Phantom Energy Mediates a Long-Range Repulsive Force

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Scalar field models with nonstandard kinetic terms have been proposed in the context of k inflation, of Born-Infeld Lagrangians, of phantom energy and, more in general, of low-energy string theory. In general, scalar fields are expected to couple to matter inducing a new interaction. In this Letter I derive the cosmological perturbation equations and the Yukawa correction to gravity for such general models. I find three interesting results: first, when the field behaves as phantom energy (equation of state less than -1), then the coupling strength is negative, inducing a long-range repulsive force; second, the dark-energy field might cluster on astrophysical scales; third, applying the formalism to a Brans-Dicke theory with a general kinetic term it is shown that its Newtonian effects depend on a single parameter that generalizes the Brans-Dicke constant.

There has been in recent years a renewal of interest in scenarios that propose alternatives or corrections to Einstein's gravity. These proposals are rather heterogeneous: they find their roots in multidimensional theories in which gravity propagates in more than three dimensions [1], in scalar fields predicted in superstrings that mediate long-range interactions [2], and in various mechanisms that might give cosmologically observable effects [3] and violate the equivalence principle [4] or Newton's inverse square law (see, e.g., [5]). Moreover, the cosmological observations of dark energy [6] can be explained in terms of an almost massless scalar field that, if coupled to matter, would again modify gravity [3,4,7–9]. Although the scale of the dark-energy interaction is expected to be of cosmological size, observable effects in clusters and galaxies [10] have been investigated.

Along with the increase of interest in the theory, new experimental tests of gravity have been performed (see the reviews in Ref. [11] and recent results in [12]). Most laboratory experiments interpret the results in terms of a possible Yukawa addition to the gravitational potential

$$V(r) = -G\frac{m_1 m_2}{r}[1 + \alpha e^{-r/\lambda_s}]. \tag{1}$$

This form is the static limit of interactions from exchange of bosons of mass $m_s = \lambda_s^{-1}$ (in Planck units); the interaction strength α is proportional to the square of the couplings. In view of the results below, it is important to remark that for spin-0 particles with a standard Lagrangian the strength α is always positive. A negative amplitude α (i.e., a repulsive effect) can be obtained only via the exchange of vector bosons [11] or in models with finite size of gravitons [5]. The sign of α gives therefore an important indication of the underlying physics.

The aim of this Letter is to evaluate the Yukawa correction to gravity when the scalar field ϕ has a nonstandard kinetic term, i.e., when the action is generalized as follows ($c = 8\pi G = 1$, signature $- + + +$):

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + p[X, U(\phi)]\right] + S_m(\rho_m, e^{2\omega(\phi)}g_{\mu\nu}),$$

where $X = -\phi_{,\mu}\phi^{,\mu}/2$, $U(\phi)$ is any function of ϕ, and where field and matter are coupled in the action S_m through the scalar-tensor metric $e^{2\omega(\phi)}g_{\mu\nu}$; we are working therefore in the Einstein frame (the subscript m denotes quantities that refer to matter). With the same formalism we derive the equations of cosmological perturbations. In the standard case, $p = X - U(\phi)$. Then the functions α and λ_s in the Yukawa correction are [13]

$$\alpha = 2[d\omega(\phi)/d\phi]^2 \equiv 2\omega_{,\phi}^2,$$
$$\lambda_s = m_s^{-1} = (d^2U/d\phi^2)^{-1/2} \equiv (U_{,\phi\phi})^{-1/2} \tag{2}$$

(there is an extra factor of 2 in α with respect to [13] because they assume $2X$ as the kinetic term).

Higher-order kinetic terms appear quite generically in superstrings (see, e.g., [14,15]). Moreover, they have been used in cosmology to model an accelerated phase without potential, the so-called k inflation or k essence [16,17]. General kinetic terms can also reproduce the behavior of "phantom energy," i.e., a field with an equation of state $w = p/\rho < -1$, which is realized with a negative kinetic term [18]. Finally, they have been introduced also in the form of the Born-Infeld Lagrangian $L_{\rm BI} = -U(\phi)(1 - 2M^{-4}X)^{1/2}$ [19]. Writing the energy-momentum tensor as

$$T_{\mu\nu}^{(\phi)} = g_{\mu\nu}p + p'\phi_{,\mu}\phi_{,\nu}, \tag{3}$$

where $p' = \partial p/\partial X$, one can readily identify the pressure with p and the energy density with [16] $\rho = 2Xp' - p$. We find it convenient to write the potential as $U = U_0 e^{-\sqrt{2/3}f(\phi)\phi}$, so that $U_{,\phi} = -\sqrt{2/3}f_1(\phi)U$ and the nth derivative

$$U_{,\phi}^{(n)} = (-1)^n(2/3)^{n/2}f_n U, \tag{4}$$

where $f_0 = 1$ and $f_n = f_{n-1}f_1 - (df_{n-1}/d\phi)/(\sqrt{\frac{2}{3}})$. All

general expressions below [i.e., those in which we do not assume a specific form of $p(X, U)$] could be simplified by a redefinition of the field $\phi^* \equiv f(\phi)\phi$ such that the potential becomes exponential; in practice this amounts to setting $f = \mu = $ const and $f_n = \mu^n$. The Klein-Gordon equation of motion in a flat Friedmann-Robertson-Walker metric with scale factor a written in the e-folding time $\eta = \log a$ is

$$x' + \left(3 + \frac{H'}{H}\right)x - 18Ax^3H^2p^{2,0}$$
$$+ Ay^2f_1\mu(p^{0,1} - 6x^2H^2p^{1,1}) = A\beta\Omega_m, \quad (5)$$

where we introduced the variables $x^2 = \phi'^2/6$ and $y^2 = U/3H^2$, and where $A = (p^{1,0} + 6H^2x^2p^{2,0})^{-1}$. (Note that x^2, y^2 are the kinetic and potential energy density fractions in the standard case.) We denote with $p^{i,j}$ the derivative $\partial^{i+j}p(X, U)/\partial^i X\partial^j U$ (but we also use the shorthand $p' = \partial p/\partial X$, $p'' = \partial^2 p/\partial X^2$; in all other cases the prime denotes $d/d\eta$). The standard case is recovered when $p^{1,0} = 1$, $p^{0,1} = -1$, and $p^{i,j} = 0$ for any $i + j > 1$. To conform to the previous notation (e.g., [20]), we defined instead of α the coupling $\beta = (3/2)^{1/2}\omega_{,\phi}$. The Einstein equations are

$$1 = \frac{6H^2x^2p' - p}{3H^2} + \Omega_m, \quad \frac{H'}{H} = -\frac{3}{2}\Omega_m - 3x^2p'. \quad (6)$$

Notice that $X = 3H^2x^2$. The conservation equation for matter is the same as in the standard coupled case (e.g., [20]). From the definitions of p and ρ we define the field sound speed [16]

$$c_s^2 = \frac{\partial p/\partial X}{\partial \rho/\partial X} = \frac{p'}{p' + 6H^2x^2p''} = Ap' \quad (7)$$

and the equation of state

$$w = \frac{p}{\rho} = -1 + \frac{2x^2p'}{1 - \Omega_m}. \quad (8)$$

Phantom dark energy, $w < -1$, requires therefore $p' < 0$. For instance, if $\Omega_m \approx 0.3$ and $w \approx -1.3$ [6] then $p' \approx -0.1/x^2$.

We wish to keep the scalar Lagrangian as general as possible. However, it is instructive to derive the conditions of classical and quantum stability. The classical stability is ensured by the condition $\rho > 0$ and $c_s^2 > 0$, the quantum stability by the positivity of the Hamiltonian, i.e., [15] $A > 0$, $p' > 0$, and $p^{0,2} < 0$. Altogether, the conditions are $p < \frac{p'}{2X}$, $p' > 0$, $p'' > -\frac{p'}{2X}$, and $p^{0,2} < 0$.

Generally speaking, the scalar field can couple to matter with different couplings [4]. The observational constraints on the baryon coupling are rather tight [21], while those on dark matter are much weaker and depend on the cosmic evolution itself [22]. We assume in this Letter a simplified model containing only one species of pressureless matter.

To derive the weak field limit, we use the longitudinal gauge metric in $\eta = \log a$:

$$ds^2 = -(1 + 2\Psi)\frac{d\eta^2}{H^2} + e^{2\eta}(1 - 2\Phi)dx_i dx^i. \quad (9)$$

Moreover, we redefine $\delta\phi = \sqrt{6}\varphi$ and introduce the matter density contrast $\delta_m = \delta\rho_m/\rho_m$, the dimensionless matter velocity divergence $\theta_m = \nabla_i v_{(m)i}/H$, and the field density contrast

$$\Omega_\phi\delta_\phi \equiv \frac{\delta\rho}{3H^2}$$
$$= 2\varphi y^2f_1\mu(p^{0,1} - 6x^2H^2p^{1,1}) - \frac{2x^2}{A}\Phi + \frac{2x}{A}\varphi'.$$

In the absence of anisotropic stress $\Phi = \Psi$. The metric perturbation in Fourier space with wavelength $\lambda = Ha/k$ (not to be confused with λ_s) is therefore

$$\Phi = -\frac{3}{2}\lambda^2[(\delta_m + 3\lambda^2\theta_m)\Omega_m + \delta_\phi\Omega_\phi + 6x\varphi p^{1,0}].$$

From now on we assume $\beta = $ const although the generalization to $\beta(\phi)$ is straightforward (see, for instance, [20]). The matter equations are

$$\delta_m' = -\theta_m + 3\Phi' - 2\beta\varphi', \quad (10)$$

$$\theta_m' = -\left(2 + \frac{H'}{H} - 2\beta x\right)\theta_m + \lambda^{-2}(\Phi - 2\beta\varphi). \quad (11)$$

Let us now go to the Newtonian limit of small scales, i.e., $k \rightarrow \infty$ or $\lambda \rightarrow 0$. At the first nontrivial order the perturbed scalar field equation is

$$\varphi'' + F(\phi)\varphi' + \hat{m}(\phi)^2\varphi + c_s(\phi)^2\lambda^{-2}\varphi = A\beta\Omega_m\delta_m. \quad (12)$$

The function F reduces to $(3 + H'/H)$ in the standard case and in the static limit $x \rightarrow 0$; its general expression depends on higher-order derivatives, up to $p^{3,0}$ but is very complicated and unuseful for what follows (detailed calculations will be published elsewhere). The dimensionless effective mass $\hat{m}$ in units of H^{-1} is given by

$$A^{-1}\hat{m}^2 = 2y^2p^{0,1}(3ABf_1^2H^2y^2 - f_2) + 3x^2p^{1,0}(3p^{1,0}$$
$$+ A^{-1}) - 6f_1^2H^2y^4p^{0,2} + 18ABH^2xy^2f_1p^{1,0}$$
$$- 6H^2y^2[6ABf_1^2H^2x^2y^2p^{1,1} + ABf_1\beta\Omega_m$$
$$+ xp^{1,1}(3f_1 - 2f_2x) - 6f_1^2H^2x^2y^2p^{1,2}], \quad (13)$$

where $B = p^{1,1} + 6H^2x^2p^{2,1}$. Consider now Eq. (12) in the absence of matter, $\Omega_m = 0$, i.e., for a self-gravitating boson fluid. For a standard kinetic term, $c_s = 1$ and $\hat{m}^2 = U_{,\phi\phi}/H^2 - 12x^2$. The effective mass is of order unity or less if the field is identified as dark energy (since $x^2 \leq 1$ from the constraints on w, see Eq. (8), and U is expected to be slow rolling with respect to the expansion rate); therefore, the growth of the scalar field fluctuations is prevented at all subhorizon scales. Scalar field clustering is of course possible for large and negative potentials or rapid oscillations [23], adding internal degrees of freedom [24] and, in general, at near-horizon scales [25]. In the

general kinetic case, however, different values and signs of c_s and $\hat{m}$ are possible. If $\hat{m}^2$ is negative, then the field fluctuations grow at scales (in units of H^{-1}) $\lambda > \hat{\lambda}_s = |c_s/\hat{m}|$; if $c_s^2 < 0$, the instability is at scales smaller than $\hat{\lambda}_s$; finally, if both are negative, then the instability is at all subhorizon scales.

Let us restrict now our attention to the case $\hat{m}^2$, $c_s^2 > 0$. Then the solution of the homogeneous equation associated to (12) is a fast oscillation around zero; the amplitude of φ is driven by the right-hand side forcing and is given by the solution of

$$\hat{m}(\phi)^2 \varphi + c_s(\phi)^2 \lambda^{-2} \varphi = A\beta\Omega_m \delta_m; \qquad (14)$$

that is,

$$\varphi = \frac{A\beta\Omega_m \delta_m a^2 H^2}{\hat{m}^2 a^2 H^2 + c_s^2 k^2} = \frac{\beta}{p'} \frac{k^2 \lambda^2 \Omega_m \delta_m}{a^2(\frac{\hat{m}^2 H^2}{c_s^2}) + k^2}. \qquad (15)$$

From the equation for θ'_m and from (15) we see that the potential acting on matter is the Newton-Yukawa potential $\Phi^* = \Phi - 2\beta\varphi$ that, upon Fourier transform, obeys the Poisson equation

$$\nabla_i \nabla^i \Phi^* = -\tfrac{3}{2} H^2 \Omega_m \delta_m (1 + \tilde{\alpha} e^{-r/\tilde{\lambda}}), \qquad (16)$$

where

$$\tilde{\alpha} = \frac{4}{3} \frac{\beta^2}{p'} = \frac{\alpha}{p'}, \qquad (17)$$

$$\tilde{\lambda} = \frac{\hat{\lambda}_s}{H} = \frac{c_s}{\hat{m} H}. \qquad (18)$$

These expressions generalize Eqs. (2) and are the main result of this Letter. From $\tilde{\alpha}$ we see that a phantom field, $p' < 0$, coupled to matter mediates a *repulsive* force, contrary to bosons with standard kinetic terms whose interaction is always attractive. The force is long range if $\hat{m} \leq c_s$: this depends on the particular Lagrangian but for a field that drives the cosmic acceleration this is what one expects. In general, the repulsive scalar gravity may balance and overcome gravity at small scales. From the general expressions for $\tilde{\alpha}$ and $\tilde{\lambda}$ we derive now two limits that we can call the *laboratory* limit (i.e., the static interaction) and the *cosmological* limit (i.e., a massless or "dark-energy" field). Finally, we apply the results to the Brans-Dicke model.

Laboratory limit.—We specify the Yukawa correction to the case in which the background solution is static, $\phi'' = \phi' = 0$, i.e., $x \rightarrow 0$ (assuming that p does not diverge, e.g., that it does not contain inverse powers of X). Then we have $c_s = 1$ and

$$\hat{m}^2 = \frac{\hat{m}_s^2}{p^{1,0}} g(p),$$

$$g \equiv 3\frac{H^2 f_1}{f_2}\left[y^2 f_1 \left(\frac{p^{1,1} p^{0,1}}{p^{1,0}} - p^{0,2} \right) - \frac{\beta\Omega_m p^{1,1}}{p^{1,0}} \right] - p^{0,1}, \qquad (19)$$

where $\hat{m}_s^2 = 2y^2 f_2 = U_{,\phi\phi}/H^2$ is the *dimensionless* standard mass, and therefore

$$\tilde{\lambda}^{-2} = \frac{U_{,\phi\phi}}{p'} g(p). \qquad (20)$$

In general, $g(p)$ can be given any value. For instance, if

$$p(X, U) = P_1(X) + UP_2(X/U), \qquad (21)$$

where $P_{1,2}$ are any function for which $P_{1,2}(0) = 0$, then $g = 0$ (i.e., an infinite-range force) in the static limit. If instead the Lagrangian p is in the form $p = K(X) - U$ (additive Lagrangian), then $g = 1$ and $\tilde{\lambda} = \sqrt{p'}/m_s$. It follows that the effect of the generalized kinetic term on the Yukawa correction can be neatly absorbed into the two Yukawa parameters α, λ as $\tilde{\alpha} = \alpha/p'$ and $\tilde{\lambda}^2 = \lambda_s^2/p'$. A phantom field with an additive Lagrangian and small kinetic energy x therefore has an imaginary effective mass, unless of course $U_{,\phi\phi}$ is negative as well. As already remarked this implies an instability of the dark-energy fluctuations (whether or not the field is coupled). It may be conjectured that if such an instability leads to nonlinearity at large scales before the final singularity ("big rip"), then the singularity itself might be avoided.

Cosmological limit.—If the field ϕ is a dark-energy field its effective mass $\hat{m}$ is expected to be negligible. Although in the general case this is not necessarily true, we adopt now this simplification. Then from (15) we have

$$\varphi = \frac{\beta}{p'} \lambda^2 \Omega_m \delta_m. \qquad (22)$$

From this we see that the Yukawa correction to Newton's constant is $G = G_N(1 + \frac{4}{3}\frac{\beta^2}{p'})$. Derivating δ'_m and inserting (22) we obtain finally

$$\delta''_m + \delta'_m\left(2 + \frac{H'}{H} - 2x\beta\right) - \frac{3}{2}\Omega_m \delta_m\left(1 + \frac{4}{3}\frac{\beta^2}{p'}\right) = 0. \qquad (23)$$

This equation generalizes the similar one given in Ref. [20] and shows how the kinetic term enters the growth of the matter perturbations. Although for simplicity it has been derived for $\beta = \text{const}$ it is actually valid also for $\beta = \beta(\phi)$. The solution depends of course on the cosmological history and on the particular form of the field Lagrangian. This will be studied in subsequent work. Here we notice only a few interesting points. First, as already pointed out, a phantom field has $p' < 0$ and therefore its effect on δ_m counteracts gravity. This could be used to set stringent limits on phantom fields coupled to dark matter. Second, since in general p' evolves in time, then also the term β^2/p' is time dependent, even if the coupling β itself is constant. In particular, if p contains higher-order terms in X then the effective coupling β^2/p' will change when the kinetic terms become subdominant, i.e., when the acceleration sets is. This might introduce novel features as, e.g., a strong non-Gaussianity in the galaxy distribution [26].

Brans-Dicke model.—As an application of the results let us now consider a generalization of the Brans-Dicke

Lagrangian,

$$L = \frac{\sqrt{g}}{2}\left[\Phi R + \frac{2\omega_{\mathrm{BD}}}{\Phi}P(Y)\right], \qquad (24)$$

where P is any function of $Y = -g^{\mu\nu}\Phi_{;\mu}\Phi_{;\nu}/2$ and where ω_{BD} is the Brans-Dicke constant [a nonconstant ω_{BD} can be absorbed in $P(Y)$]. The Brans-Dicke scalar field Φ should not be confused with the gravitational potential introduced in Eq. (9). Let us call this model kinetic Brans-Dicke (kBD). Under the conformal transformation $\tilde{g}_{\mu\nu} = e^{2\omega}g_{\mu\nu}$ the Lagrangian assumes the canonical form with

$$p(X, U) = \frac{X}{3 + 2\omega_{\mathrm{BD}}}\left[3 + 2\omega_{\mathrm{BD}}\frac{U}{X}P\left(\frac{X}{U}\right)\right], \qquad (25)$$

if $\Phi = e^{2\omega}$ and if we define $X = -\omega_{;\mu}\omega^{;\mu}(3 + 2\omega_{\mathrm{BD}})$, $U = (3 + 2\omega_{\mathrm{BD}})e^{-6\omega}/2$, and $\omega(\phi) = \phi[2(3 + 2\omega_{\mathrm{BD}})]^{-1/2}$. It follows finally from Eq. (17) that

$$\tilde{\alpha} = \frac{2\omega_{,\phi}^2}{p^{1,0}} = \frac{1}{3 + 2\omega_{\mathrm{BD}}P_{,Y}}, \qquad (26)$$

where $P_{,Y} = dP/dY$. Moreover, since $p(X, U)$ has the form (21) then $\tilde{m} = 0$ in the static limit [if $P(Y = 0) = 0$, for instance, $P = \sum_n a_n Y^n$, $n > 0$]. This shows that the Newtonian effects of the kinetic Brans-Dicke theory are parametrized uniquely by $\omega_{\mathrm{kBD}} = \omega_{\mathrm{BD}}P_{,Y}$ and, therefore, all the experimental constraints on ω_{BD} in the literature [4] can be read equivalently as constraints on ω_{kBD}. A order of unity ω_{BD} is then acceptable if $P_{,Y}$ is very large.

Conclusions.—A scalar-tensor metric $e^{2\omega(\phi)}g_{\mu\nu}$ is arguably the simplest, oldest, most studied, and most motivated extension of Einstein's gravity. So far, however, only a standard kinetic term has been employed in scalar-tensor theories (see, e.g., [3,4]). On the other hand, a generalized kinetic term has been proposed to account for a phantom equation of state and kinetic-driven inflation and on theoretical grounds. This Letter lies at the crossroad of these two streams: it investigates a kinetic scalar-tensor gravity, focusing on the properties that do not depend on the particular model. Perhaps the most interesting result is that a field with equation of state $w < -1$ (a phantom field) mediates a repulsive force, contrary to ordinary spin-0 bosons. We also derived the field effective mass in the general case and showed that it may induce clustering of dark energy. We also have shown that the Newtonian effects of a kinetic Brans-Dicke model are parametrized by a single quantity, $\omega_{\mathrm{kBD}} = \omega_{\mathrm{BD}}P_{,Y}$, that generalizes the Brans-Dicke constant.

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