# Fluctuations of the Initial Conditions and the Continuous Emission in the Hydrodynamical Description of Two-Pion Interferometry

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Within the hydrodynamical approach, we study the Bose-Einstein correlation of identical pions by taking into account both event-by-event fluctuating initial conditions and continuous pion emission during the whole development of the hot and dense matter formed in high-energy collisions. Important deviations occur, compared to the usual hydrocalculations with smooth initial conditions and a sudden freeze-out on a well defined hypersurface. Comparison with data at the BNL Relativistic Heavy Ion Collider (RHIC) shows that, despite the rather rough approximation we used here, this description can account for the $m_T$ dependence of $R_L$ and $R_s$, and produces a significant improvement for $R_o$ with respect to the usual version.

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*Introduction.*—When describing ultrarelativistic heavy-ion collisions in the hydrodynamical approach [1], a simple picture has been extensively adopted. It is usually considered that, after the initial very complicated interaction between two incident nuclei, at some early instant of time, a local thermal equilibrium is attained. Such a state is usually described in terms of a set of highly symmetric and smooth distributions of velocity and thermodynamical quantities. These are the initial conditions (IC) for the hydrodynamic equations, which must be complemented with reasonable equations of state (EoS). Then, as the thermalized matter expands, the system gradually cools down and, when the temperature reaches a certain freeze-out (FO) value $T_{\rm FO}$, it suddenly decouples. Every observed quantity is then computed on the hypersurface $T = T_{\rm FO}$. For instance, the momentum distribution of the produced hadrons are obtained by using the Cooper-Frye integral [2] extended over this hypersurface.

Though operationally simple and actually useful for obtaining a nice comprehension of several aspects of the phenomena, such a scenario is clearly highly idealized when applied to finite-volume and finite-lifetime systems as those formed in high-energy heavy-ion collisions. In this Letter, we examine modification of two ingredients of such a description, namely, (i) effects of fluctuations in the IC, and (ii) consequences of continuous emission (CE) of particles, regarding two-pion correlation.

The identical-particle correlation, also known as Hanbury-Brown–Twiss (HBT) effect [3,4] is a powerful tool for probing the geometry of the space-time region from which they were emitted. If the source is static like a star, it is directly related to spatial dimensions. When applied to a dynamical source, however, several nontrivial effects appear [5,6], reflecting its time evolution as happens in high-energy heavy-ion collisions. Being so, the inclusion of IC fluctuations and of the continuous emission may affect considerably the so-called HBT radii, because both of them modify in an essential way the particle emission zone in the space-time.

The usual symmetric, smooth IC may be understood as representing the mean distributions of hydrodynamic variables over several events. However, since our systems are not large enough, large fluctuations varying from event to event are expected. In previous papers [7,8], we showed that indeed the effects of these fluctuations on the observed quantities are sizeable and moreover in [8], by studying the rapidity distributions of pions, we showed that the average multiplicity of pions decreases if we consider the event-by-event fluctuations, in comparison with the one given by the averaged IC [9]. Concerning two-pion correlation, as the IC in the event-by-event base often show small high-density spots in the energy distribution, we expect that these spots manifest themselves at the end when particles are emitted, giving smaller HBT radii.

As for the decoupling process, it has been proposed [10,11] an alternative picture where the emission occurs not only from the sharply defined freeze-out hypersurface, but continuously from the whole expanding volume of the system at different temperatures and different times. According to this picture, the large-transverse-momentum ($k_T$) particles are mainly emitted at early times when the fluid is hot and mostly from its surface, whereas the small-$k_T$ components are emitted later when the fluid is cooler and from larger spatial domain. Mostly by using a simple scaling solution, we showed in the previous papers that this picture gives several nice results, namely, (i) CE enhances the large-$m_T$ component of the heavy-particle ($p, \Lambda, \Xi, \Omega, \dots$) spectra, (ii) it gives a concave shape for the pion $m_T$ spectrum even without transverse expansion of the fluid [10,11], (iii) it can lead to the correct hyperon production ratios and spectrum shapes with a conceptually reasonable choice of parameters [10,12,13], and (iv) it reproduces the observed mass dependence of the slope parameter $T$ [14]. Concerning

  

HBT correlation, we showed [15], within the same approximation, that whereas the so-called *side* radius is independent of the average $k_T$, the *out* radius decreases with $\langle k_T \rangle$ because of the reason mentioned above. This behavior is expected to essentially remain in the general case we are going to discuss below and shown by data [16].

*Initial Conditions.*— In order to produce event-by-event fluctuating IC, we use the NeXus event generator [17], based on Gribov-Regge model. Given the incident nuclei and the incident energy, it produces the energy-momentum tensor distribution at the time $\sqrt{t^2 - z^2} =$ 1 fm, in event-by-event basis. This, together with the baryon-number density distributions, constitute our fluctuating IC. The strangeness has not been introduced in the present calculations. As mentioned in the Introduction, we understand that the usual symmetric, smooth IC may be obtained from these by averaging over many events. In Fig. 1, we show an example of such an event for a central Au + Au collision at 130 A GeV, compared with an average over 30 events. As can be seen, the energy-density distribution for a single event (left), at the midrapidity plane, presents several blobs of high-density matter, whereas in the averaged IC (right), the distribution is smoothed out, even though the number of events is only 30. In the results below, we show that the fingerprints of such high-density spots remain until the freeze-out stage of the fluid, giving smaller HBT radii.

*Hydrodynamic Equations.*— The resolution of the hydrodynamic equations deserves special care since our initial conditions do not have, in general, any symmetry nor they are smooth. We adopt the recently developed SPheRIO code [18], based on the so-called smoothed-particle hydrodynamics (SPH), first used in astrophysics and which we have adapted for nuclear collisions [19], a method flexible enough, giving a desired precision. The peculiarity of SPH is the use of discrete Lagrangian coordinates attached to small volumes ("particles") with some conserved quantities. Here, we take the entropy and the baryon number as such quantities. Then, the entropy density, for example, is parametrized as

$$s(\mathbf{x}, t) = \frac{1}{\gamma}\sum_i \nu_i W(\mathbf{x} - \mathbf{x}_i(t); h), \tag{1}$$

where $W[\mathbf{x} - \mathbf{x}_i(t); h]$ is a normalized kernel; $\mathbf{x}_i(t)$ is the

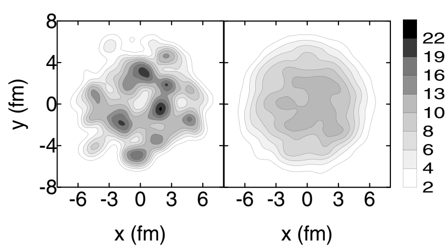


FIG. 1. Examples of initial conditions for central Au + Au collisions given by NeXus at midrapidity plane. Energy density is plotted in units of GeV/fm$^3$. Left: one random event. Right: average over 30 random events.

$i$-th particle position so the velocity is $\mathbf{v}_i = d\mathbf{x}_i/dt$; $h$ is the smoothing scale parameter, and we have

$$S = \int d^3\mathbf{x}\,\gamma s(\mathbf{x}, t) = \sum_i^N \nu_i. \tag{2}$$

Observe that, since an entropy $\nu_i$ is attached to the $i$-th SPH particle, and so is a baryon number, the total entropy $S$ and the baryon number $N$ are automatically conserved. By rewriting the usual energy-momentum conservation equation $\partial_\mu T^{\mu\nu} = 0$, we get a set of coupled ordinary equations [19]

$$\frac{d}{dt}\left(\nu_i \frac{P_i + \varepsilon_i}{s_i}\gamma_i \mathbf{v}_i\right) = -\sum_j \nu_i \nu_j \left[\frac{P_i}{\gamma_i^2 s_i^2} + \frac{P_j}{\gamma_j^2 s_j^2}\right]\nabla_i W(\mathbf{x}_i - \mathbf{x}_j; h). \tag{3}$$

*Equations of State.*— For EOS, we consider ones with a first-order phase transition between quark-gluon plasma (QGP) and a hadronic resonance gas, with baryon-number conservation taken into account. In the QGP, we consider an ideal gas of massless quarks $(u, d, s)$ and gluons, with the bag pressure $B$ taken to give the critical temperature $T_c = 160$ MeV at zero chemical potential. In the resonance gas phase, we include all the resonances below 2.5 GeV, with the excluded volume effect taken into account.

*Two-Pion Correlation.*— We assume that all pions are emitted from a chaotic source and neglect the resonance decays. It is argued [20] that, since resonance decays contribute to the correlations with very small $q$ values ($q \lesssim q_{\min}$, where $q_{\min}$ is the minimum measurable $q$), the experimentally determined HBT radii are essentially due to the direct pions. Then the correlation function is expressed in terms of the distribution function $f(x, k)$ as

$$C_2(q, K) = 1 + \frac{|I(q, K)|^2}{I(0, k_1)I(0, k_2)} \tag{4}$$

where $K = (k_1 + k_2)/2$ and $q = (k_1 - k_2)$ and $k_i$ is the momentum of the $i$-th pion. Usually

$$I(q, K) \equiv \langle a^+_{k_1} a_{k_2} \rangle = \int_{T_{\rm FO}} d\sigma_\mu K^\mu f(x, K) e^{iqx}. \tag{5}$$

In SPH representation, we write $I(q, K)$ as

$$I(q, K) = \sum_j \frac{\nu_j n_{j\mu} K^\mu}{s_j |n_{j\mu} u_j^\mu|} e^{iq_\mu x_j^\mu} f(u_{j\mu} K^\mu), \tag{6}$$

where the summation is over all the SPH particles. In the Cooper-Frye freeze-out, these particles should be taken where they cross the hypersurface $T = T_{\rm FO}$ and $n_{j\mu}$ is the normal to this hypersurface.

*Continuous Emission Model (CEM).*— In CEM [10], it is assumed that, at each space-time point $x^\mu$, each particle has a certain escaping probability

$$\mathcal{P}(x, k) = \exp\left[-\int_{\tau}^{\infty} \rho \sigma v d\tau'\right], \tag{7}$$

due to the finite dimensions and lifetime of the thermalized matter. The integral above is evaluated in the proper frame of the particle. Then, the distribution function $f(x, k)$ of our system has two components, one representing the portion of the fluid already free and another corresponding to the part still interacting, *i.e.*,

$$f(x, k) = f_{\rm free}(x, k) + f_{\rm int}(x, k). \tag{8}$$

We may write the free portion as

$$f_{\rm free}(x, k) = \mathcal{P} f(x, k). \tag{9}$$

The integral (5) is then rewritten in CEM as

$$I(q, K) = \int_{\sigma_0} d\sigma_\mu K^\mu f_{\rm free}(x_0, K) e^{iqx} + \int d^4x \partial_\mu [K^\mu f_{\rm free}(x, K)] e^{iqx}, \tag{10}$$

where the surface term corresponds to particles already free at the initial time.

The problem of this description is its complexity in handling, because $\mathcal{P}$ depends on the momentum of the escaping particle and, moreover, on the future of the fluid as seen in Eq. (7). In order to make the computation practicable, here we first take $\mathcal{P}$ on the average, *i.e.*, $\mathcal{P}(x, k) \Rightarrow \langle \mathcal{P}(x, k) \rangle \equiv \mathcal{P}(x)$. Then, approximate linearly the density $\rho(x') = \alpha s(x')$ in Eq. (7). Thus,

$$\mathcal{P}(x, k) \Rightarrow \mathcal{P}(x) = \exp\left(-\kappa \frac{s^2}{|ds/d\tau|}\right), \tag{11}$$

where $\kappa = 0.5 \alpha \langle \sigma v \rangle$.

Now, Eq. (10) is translated into SPH language, by computing the sum (6) not over $T = T_{\rm FO}$ but picking out SPH particles according to this probability, with $n_{j\mu}$ pointing to the 4-gradient of $\mathcal{P}$. Thus, our approximation includes also emission of particles of any momentum once a SPH particle has been chosen. However, since our procedure favors emission from fast outgoing SPH particles, because $\rho$ decreases faster and so does $s$ in this case making $\mathcal{P}$ larger, we believe the main feature of CEM is preserved.

*Results.*— We first assume sudden freeze-out at $T_{\rm FO} = 128$ MeV. This temperature was previously found by studying the energy dependence of kaon slope parameter $T^*$ [21]. It has been also shown [8] that $T^*$ is not sensitive to IC fluctuations.

In Fig. 2, we compare $C_2$ averaged over 15 fluctuating events with those computed from the averaged IC (so, without fluctuations). One can see that the IC fluctuations are reflected in large fluctuations also in the HBT correlations. When averaged, the resulting $C_2$ are broader than those computed with averaged IC, so giving smaller radii as expected. Also the shape of $C_2$ changes. We plot the $m_T$ dependence of HBT radii, with Gaussian fit of $C_2$, in

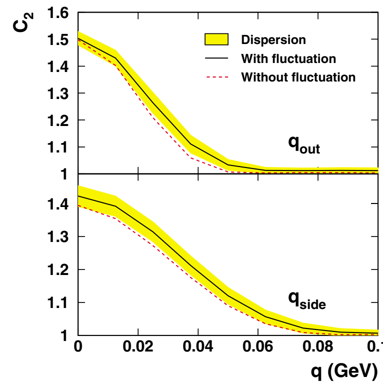


FIG. 2 (color online). Correlation functions from fluctuating IC and averaged IC. Sudden FO is used here. The rapidity range is $-0.5 \leq Y \leq 0.5$ and $q_{o,s,l}$, which do not appear in the horizontal axis, are integrated over $0 \leq q_{o,s,l} \leq 35$ MeV.

Fig. 3, together with RHIC data [16,22] and results with CEM. It is seen that the smooth IC with sudden FO makes the $m_T$ dependence of $R_o$ flat or even increasing, in agreement with other hydrocalculations[23] but in conflict with the data. The fluctuating IC make the radii smaller, especially $R_o$, without changing the $m_T$ dependence.

Let us now consider CEM. In this Letter, we estimated $\kappa$ as being 0.3, corresponding to $\langle \sigma v \rangle = 2$ fm$^2$. In Fig. 4, we show the charged $m_T$ distribution to ensure that the estimate above does reproduce correctly these data. Now, look at the $m_T$ dependence of the HBT radii, shown in Fig. 3. Comparing the averaged IC case, with CEM (CE1), with the corresponding freeze-out (FO1), one sees that, while $R_L$ remains essentially the same, $R_s$ decreases

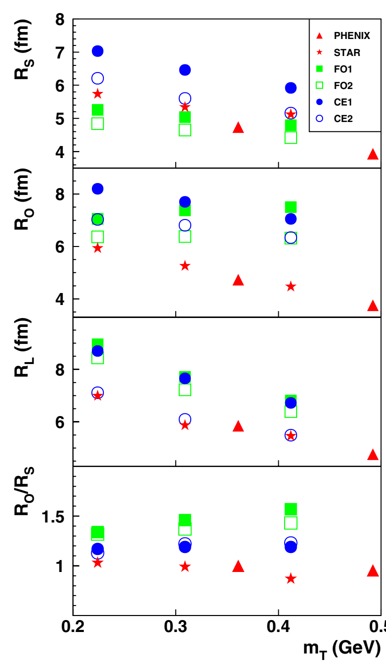


FIG. 3 (color online). HBT radii and the ratio $R_o/R_s$ for sudden freeze-out (FO) and CE. 1 stands for averaged IC and two fluctuating IC. Data are from [16,22]: $(\pi^+ + \pi^-)/2$.

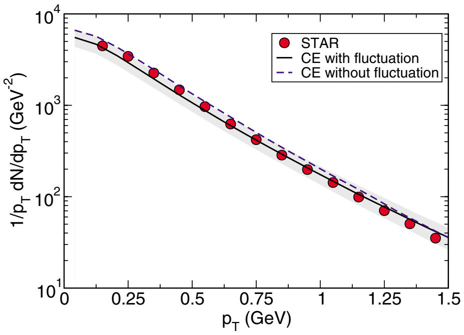


FIG. 4 (color online). Charged-particle $p_T$ distributions in CEM for the most central Au + Au at 130 A GeV. The data are from [26].

faster, and as for $R_o$, it decreases now, inverting its $m_T$ behavior. The account of the fluctuating IC in addition (CE2) makes all the radii smaller as in FO case, obtaining a nice agreement with data for $R_L$ and $R_s$, and improving considerably the results for $R_o$ with respect to the usual hydrodescription.

*Conclusions and Outlooks.*— In this Letter, we showed that both the event-by-event fluctuations of the IC and the CE instead of sudden freeze-out largely modify the HBT correlation of produced pions, so they should be included in more precise analyses of data. The IC fluctuations give smaller radii, without changing the $m_T$ dependence, which is a natural consequence of the presence of high-density spots at the early times. Continuous particle emission, on the other hand, does not change $R_L$ but enhances the $m_T$ dependence of $R_s$ and inverts the $m_T$ behavior of $R_o$, which now decreases with $m_T$ in accord with data. This is because, in CEM, large-$k_T$ particles appear mostly at the early stage of the expansion from a thin hot shell of the matter, whereas small-$k_T$ particles appear all over the expansion, and from larger portion of the fluid. The combination of these two effects can give account of the $m_T$ dependence of $R_L$ and $R_s$ and improves considerably the one for $R_o$ with respect to the usual version.

We shall emphasize that these conclusions could only be reached because we have explicitly solved 3 + 1 dimensional hydrodynamic equations and not simply parametrized the final flow as often done [24]. The results are preliminary. In applying the CEM we had to make a drastic approximation, expressed by Eq. (11), in order to make it feasible. It is likely that this is the reason why the discrepancy in $R_o$ still persists. In addition, there exist certainly dissipation effects. Since SPH is an effective description in terms of parameters $\nu_i$ appearing in Eq. (1), some smoothing of short wavelength Fourier components is taken into account through the kernel $W[\mathbf{x} - \mathbf{x}_i(t); h]$. We preferred not to explicitly include the viscosity at this stage, as it is still an open problem of hydrodynamics (see more details in Ref. [25]).

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