Stripe Formation in Granular Mixtures due to the Differential Influence of Drag

P. Sánchez, Michael R. Swift, and P. J. King

School of Physics and Astronomy, University of Nottingham, Nottingham, NG7 2RD, United Kingdom
(Received 10 December 2003; revised manuscript received 20 May 2004; published 28 October 2004)

We describe an investigation of fluid-immersed granular mixtures subjected to horizontal vibration. For sufficiently large amplitudes of vibration, a mixture of equal-sized glass and bronze particles in water is found to separate into a striped pattern. Numerical simulations based on soft-sphere molecular dynamics coupled to the interstitial fluid are able to capture many of the features observed experimentally. We propose a general pattern-formation mechanism based on the differential influence of drag on the components of the mixture. An expression for the number of stripes as a function of the system parameters is derived and shown to be in good agreement with experiments.

DOI: 10.1103/PhysRevLett.93.184302 PACS numbers: 45.70.Qj, 02.70.Ns, 45.70.Mg

Separation and pattern formation in granular mixtures are currently areas of intense research activity within the physics [1] and engineering [2] communities. Granular separation is important in many industrial processes and techniques based on sedimentation, rotation, or vibration have been developed to process a wide range of materials, based on differences in properties such as particle size, shape, or density [3]. In the natural world, granular separation and pattern formation are widely observed, for example, in rock formation by sedimentation, in sand ripples, and on patterned ground [4].

Recently it has been noted that fluid-immersed granular mixtures spontaneously separate under vertical vibration [5]. The morphology of the separated state has been found to depend upon the vibratory conditions [5], on the composition of the mixture [6], and on the nature of the fluid [7]. The mechanism behind this form of separation has been traced to the influence of the fluid on the grains as it is driven through the granular bed by vibration [8,9]. A seemingly unrelated separation into periodic stripes has been observed in a binary granular mixture shaken horizontally on a tray [10]. This behavior was found to be uninfluenced by the surrounding air and a mechanism based on entropic ordering has been proposed [11].

In this Letter we describe experimental observations of fluid-driven stripe formation in horizontally vibrated granular mixtures. We investigate the dependence of this behavior on the vibratory conditions and on the size and density differences of the components of the mixture. We have carried out molecular dynamics simulations which capture the key features observed experimentally and suggest a pattern-formation mechanism based on the differential influence of drag. We derive an expression for the number of stripes in terms of the system parameters and verify our predictions by experimentation and simulation. We argue that the stripe and pattern formation observed in a wide range of systems [4,10–13] may be based on a common mechanism.

The initial experimental system consists of a mixture of equal-sized glass and bronze spheres, fully immersed in water. The particles were sieved from a broad size distribution to extract those within the size-range 1.00–1.18 mm. A dry mixture of bronze and glass spheres, 33%:67% by volume, was poured into a cylindrical glass tube 11 cm long and of 5 mm internal diameter. The tube was then filled fully with water and closed by two rubber stoppers. This procedure ensured that the initial configuration was well mixed. The tube was then attached to an electromechanical vibrator, the axis of vibration being accurately aligned to the horizontal.

Figure 1 shows the time development of the system under sinusoidal vibration at $f = 50$ Hz and amplitude of vibration $A = 2$ mm. The corresponding maximum acceleration relative to gravity, $\Gamma = A(2\pi f)^2/g$, is 20. Initially, as the mixture becomes fluidized by vibration, the heavier bronze component slowly sinks to the bottom of the tube. During this process, bronze-rich regions begin to develop. Subsequently, these regions merge and form a striped pattern *perpendicular* to the direction of vibration. Within a few minutes, the configuration con-

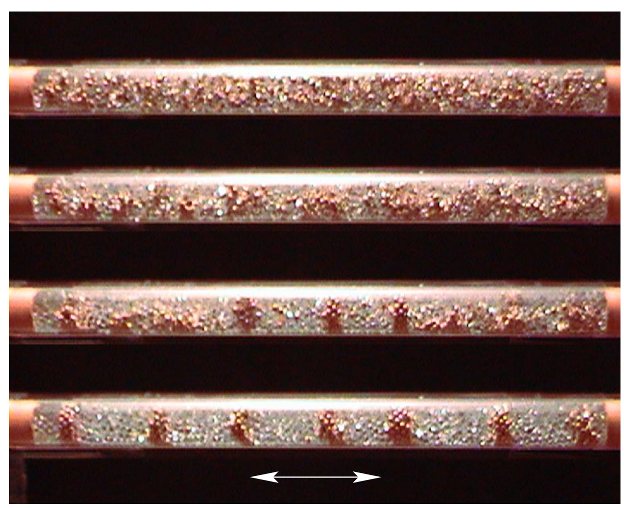

FIG. 1 (color online). Snapshots showing the time evolution of the granular mixture starting from a well-mixed initial configuration. From top to bottom, the times correspond to $t =$ 1.0, 2.0, 2.3, 3.0 min. The darker spheres represent the bronze component. The arrows indicate the axis of vibration.

sists of stripes entirely of bronze separated by regions of pure glass. Such a state appears to be stable, at least on the time scale of a few hours. The striped pattern is approximately periodic and is reproducible between different experimental runs. However, this periodicity is sensitive to the initial configuration and only a well-mixed initial state produces stripes which are close to periodic.

Clear stripe formation is observed over a range of values of f and Γ. We have investigated these phenomena for different particle sizes and container geometries, using both air and water as the interstitial fluid. Under horizontal vibration, stripe formation occurs in those systems that separate well under vertical vibration, as discussed in [6,7]. Specifically, for fine grains that are strongly influenced by the surrounding fluid, a mixture of particles having densities ρ_1 and ρ_2, and radii r_1 and r_2, respectively, will form a striped pattern if the ratio $\rho_1 r_1^2/\rho_2 r_2^2$ is sufficiently different from unity. The pattern-forming behavior is robust and does not require precise horizontal sinusoidal vibration. Indeed, the formation of stripes can be observed if a tube containing a water-immersed mixture of lead shot and plastic beads is vigorously shaken horizontally by hand. However, in all the systems we have investigated, if the fluid is removed stripe formation is no longer observed.

In order to understand these phenomena in more detail, we have performed numerical simulations that are able to reproduce the behavior observed experimentally. The granular motion is simulated using soft-sphere molecular dynamics [14]. The particles are confined in a container that is vibrated sinusoidally with amplitude A and angular frequency ω. The interaction between the grains and the fluid is modeled by adding an effective Stokes' drag force to each particle of the form

$$F_{\mathrm{drag}} = -\alpha r(V - V_f), \quad (1)$$

where V is the velocity of the particle, V_f is the fluid velocity, and α is determined from the laminar term of the Ergun equation and depends upon the fluid viscosity and mean porosity of the bed [8,15]: it models the enhanced fluid drag on a particle when it is part of a porous medium [16]. We have also carried out some simulations based on the full Ergun equation; the nonlinear corrections do not substantially modify the observed behavior. We also assume that the fluid, being incompressible, is driven backwards and forwards by the motion of the container; $V_f = A\omega\cos(\omega t)$ in the horizontal direction. This accelerated motion also introduces buoyancy forces in the horizontal direction. Further details of the model can be found in [8].

Figure 2 shows the time development of the simulations in 3D. The bronze and glass components are equal sized and contained in a cylindrical tube 10 cm long with internal diameter 5 mm, vibrated at 50 Hz and an amplitude of 2 mm. As is observed experimentally, an initially well-mixed configuration evolves into a periodic striped pattern. The resulting bronze and glass stripes are extremely pure. Such a configuration appears to be stable over time scales accessible to simulation. During every cycle, gaps open up between the bronze and glass regions, as is observed in our experiments.

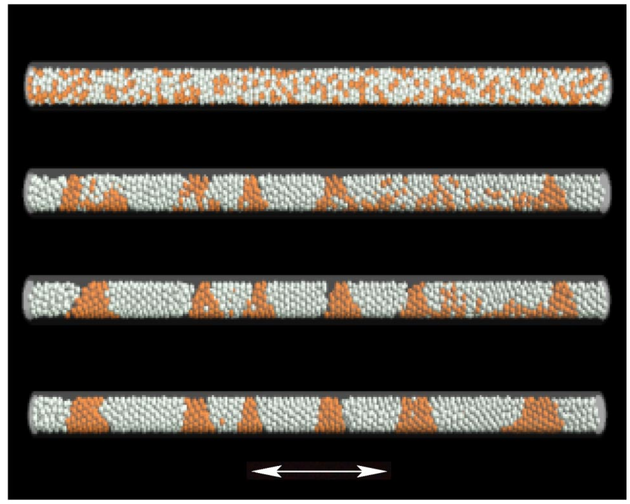

FIG. 2 (color online). Snapshots from simulations showing the time evolution of the granular mixture starting from a well-mixed initial configuration. From top to bottom, the times correspond to $t = 0.0$, 0.5, 2.0, 5.0 min. The darker spheres represent the bronze component.

If a force of gravity, *parallel* to the direction of vibration, is introduced into the model, then the individual stripes merge and coarsen until a single bronze-rich region remains. This is the behavior observed experimentally if the apparatus is rotated so that the direction of vibration is vertical. Depending on the vibratory conditions, a single bronze stripe may then form as a sandwich between upper and lower glass stripes, or it may rise to the top of the bed lying above a single glass stripe [8]. These experiments and simulations show that the mechanism for stripe formation observed in horizontally vibrated granular mixtures immersed in a fluid is closely related to that for fluid-driven separation observed in vertically vibrated granular mixtures [5,6].

In order to remove the bias caused by gravity, we have carried out simulations and experiments in 2D. As in 3D, the fluid-driven model exhibits separation under vibration, and stripe formation is observed at high amplitudes. In general, the morphology of the separated state depends upon the amplitude of vibration, as shown in Fig. 3. For small vibration amplitudes, a well-mixed initial configuration remains mixed for all times. At higher amplitudes, clusters of bronze are seen to form, and the system reaches a steady state characterized by a mean cluster size. We have taken two particles to be in the same cluster if they are of the same type and if their centers are within $3r$. We define a cluster size by the particle radius times the

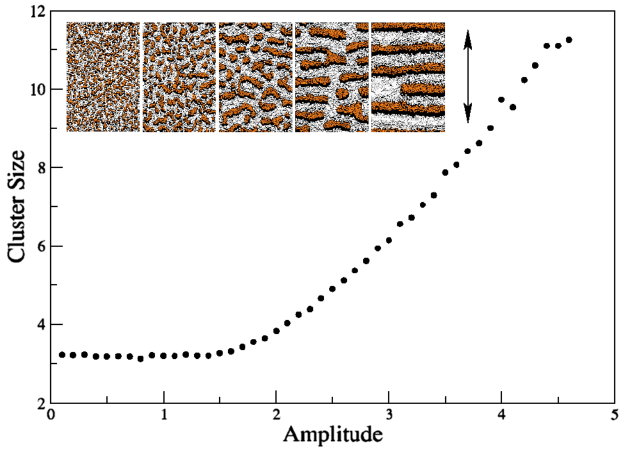

FIG. 3 (color online). The dependence of the steady-state mean bronze cluster size, in units of particle diameters, on amplitude for a mixture vibrated at 50 Hz in 2D. Inset: Snapshots from simulations showing the cluster morphology for amplitudes $A = 1.0$, 2.0, 2.7, 3.5, and 4.5 grain diameters, with periodic boundary conditions.

square root of the mean number of particles within each cluster. It provides a measure of how well mixed or separated the system is. Its dependence on amplitude shows a distinct change in behavior at an amplitude comparable to the size of the grains.

Simulations allow us to make a detailed study of the separation process and to suggest a simple microscopic separation mechanism. The two key ingredients are (1) the fluid is driven backwards and forwards through the bed by vibration of the container, and (2) the fluid influences the two components of the mixture differently. In our experiments with equal-sized bronze and glass particles, the fluid flow will influence the lighter glass component more than the bronze. Consequently, it will tend to move the glass with respect to the bronze in the direction of vibration. Over one cycle of vibration, glass is initially dragged out of bronze-rich regions. On the return part of the cycle, the bronze particles left behind are less likely to reenter glass-rich regions because these regions are now more compact due to the greater fluid damping of the glass. Over a few cycles, this process will result in the formation of bronze clusters, the less damped component. Once like particles come together, there will be little tendency for them to separate as they are driven in the same way by the fluid. The process will then repeat until a stable configuration is reached.

Both in simulation and in experiment, the system reaches an amplitude dependent steady state in which the particles move backwards and forwards periodically due to the fluid drag, but do not collide with each other. For particles of the same type, this motion can occur for any particle separation. However, because the more easily dragged component (glass) has a greater amplitude of motion than the less dragged component (bronze), collisions between glass and bronze can be avoided only if there is sufficient space between them. As the total available free space can be redistributed by like particles coming together, clusters of like particles merge until there is enough free space around them to avoid collisions. This is why we observe gaps between bronze and glass regions.

For sufficiently small amplitudes of vibration, each grain has enough space surrounding it and a well-mixed configuration remains mixed. As the amplitude of vibration is increased, more space is needed around clusters because the difference in motion of the bronze and glass increases. This extra space may be provided by the formation of larger clusters, which is why the cluster size grows with amplitude, as shown in Fig. 3.

For high amplitudes of vibration, the steady state is a periodic stripe pattern, as shown in the inset of Fig. 3. In this limit, the 1D nature of the pattern can be analyzed using a simple geometrical model which relates the number of stripes to the vibratory conditions. In order for collisions between bronze and glass stripes to be avoided, the maximum gap between them must be greater than or equal to $2\Delta a$, where Δa is the *difference* in the amplitudes of the bronze and glass motions. Given this gap, the number of stripes in the steady state can then be calculated. Consider a mixture of n_G and n_B glass and bronze spheres of radii r_G and r_B, respectively, which has separated into an equal number of bronze and glass stripes N. To fit them into an area of length L and width W, N must satisfy

$$WL = \frac{n_G \pi r_G^2}{C_G} + \frac{n_B \pi r_B^2}{C_B} + W(2\Delta a N + 2\Delta e), \qquad (2)$$

where the local area filling fractions within the bronze and glass stripes are C_B and C_G. Here Δe is an end correction equal to the difference in amplitudes of the box and glass. If the area ratio of bronze to glass particles is $R = n_B \pi r_B^2 / n_G \pi r_G^2$, and C is the overall area filling fraction, then, from Eq. (2), N may be written as

$$N = \frac{L}{2\Delta a}\left[1 - \frac{C(C_B + RC_G)}{C_B C_G (1 + R)}\right] - \frac{\Delta e}{\Delta a} \qquad (3)$$

in terms of the vibrational parameters and the unknown quantities C_B and C_G. We note that, in the case of an extra end glass stripe as is frequently seen in our experiments, Eq. (3) is unchanged if N is taken to be the number of *bronze* stripes. The periodic nature of the patterns which we have observed results from random initial conditions, while deviations from initial randomness lead to variations in the stripe widths. The derivation of Eq. (3), however, is general, not requiring a *periodic* striped pattern.

In order to test this theoretical prediction, we have carried out quasi-2D experiments in a shallow water-filled box of internal dimensions 142 mm × 32 mm × 2.7 mm, filled with a mixture of 2.3 mm diameter lead

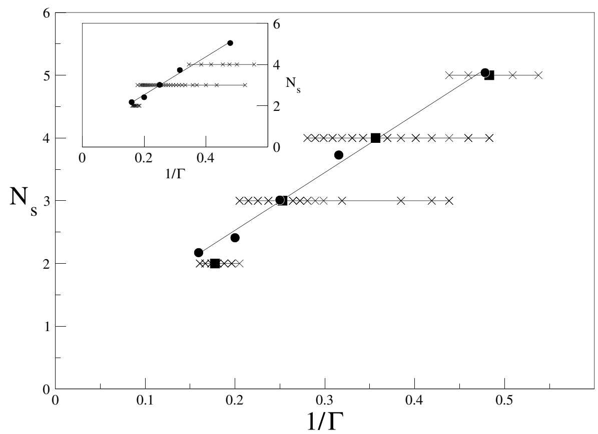

FIG. 4. The stable number of stripes N_s vs $1/\Gamma$ obtained in the quasi-2D experiments. The crosses represent the experimentally observed number of stripes, while the squares represent the average for each value of N_s. The continuous line is a linear fit through the circular points which were derived from the theoretical predictions of Eq. (3). The inset shows a comparison between simulations and this line.

shot and 1.98 mm glass spheres. The box was aligned with its largest face perpendicular to gravity and vibrated horizontally along its principal axis. Such a geometry eliminates the effects of gravity by restraining vertical motion. Figure 4 shows the number of stripes as a function of $1/\Gamma$ obtained experimentally under sinusoidal vibration at 10 Hz. The crosses show the stable number of stripes obtained for different values of Γ, each for well-mixed initial conditions. The squares represent a weighted average of Γ for obtaining a particular number of stripes. The continuous line is a linear fit to the circular points. These points have been derived from Eq. (3), with the values of Δa, R, C, C_G, and C_L obtained from the experiments [C_L, the filling fraction of lead, replaces C_B in Eq. (3)]. For the system considered here, $R = 0.37$ and $C = 0.64$. C_G and C_L are found in general to have values close to 0.8 and 0.72, respectively, the measured values being used to obtain the line. It should be noted that the observed number of stripes, N_s, is an integer, while in Eq. (3), N is a continuous function of the system parameters. Small variations in C_G and C_L about their mean values allow N to be an integer. This is the reason for the steplike structure of the data points in Fig. 4. There is good agreement between the predicted and observed mean values. The inset of Fig. 4 shows results from simulations using the linear drag model, Eq. (1). The value of α was chosen to best match the lead and glass motions with the experimentally obtained values [16]. The agreement between our simulations of stripe formation and theoretical prediction is also reasonably good.

Our experiments and simulations demonstrate a pattern-forming mechanism which results from the differential influence of fluid drag. It is clearly distinct from conventional separation mechanisms based only on size and/or density differences as it requires a fluid to be driven through the bed by vibration. This work suggests that separation should occur in many systems in which the individual components are influenced differentially by periodic forcing [4,12].

In recent experiments by Mullin *et al.* [10,11], stripe formation has also been observed in binary granular mixtures vibrated horizontally on a flat surface. It was shown that experiments carried out in vacuum still exhibit separation and stripe formation. Consequently, a separation mechanism based on fluid effects has been ruled out. However, the similarity of the patterns observed in [10] and those seen in our experiments and simulations suggest a common underlying physical mechanism based on the differential influence of drag. In our experiments the drag is provided by the fluid as it is forced through the bed by vibration. In Mullin's experiments the differential drag may be provided by friction with the tray [13].

We are grateful to Makin Powders Limited for providing materials. P. S. acknowledges the ORS for financial support.

[1] For a review, see H. M. Jaeger, S. R. Nagel, and R. P. Behringer, Rev. Mod. Phys. **68**, 1259 (1996).

[2] J. M. Ottino and D.V. Khakhar, Annu. Rev. Fluid Mech. **32**, 55 (2000), and references therein.

[3] J. Bridgewater, Powder Technol. **15**, 215 (1976); B. A. Wills, *Mineral Processing Technology* (Butterworth and Heinemann, Oxford, 1997).

[4] P. A. Mulheran, J. Phys. I (France) **4**, 1 (1994); M. A. Kessler and B. T. Werner, Science **299**, 380 (2003).

[5] N. Burtally, P. J. King, and M. R. Swift, Science **295**, 1877 (2002).

[6] N. Burtally *et al.*, Granular Matter **5**, 57 (2003).

[7] M. Leaper *et al.*, Granular Matter (to be published).

[8] P. Biswas *et al.*, Phys. Rev. E, **68**, 050301(R) (2003).

[9] M. A. Naylor, M. R. Swift, and P. J. King, Phys. Rev. E **68**, 012301 (2003).

[10] T. Mullin, Phys. Rev. Lett. **84**, 4741 (2002);T. Mullin and P. Reis, Phys. Rev. Lett. **89**, 244301 (2002).

[11] P. Reis *et al.*, Europhys. Lett. **66**, 357 (2004).

[12] H. Lowen and J. Dzubiella, Faraday Discuss. **123**, 99 (2003); A. M. Gaudin, *Principles of Mineral Dressing* (McGraw-Hill, New York, 1939), p. 257.

[13] G. C. M. A. Ehrhardt, A. Stephenson, and P. Reis, cond-matt/0403273.

[14] H. J. Herrmann and S. Luding, Continuum Mech. Thermodyn. **10**, 189 (1998).

[15] S. Ergun, Chem. Eng. Prog. **48**, 89 (1952).

[16] This linear drag force represents the simplest model for grain-fluid coupling that appears to capture the basic physics behind stripe formation. In our 2D experiments, we have observed a linear relationship between the grain amplitudes and Γ, justifying the use of a linear drag force.