Precise Measurement of the $\pi^+ \to \pi^0 e^+ \nu$ Branching Ratio

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(Received 17 December 2003; revised manuscript received 19 July 2004; published 25 October 2004)

Using a large acceptance calorimeter and a stopped pion beam we have made a precise measurement of the rare $\pi^+ \to \pi^0 e^+ \nu$ (π_β) decay branching ratio. We have evaluated the branching ratio by normalizing the number of observed π_β decays to the number of observed $\pi^+ \to e^+ \nu$ (π_{e2}) decays. We find the value of $\Gamma(\pi^+ \to \pi^0 e^+ \nu)/\Gamma(\text{total}) = [1.036 \pm 0.004(\text{stat}) \pm 0.004(\text{syst}) \pm 0.003(\pi_{e2})] \times 10^{-8}$, where the first uncertainty is statistical, the second systematic, and the third is the π_{e2} branching ratio uncertainty. Our result agrees well with the standard model prediction.

DOI: 10.1103/PhysRevLett.93.181803 PACS numbers: 13.20.Cz, 11.40.–q, 12.15.Hh, 14.40.Aq

The rare pion beta decay, $\pi^+ \to \pi^0 e^+ \nu$ (branching ratio $R_{\pi\beta} \simeq 1 \times 10^{-8}$), is one of the most basic semileptonic electroweak processes. It is a pure vector transition between two spin-zero members of an isospin triplet and is therefore analogous to superallowed Fermi (SF) transitions in nuclear beta decay. Because of its simplicity and accuracy, the theory of Fermi beta decays is one of the most precise components of the standard model (SM) of electroweak interactions.

The conserved vector current (CVC) hypothesis [1,2] and quark-lepton universality relate the rate of pure vector beta decay (both pion and nuclear) to that of muon decay via the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix element V_{ud} [3,4]. Including loop corrections, δ, the rate of pion beta decay is given by [5,6]

$$\Gamma_{\pi\beta} = \frac{G_\mu^2 |V_{ud}|^2}{30\pi^3}\left(1 - \frac{\Delta}{2M_+}\right)^3 \Delta^5 f(\epsilon, \Delta)(1 + \delta), \qquad (1)$$

where G_μ is the Fermi weak coupling constant, $\Delta = M_+ - M_0$, $\epsilon = (m_e/\Delta)^2$, and M_+, M_0, and m_e are the masses of the π^+, π^0, and the electron, respectively, while f, the Fermi function, is given by

$$f(\epsilon, \Delta) = \sqrt{1-\epsilon}\left[1 - \frac{9}{2}\epsilon - 4\epsilon^2 + \frac{15}{2}\epsilon^2 \ln\left(\frac{1+\sqrt{1-\epsilon}}{\sqrt{\epsilon}}\right) - \frac{3}{7}\frac{\Delta^2}{(M_+ + M_0)^2}\right]. \qquad (2)$$

The main experimental source of uncertainty in $\Gamma_{\pi\beta}$ amounts to just 0.05%; it comes from the measurement of Δ [7]. The combined radiative and short-range physics corrections amount to $\delta \simeq 0.033$ and are exceptionally well controlled, yielding an overall theoretical uncertainty of $\Gamma_{\pi\beta}$ of $\lesssim 0.1\%$ [6,8–10]. Hence, pion beta decay presents an excellent means for a precise experimental determination of the CKM matrix element V_{ud}, hindered only by the low branching ratio of the decay.

The CKM quark mixing matrix has a special significance in modern physics as a cornerstone of a unified description of the weak interactions of mesons, baryons, and nuclei. In a universe with three quark generations, the 3×3 CKM matrix must be unitary, barring certain classes of hitherto undiscovered processes not contained in the standard model. Thus, an accurate experimental evaluation of the CKM matrix unitarity provides an independent check of possible deviations from the SM. As the best studied element of the CKM matrix, V_{ud} plays an important role in all tests of its unitarity. However, evaluations of V_{ud} from neutron decay have, for the most part, not been consistent with results from nuclear SF decays [11]. Clearly, a precise evaluation of V_{ud} from pion beta decay, the theoretically cleanest choice, is of interest.

The most precise measurement of the pion beta decay rate on record was made by McFarlane *et al.*, by detecting in-flight π^+ decays [12]. This work reported $\Gamma_{\pi\beta} = 0.394 \pm 0.015\ \text{s}^{-1}$, which is an order of magnitude less precise than the theoretical description of the same process. Hence, we initiated the PIBETA experiment, a program of precise measurements of the rare pion and muon decays, chief among them being the pion beta decay, at the Paul Scherrer Institute (PSI), Switzerland [13].

In this Letter, we present an analysis of the $\pi^+ \to \pi^0 e^+ \nu$ decay events recorded with the PIBETA apparatus from 1999 to 2001. As Depommier *et al.* before us [14], we have chosen to detect pion decays at rest. We tuned the

πE1 beam line at PSI to deliver $\sim 10^6\ \pi^+/\mathrm{s}$ with $p_\pi \simeq 113\ \mathrm{MeV}/c$. The pions were slowed in an active degrader detector and stopped in a segmented nine-element active target, both made of plastic scintillator. The major detector systems are shown in a schematic drawing in Fig. 1. Energetic charged decay products are tracked in a pair of thin concentric multiwire proportional chambers and a thin 20-segment plastic-scintillator veto (PV) barrel detector. Both neutral and charged particles deposit most (or all) of their energy in a spherical electromagnetic shower calorimeter consisting of 240 elements made of pure CsI. The entire detector system, its response to positrons, photons, and protons, energy and time resolution, signal definitions, along with other relevant details of our experimental method, are described at length in Ref. [15].

The measurement relies on detecting the $\pi^0 \to \gamma\gamma$ decay that immediately follows a pion beta decay event. The two photons are emitted nearly back to back, with about 67 MeV each. Thus, the experiment is set to record all large-energy (above the $\mu \to e\nu\bar{\nu}$ end point) electromagnetic shower pairs occurring in opposing detector hemispheres during an $\sim$180 ns long "pion gate," πG (nonprompt two-arm events). The πG is timed so as to include a sample of pileup events preceding the pion stop. In addition, we record a large prescaled sample of nonprompt single shower (one-arm) events. Using these minimum-bias sets, we extract the π_β and π_{e2} event sets, the latter for branching ratio normalization. In a stopped pion experiment these two channels have nearly the same detector acceptance and share many common systematic uncertainties.

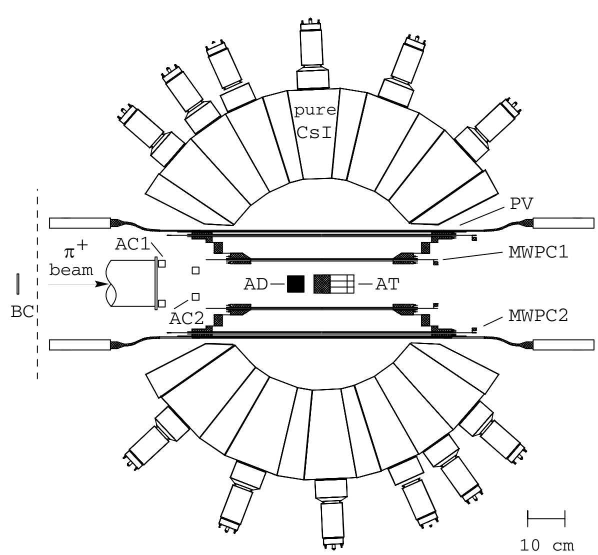

FIG. 1. A schematic cross section of the PIBETA detector system. BC: thin upstream beam counter; AC1 and AC2: active beam collimators; AD: active degrader; AT: active target; MWPC1 and MWPC2: thin cylindrical wire chambers; PV: thin 20-segment plastic-scintillator barrel. BC, AC1, AC2, AD, and AT detectors are also made of plastic scintillator.

A full complement of 12 fast analog triggers comprising all relevant logic combinations of one- or two-arm, low- or high calorimeter threshold (labeled LT and HT, respectively), prompt and delayed (with respect to π^+ stop time), as well as a random and a three-arm trigger, were implemented in order to obtain maximally comprehensive and unbiased data samples.

Signal definition and accurate counting of the π_{e2} events for normalization present a major challenge in this work. As in all previous studies, our π_{e2} data include undiscriminated soft-photon $\pi_{e2\gamma}$ events. Because of positron energy straggling in the target, accidental coincidences of multiple muon-decay events, and the calorimeter energy resolution function, the π_{e2} events are superimposed on a non-negligible muon-decay background. This background was removed by fitting the measured e^+ timing spectra with the functions for pion decay (signal), muon decay (background), plus the associated pileups (see Fig. 2, top panel). We also extracted the absolute π_{e2} branching ratio using this method and normalizing to the number of pion stops in the target. The results were in agreement with the recommended Particle Data Group (PDG) value [11] at a subpercent level, with the precision limited by the systematic uncertainties of counting the stopped pions. The latter is absent in our determination of $R_{\pi\beta}$. The π_{e2} energy spectrum after background subtraction is given in Fig. 2, bottom panel. The statistical uncertainty of the extracted number of π_{e2} events, $N_{\pi e2}$, is negligible.

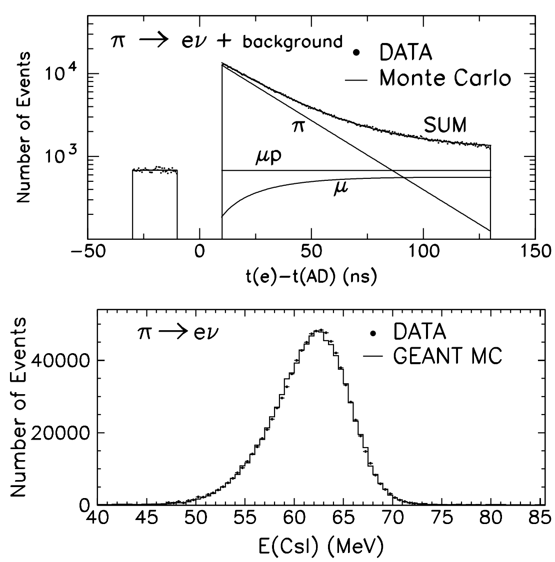

FIG. 2. Top panel: A typical histogram (dots) of differences between $t(e)$, one-arm HT event time, and t(AD), beam pion stop time, compared with a sum of the Monte Carlo–simulated responses for π_{e2} decay (π), muon decay (μ), and muon pileup events (μp). The π_{e2} pileup background, being much lower, is off scale in the plot. Prompt events are suppressed. Bottom panel: CsI calorimeter energy spectrum for the π_{e2} decay events, after background subtraction.

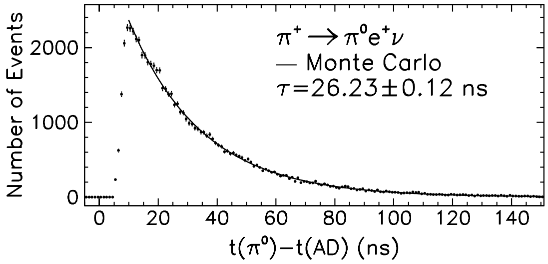

FIG. 3. Histogram of time differences between the beam pion stop and the π_β decay events (dots); curve: pion lifetime.

The π_β signal definition was more straightforward, as seen in Figs. 3 and 4, which show the pion decay time spectrum and γ-γ relative timing histogram, respectively, for π_β events, both free of backgrounds. Finally, the histogram of recorded γ-γ opening angles for pion beta events, shown in Fig. 5, provides a sensitive test of the accuracy of our reconstruction of the spatial distribution of beam pion stops, an important contributor to the acceptance uncertainty.

The π_β branching ratio $R_{\pi\beta}$ was evaluated from

$$R_{\pi\beta} = \frac{N_{\pi\beta}}{N_{\pi^+} f_{\pi G} A^{\rm HT}_{\pi\beta} \tau_1 f_{\rm CPP} f_D f_{\rm ph}}, \tag{3}$$

where $N_{\pi\beta}$ is the number of detected π_β events corrected for the number $N^{\rm accid}_{\pi\beta}$ of accidental background events, N_{π^+} is the number of the decaying π^+'s, $f_{\pi G}$ is the delayed pion gate fraction, $A^{\rm HT}_{\pi\beta}$ is the high-threshold detector acceptance evaluated by GEANT simulation, τ_1 is the detector live time, $f_{\rm CPP}$ is the correction due to the charged particle veto system pileup, $f_D = R_{\pi^0\to\gamma\gamma}$ is the π^0 Dalitz decay correction, and $f_{\rm ph}$ is the photonuclear absorption correction.

The $\pi \to e\nu$ branching ratio $R_{\pi e2}$ is given by

$$R_{\pi e2} = \frac{N_{\pi e2} p_{\pi e2}}{N_{\pi^+} f_{\pi G} A^{\rm HT}_{\pi e2} \tau_1 \epsilon_{\rm PV} \epsilon_{\rm C1} \epsilon_{\rm C2}}, \tag{4}$$

where $p_{\pi e2} = N^{\rm tot}_{\pi e2}/N_{\pi e2}$ is the prescaling factor applied to π_{e2} triggers, $A^{\rm HT}_{\pi e2}$ is the HT detector acceptance for $\pi \to e\nu$ decay events, including radiative corrections, while $\epsilon_{\rm PV}$, $\epsilon_{\rm C1}$, and $\epsilon_{\rm C2}$ denote the plastic veto and wire chamber efficiencies, respectively. Clearly, taking the ratio $R_{\pi\beta}/R_{\pi e2}$ leads to cancellations of many common factors, apart from small corrections taking into account slight differences in thresholds, trigger timing (two arm versus one arm), weighting of the efficiencies, and similar effects. Most importantly, N_{π^+}, the number of stopped pions, drops out. The main sources of uncertainty are listed with their values in Table I.

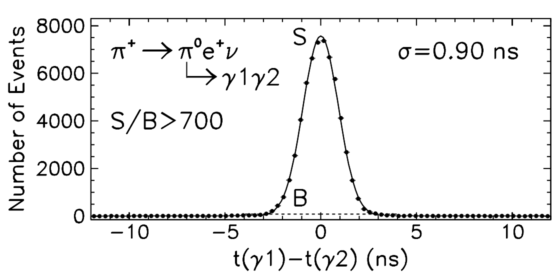

FIG. 4. Histogram of γ-γ time differences for π_β decay events (dots); curve: fit with a Gaussian function plus a constant.

As the external systematic uncertainties are self-explanatory, we focus on the internal ones. The systematic uncertainty in $N_{\pi e2}$ comes from the muon-decay background subtraction discussed above, and reflects the propagated error limits of the method. The precision of $A^{\rm HT}_{\pi\beta}/A^{\rm HT}_{\pi e2}$ is dominated by the uncertainty of the x-y-z distribution of pion stops in the target. The latter was determined with better than 50 μm accuracy by tomographic backtracing of π_{e2} and muon-decay positrons into the target [16]. Corrections due to the undetected low portions of the e and γ energy spectra in the calorimeter (the energy "tail") contribute weakly to the acceptance uncertainty due to strong correlations between the energy responses to the two decay channels. This experiment has a unique advantage over its predecessors: it measures branching ratios as well as differential angular and energy distributions of decay products for all rare pion and muon decays simultaneously. This provides multiple redundant consistency checks of the evaluated and simulated acceptances (cf., e.g., Ref. [17]). In the present analysis the largest internal contribution to the systematic uncertainty comes from the ratio of gate fractions, $r_{\pi G}$, due to our decision to include even the earliest π_β decay events, thus maximizing the useful event statistics. The inherent resolution in the zero time point is excellent—it relies on the prompt $A(\pi^+, \pi^0)B$ signal and the accelerator rf pulse, providing timing calibration at the level of $\sim$20 ps or better, and room for further improvement of the $r_{\pi G}$ precision. The pileup correction $f_{\rm CPP}$ was evaluated using a random trigger, and confirmed by simulations. We modified our GEANT3 code to calculate the photonuclear correction $f_{\rm ph}$, and conservatively assigned it a 50% uncertainty (details are given in Ref. [18]). Efficiencies $\epsilon_{\rm PV}$, $\epsilon_{\rm C1}$, and $\epsilon_{\rm C2}$, not listed in Table I, were measured with an accuracy of 0.01% [16].

Using the above method and the PDG 2004 recommended value of $R^{\rm exp}_{\pi e2} = 1.230(4) \times 10^{-4}$ [11], we extract

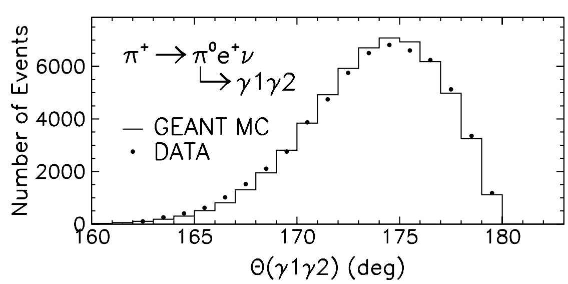

FIG. 5. Histogram of the γ-γ opening angle in π_β decay.

TABLE I. Summary of the main sources of uncertainty $\Delta R_{\pi\beta}$ in the extraction of the $\pi\beta$ branching ratio, given in % (see text for discussion).

Uncertainty type	Quantity	Value	$\Delta R_{\pi\beta}$ (%)
External	$R^{\rm exp}_{\pi e2}$	1.230×10^{-4}	0.33
	$R^{\rm exp}_{\pi^0 \to \gamma\gamma}$	0.9880	0.03
	π^+ lifetime	26.033 ns	0.02
Combined external			0.33
Internal	$N^{\rm tot}_{\pi e2}$ (syst)	6.779×10^8	0.19
	$A^{\rm HT}_{\pi\beta}/A^{\rm HT}_{\pi e2}$	0.9432	0.12
	$r_{\pi \rm G} = f^{\pi\beta}_{\pi \rm G}/f^{\pi e2}_{\pi \rm G}$	1.130	0.26
	$N^{\rm accid}_{\pi\beta}$	0	<0.1
	$f_{\rm CPP}$ correction	0.9951	0.10
	$f_{\rm ph}$ correction	0.9980	0.10
Combined internal			0.38
Statistical	$N_{\pi\beta}$	64 047	0.395

our main result, the pion beta decay branching ratio:

$$R^{\rm exp}_{\pi\beta} = [1.036 \pm 0.004({\rm stat}) \pm 0.005({\rm syst})] \times 10^{-8}, \quad (5)$$

or, in terms of the decay rate,

$$\Gamma^{\rm exp}_{\pi\beta} = [0.3980 \pm 0.0015({\rm stat}) \pm 0.0019({\rm syst})]\ {\rm s}^{-1}, \quad (6)$$

which represents a sixfold improvement in accuracy over the previous measurement [12]. Alternatively, the normalization can be tied to the theoretical value $R^{\rm theor}_{\pi e2} = (1.2352 \pm 0.0005) \times 10^{-4}$ [19], which would increase the extracted $R^{\rm exp}_{\pi\beta}$ by 0.42% to 1.040×10^{-8}. In a direct evaluation of the pion beta decay branching ratio using Eq. (3), i.e., normalizing to the number of beam pion stops, we obtain $R_{\pi\beta} \times 10^8 = 1.042 \pm 0.004({\rm stat}) \pm 0.010({\rm syst})$, consistent with our main result given in Eq. (5).

Whether scaled to the experimental or theoretical $R_{\pi e2}$, our result for $R^{\rm exp}_{\pi\beta}$ is in excellent agreement with predictions of the SM and CVC given the PDG recommended value range for V_{ud} [11],

$$R^{\rm SM}_{\pi\beta} = (1.038 - 1.041) \times 10^{-8} \quad (90\%{\rm C.L.}), \quad (7)$$

and represents the most accurate test of CVC and Cabibbo universality in a meson to date. Our result confirms the validity of the radiative corrections for the process at the level of $4\sigma_{\rm exp}$, since, excluding loop corrections, the SM would predict $R^{\rm no\ rad.\ corr.}_{\pi\beta} = (1.005 - 1.007) \times 10^{-8}$ at 90% C.L.

Using our result, Eq. (5), we can calculate a new value of V_{ud} from pion beta decay, $V^{\rm (PIBETA)}_{ud} = 0.9728(30)$, which is in excellent agreement with the PDG 2004 average, $V^{\rm (PDG\,'04)}_{ud} = 0.9738(5)$. We will continue to improve the accuracy of the π_β decay branching ratio by further refining the experiment simulation and analysis and by adding new data.

We thank W. A. Stephens, Z. Hochman, and the PSI experimental support group for invaluable help in preparing and running the experiment. This work has been supported by the U.S. National Science Foundation, the U.S. Department of Energy, the Paul Scherrer Institute, and the Russian Foundation for Basic Research.

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