows) at $t = 2.998$ shows that, after the complex fourfold collision, the second pair survives. In fact, it still has enough energy to turn back and complete the entire rotation at $t = 3.984$; there the phase flattens and a new pair prepares to enucleate at point A (the latter is shown at $t = 4.980$). During the collision the second incoming pair gains energy at the expenses of the former oscillating pair, avoids annihilation, emerges from the collision, and turns back. From the above analysis we see that the mechanism is hybrid: during a single period, it involves both a full rotating F-AF pair and a half propagating F-AF pair. From this point of view, this dynamical state shares the nature of a Fiske resonance (the second Fiske resonance at V_2 in which the two pairs both annihilate) with that of a zero-field resonance (the second zero-field resonance at V_4 in which the two pairs both propagate around the junction making a complete turn). Numerical simulations show that hybrid dynamics of the third resonance is stable in all range of normalized length $\bar{l}$ between about 4.5 and 2π. The hybrid lobes for shorter length junctions tend to become smaller with respect to Kulik subsequent lobes until they disappear when the junction length is less than about 4.5. On the other hand, for $\bar{l} > 2\pi$, the junction dynamics switches to Samuelsen mechanism; i.e., three F-AF pairs are enucleated on one side and annihilated on the other. In principle, hybrid dynamics could exist whenever the length of the junction is not sufficient to accommodate the required number of pairs to achieve the correct voltage.

In conclusion, we have observed a novel dynamical resonant state in a perturbed sine-Gordon system. This resonant state has been observed as a singularity in the dc current-voltage characteristic of an annular Josephson junction whose dimensions are comparable with the F-AF pair size. In this state, both half-propagating motion of a F-AF pair, due to the presence of external magnetic field, and propagation around the whole junction of another F-AF pair coexist. It is also easy to figure out how this hybrid dynamics could be extended to involve both asymmetric propagation and the back and forth motion of fluxons in standard long linear junctions of critical size. Whenever the condition $L \gg \lambda_j$ is not amply satisfied, i.e., when the size of the nonlinear excitations are comparable to the size of the system, one can expect to observe such hybrid states, which is the modality of the system to cope with an "overcrowded" condition.

M. P. L. is supported by MIUR project *Sviluppo di componentistica avanzata e sua applicazione a strumentazione biomedica.* A. B. is partially supported by EC(QUACS-RTN). C. N. thanks M. Torney for reading the manuscript.

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