Quantum Cloning of a Coherent Light State into an Atomic Quantum Memory

Jaromír Fiurášek,[1,2] Nicolas J. Cerf,[1] and Eugene S. Polzik[3]

[1]QUIC, Ecole Polytechnique, CP 165, Université Libre de Bruxelles, 1050 Bruxelles, Belgium
[2]Department of Optics, Palacký University, 17. listopadu 50, 77200 Olomouc, Czech Republic
[3]Niels Bohr Institute, Danish Quantum Optics Center–QUANTOP, Copenhagen University,
Blegdamsvej 17, Copenhagen Ø, Denmark

A scheme for the optimal Gaussian cloning of coherent light states at the interface between light and atoms is proposed. The distinct feature of this proposal is that the clones are stored in an atomic quantum memory, which is important for applications in quantum communication. The atomic quantum cloning machine requires only a single passage of the light pulse through the atomic ensembles followed by the measurement of a light quadrature and an appropriate feedback, which renders the protocol experimentally feasible. An alternative protocol, where one of the clones is carried by the outgoing light pulse, is discussed in connection with eavesdropping on quantum key distribution.

Quantum information processing with continuous variables provides an interesting alternative to the traditional qubit-based approach. Continuous variables (CV) seem to be particularly suitable for quantum communication applications, as, for example, quantum teleportation [1] or quantum key distribution (QKD) [2]. Another important feature of CV is the feasibility of the light-atoms quantum interface [3,4], which unlike its qubit analogue does not require strongly coupled cavity QED regime for deterministic operations. Along these lines, the prospect of developing a quantum memory for light with macroscopic atomic ensembles has been explored [5–8]. Such a quantum memory is crucial for applications such as quantum repeaters or quantum secret sharing.

In this Letter, we show that the optimal Gaussian cloning [9,10] of a coherent state of a traveling light beam can be achieved via its off-resonant interaction with atomic ensembles. In the envisaged experiment, the light beam interacts with two atomic ensembles A and B (see Fig. 1). The resulting (approximate) clones are stored in the quantum states of the collective atomic spins of clouds A and B; we thus achieve cloning into an atomic quantum memory. This is fairly distinct from the all-optical setup for CV quantum cloning based on the use of a parametric optical amplifier [11,12]. A variation of our approach allows one of the clones to be stored in the atomic cloud, the second one being carried by the outgoing light pulse. This feature makes this second scheme particularly attractive for eavesdropping on the QKD schemes utilizing coherent states [2]. An eavesdropper, Eve, who intercepts the quantum signal, may keep one clone in the atomic memory and wait until the receiver announces the measurement basis (x or p quadrature) before performing the corresponding measurement on her clone. Note that such Gaussian cloning attacks are the optimal finite-size attacks on a certain class of QKD schemes with coherent states [13].

The off-resonant interaction of light with an atomic ensemble can be described by the effective unitary evolution operator $U = \exp(-iaS_z J_z)$, where S_z denotes the z component of the Stokes operator S describing the polarization state of light, and J_z stands for the z component of the collective atomic spin operator J [6,14]. The elements of the vectors S and J satisfy the standard angular momentum commutation relations, $[S_j, S_k] = i\epsilon_{jkl}S_l$ and $[J_j, J_k] = i\epsilon_{jkl}J_l$. The effective coupling strength a depends on the details of the level structure of the atoms, the detuning between light and the atomic transition, and the geometry of the experiment [15].

Consider the situation where the light beam contains a strong coherent component linearly polarized in the x direction and, similarly, the atomic spins are polarized along the x axis. In this case, the mean values of S_x and J_x attain macroscopic values $\langle S_x \rangle \approx N_L/2$ and $\langle J_x \rangle \approx FN_A$, where N_L and N_A denote the number of photons in the light beam and the number of atoms in the ensemble, respectively, and F is the total atomic spin quantum
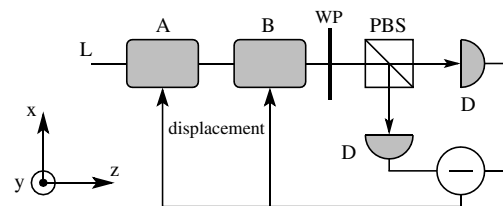

FIG. 1. Setup for the CV cloning of light into two atomic quantum memories. A light beam L having a strong coherent component polarized along the x axis propagates along the z axis and passes through two atomic ensembles A and B polarized in the x direction. The z component of the Stokes vector of the output light beam is measured using wave plates (WPs), polarizing beam splitter (PBS) and two photodetectors (D), and the atomic states are then displaced accordingly. The clones are stored in the atomic ensembles A and B.

number. This implies that we may approximate the operators S_x and J_x by their mean values. It follows from the commutation relations for S and J that the properly rescaled y and z components of S and J satisfy the canonical commutation relations for two conjugate quadrature operators x and p, namely $[x, p] = i$. We can thus introduce the effective quadrature operators for light and atoms: $x_L = S_y/\sqrt{\langle S_x \rangle}$, $p_L = S_z/\sqrt{\langle S_x \rangle}$, $x_A = J_y/\sqrt{\langle J_x \rangle}$, and $p_A = J_z/\sqrt{\langle J_x \rangle}$. Note that x_L and p_L can be interpreted as the quadratures of the optical mode linearly polarized in the y direction. This correspondence forms the basis for the implementation of CV quantum information processing using the off-resonant coupling between light and atoms. If we rewrite the unitary transformation U in terms of quadrature operators, we get

$$U = \exp(-i\kappa p_L p_A), \qquad (1)$$

where $\kappa = a\sqrt{FN_A N_L/2}$. Since the effective Hamiltonian generating U is quadratic, i.e., $H = p_L p_A$, U is a linear canonical transformation of the quadrature operators. By applying local phase shifts to the light and atoms (i.e., by rotating the vectors S and J) we may modify the effective Hamiltonian to $H = x_L p_A$ or $H = p_L x_A$. The rotation of the polarization of light is easily performed by sending the light pulse through wave plates. The polarization of the atomic cloud can be rotated by applying strongly detuned classical laser pulses [6,15].

Two-pass atomic quantum cloning.—Consider the effective unitary transformation $U = \exp(-i\kappa x_L p_A)$. In the Heisenberg picture, the quadratures evolve as

$$
\begin{aligned}
x_L^{\text{out}} &= x_L^{\text{in}}, & x_A^{\text{out}} &= x_A^{\text{in}} + \kappa x_L^{\text{in}}, \\
p_L^{\text{out}} &= p_L^{\text{in}} - \kappa p_A^{\text{in}}, & p_A^{\text{out}} &= p_A^{\text{in}}.
\end{aligned}
\qquad (2)
$$

Specifically, when $\kappa = 1$, we obtain the continuous-variable controlled-NOT (C-NOT) gate [16], $|x\rangle_L |y\rangle_A \rightarrow |x\rangle_L |y + x\rangle_A$, where $|x\rangle$ and $|y\rangle$ represent the eigenstates of the x quadratures. The mode L is the control mode while A is the target mode. Since the CV C-NOT is not its own inverse, we also introduce C-NOT†, which is obtained by choosing $\kappa = -1$ in Eq. (2). As pointed out in [9], it is possible to construct the optimal Gaussian cloning machine with a quantum network made of four such CV C-NOT gates if some particular ancillary state can be prepared.

The cloning network that we propose is depicted in Fig. 2(a). The three relevant modes labeled L, A, and B correspond, respectively, to the light beam and the two atomic ensembles (see Fig. 1). The atomic ensembles are initially prepared in the vacuum state and the light beam carries the coherent state to be cloned. The cloning can be divided into two steps. First, the information about the x quadrature is transferred from the light into the atomic samples by applying two C-NOT gates where the light is the control mode and the atomic modes are the targets. After this first passage of light, the quadratures evolve as
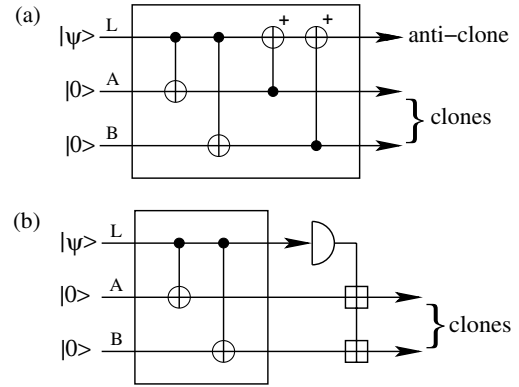
FIG. 2. (a) Network for optimal symmetric Gaussian cloning of coherent states. The lines indicate the C-NOT gates, ● labels control mode and ⊕ indicates target mode. (b) Simplified cloning network where two C-NOT gates are replaced by the measurement of the p_L quadrature followed by displacements of the atomic p quadratures, as indicated by the open boxes.

$$
\begin{aligned}
x_A' &= x_A^{\text{in}} + x_L^{\text{in}}, & p_A' &= p_A^{\text{in}}, \\
x_B' &= x_B^{\text{in}} + x_L^{\text{in}}, & p_B' &= p_B^{\text{in}}, \\
x_L' &= x_L^{\text{in}}, & p_L' &= p_L^{\text{in}} - p_A^{\text{in}} - p_B^{\text{in}}.
\end{aligned}
\qquad (3)
$$

In the second step, the information about the p_L quadrature is transmitted. This is accomplished by two C-NOT† gates where now the atomic modes play the role of the controls and the light is the target (the reverse information transfer of p from light to the atomic samples works by back action). The output quadratures can thus be expressed in terms of the input ones as

$$
\begin{aligned}
x_A^{\text{out}} &= x_L^{\text{in}} + x_A^{\text{in}}, & p_A^{\text{out}} &= p_L^{\text{in}} - p_B^{\text{in}}, \\
x_B^{\text{out}} &= x_L^{\text{in}} + x_B^{\text{in}}, & p_B^{\text{out}} &= p_L^{\text{in}} - p_A^{\text{in}}, \\
x_L^{\text{out}} &= -x_L^{\text{in}} - x_A^{\text{in}} - x_B^{\text{in}}, & p_L^{\text{out}} &= p_L^{\text{in}} - p_A^{\text{in}} - p_B^{\text{in}},
\end{aligned}
\qquad (4)
$$

which is the desired optimal Gaussian cloning transformation. To illustrate this, consider the state of a single clone, say A. For input coherent state $|\alpha\rangle_L$ in the optical mode L, the atomic mode A ends up in a mixed Gaussian state, namely, the input coherent state with superimposed thermal noise. The coherent component of clone A is equal to α, which guarantees that all coherent states are cloned with the same fidelity $\mathcal{F}$. The latter is related to the mean number $\bar{n}$ of thermal photons in A by the formula $\mathcal{F} = 1/(\bar{n} + 1)$ [12]. Equation (4) implies $\bar{n} = 1/2$ so that we obtain $\mathcal{F} = 2/3$, which is the maximal fidelity achievable by Gaussian cloning machines [17]. Hence, the proposed cloning procedure is optimal.

Single-pass atomic quantum cloning.—Let us now consider the practical realization of this procedure as illustrated in Fig. 1. The transformation (3) is accomplished by the passage of the light pulse through both atomic samples A and B with the polarization settings chosen such that the effective coupling between the light and

atomic samples is described by the Hamiltonian $H_1 = x_L(p_A + p_B)$. In practice, the pulse is very long compared to the distance between the atomic samples, so the light interacts simultaneously with both samples A and B.

The realization of the last two C-NOT† gates, however, brings complications. The light pulse should pass for a second time through the atomic samples, and, before this second passage, the polarization of the light and the atoms should be rotated to switch the effective coupling to $H_2 = -p_L(x_A + x_B)$. This may be very difficult to accomplish because, in the current experiments, the pulse is several hundred kilometers long. The leading part of the pulse, which already traveled through the atomic ensembles, would have to be stored until the tail of the pulse also passes through the atoms. Only then could the pulse be fed back into the input. Luckily, this technically challenging procedure can be avoided because the last two C-NOT† gates can be replaced by a measurement of the z component of the Stokes vector of light followed by some appropriate displacement of the atomic quadratures. The latter task can be accomplished by a tiny rotation of the polarization state of the collective atomic spin [6,15].

This crucial simplification renders our proposal experimentally feasible. The simplified cloning procedure is illustrated in Fig. 2(b). The measurement of S_z is equivalent to the measurement of p'_L. Once the classical measurement outcome p_L^{cl} is known, one has to displace the atomic p quadratures of ensembles A and B as follows:

$$p'_A \rightarrow p'_A + p_L^{cl}, \qquad p'_B \rightarrow p'_B + p_L^{cl}. \qquad (5)$$

It is immediate to see that, after displacement, the resulting quadratures of the atomic modes are equal to those given in Eq. (4), hence the optimal cloning is achieved with a single passage of light.

Cloning into atoms and light.—The above protocol produces two clones stored in two atomic memories A and B. If cloning is used as an eavesdropping attack, then Eve would like to store her clone in the memory while the second clone should be sent as a light pulse down the communication line. One option would be to transfer one of the clones from the atomic memory back to light. Such a procedure, however, would require either strong entanglement [6] or several passages of the light pulse through the atomic ensemble [3,18], which is currently infeasible. Instead, one can use the scheme depicted in Fig. 2(b) and replace the second atomic memory with a light beam B. The required C-NOT interaction between the two light beams L and B could be realized with the use of a nondegenerate optical parametric amplifier placed in between two unbalanced beam splitters [19]. After the measurement of the quadrature p'_L, the light quadrature p_B should be displaced similarly as in the CV teleportation experiments [1].

An even simpler scheme is shown in Fig. 3. The light beam L passes through a single atomic ensemble A ($H = x_L p_A$, $\kappa = 1$) and then impinges on a balanced beam

splitter BS whose second input port is in the vacuum state. Note that $\langle S_{x,L}^{out}\rangle = \langle S_{x,B}^{out}\rangle = \langle S_{x,L}^{in}\rangle/2$ and all output quadratures are defined as properly normalized y and z components of the Stokes vectors such that canonical commutation relations are satisfied. The quadrature p_L of the output beam L is measured by the homodyne detector HD, and the quadratures p_A and p_B are displaced according to

$$p_A \rightarrow p_A + \sqrt{2}p_L, \qquad p_B \rightarrow p_B + p_L. \qquad (6)$$

It is easy to show that the resulting output quadratures of the modes A and B can be expressed in terms of the input quadratures as

$$x_A^{out} = x_L^{in} + x_A^{in}, \qquad x_B^{out} = \frac{1}{\sqrt{2}}(x_L^{in} + x_B^{in}),$$
$$p_A^{out} = p_L^{in} - p_B^{in}, \qquad p_B^{out} = \sqrt{2}(p_L^{in} - p_A^{in}). \qquad (7)$$

The atomic memory A contains one clone, while the light beam B contains the other clone squeezed in the x quadrature by a factor of $\sqrt{2}$. To restore the optimal clone in the light mode B one would have to "unsqueeze" this beam using a phase-sensitive (degenerate) optical parametric amplifier with squeezing factor $\sqrt{2}$.

Asymmetric cloning.—Let us now demonstrate how to make the cloning machine asymmetric. This is particularly interesting in the context of quantum cryptography where it enables Eve to choose a trade-off between the quality of her copy (hence, the information she can extract from it) and the unavoidable noise that is added to the copy sent to the receiver. We pursue an approach inspired from [9], which relies on the "preprocessing" of the initial states of the atomic modes. Suppose that the atomic modes A and B are both initially prepared in a pure single-mode squeezed vacuum state. Mode A is squeezed in the x quadrature and mode B is squeezed in the p quadrature, and the squeezed variance is V in both cases. The scheme depicted in Fig. 2(b) then produces two asymmetric clones whose fidelities read $\mathcal{F}_A = 1/(1 + V)$ and $\mathcal{F}_B = 4V/(4V + 1)$, in accordance with [12]. This confirms that the device indeed realizes the optimal asymmetric Gaussian cloning of coherent states.

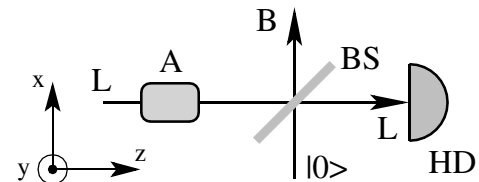The atomic ensembles can be squeezed using the procedure that was employed to entangle two

FIG. 3. Setup that produces one clone stored in an atomic quantum memory A while the second "flying" clone is carried by the light beam B. BS is a balanced beam splitter and HD denotes homodyne detector. The clone in the light beam B is squeezed in the x quadrature by a factor of $\sqrt{2}$.

atomic ensembles [4,15]. Coherent light pulses are used to perform quantum nondemolition measurements of two commuting nonlocal atomic quadratures $x_A + p_B$ and $x_A - p_B$. If the interaction strength is $\kappa = \sqrt{1/(4V) - 1/2}$, this projects the atoms onto squeezed states with squeezed variance V and a coherent component determined by the measurement outcome. By applying a properly chosen displacement, we can set the coherent component to zero, which results in a deterministic preparation of a pure single-mode squeezed vacuum state of ensembles A and B. Note that this procedure requires only a single passage of each light pulse through the atomic ensembles.

Experimental feasibility.—The feasibility of the experimental realization of atomic quantum cloning is basically determined by the relative easiness of fulfilling the condition on the coupling constant $\kappa = a\sqrt{FN_A N_L/2} = 1$. For room temperature atomic vapors, this value has been already achieved and even surpassed in [4]. For cold atomic vapor, the value of $\kappa = 1$ is also well within reach. The coupling constant can be rewritten as $\kappa = (\sigma\gamma/A\delta)\sqrt{FN_A N_L/2}$ with $\sigma = \lambda^2/2\pi$, γ—natural width of the transition and δ—detuning of the light from the transition. It is important to reduce the probability η of the spontaneous emission, caused by the probe, so that this source of decoherence does not affect the cloning protocol. It has been shown [20] that spontaneous emission can be neglected if $\eta \ll 1/(1 + \kappa^2)$. This condition translates into a usual—for the free space coupling protocols—condition on the optical density of the atomic sample, $\alpha \gg 1$, since $\kappa^2 = \alpha\eta$. This condition can be easily met with dense cold atomic samples.

Ground magnetic or hyperfine states of alkali atoms are good candidates for an atomic memory for cloning. For gas samples in paraffin coated cells at room temperature, entanglement lifetime of up to 1 msec has been demonstrated in [4]. An important source of decoherence in this system is the collisional dephasing which limits the coherence life time of atoms in a cell to about 10 msec. The storage time can be increased by using ensembles of cold trapped atoms whose ground state coherence time can be of the order of 0.1 sec or even higher. The life time of atomic clones can, in principle, be of the same order.

The last stage of the envisaged experiment is the probing of the quantum state of the atomic ensembles, in order to determine the experimentally attained cloning fidelity. In the geometry shown in Fig. 1, the pulse propagating along z axis unavoidably interacts with both ensembles A and B. It is nevertheless possible to address the ensembles individually by shining the light from a perpendicular direction, along the y axis. In this way one can measure the statistics of arbitrary quadrature and determine the quadrature variances or even perform the full tomographic reconstruction of the quantum state of the clones. Alternatively, atomic spins can be rotated between the measurements by applying pulses of magnetic field.

In summary, we have proposed an experimentally feasible method for preparing long-lived atomic or atom-light clones for continuous quantum variables of light. The protocols described here can be used in various quantum communication protocols, e.g., for the optimal eavesdropping of a quantum key distribution scheme.

Note added.—The recent experimental demonstration of a quantum memory for light [21] uses similar tools, and hence provides evidence that the proposal is feasible.

[1] A. Furusawa *et al.*, Science **282**, 706 (1998).
[2] F. Grosshans *et al.*, Nature (London) **421**, 238 (2003).
[3] A. Kuzmich and E. S. Polzik, in *Quantum Information with Continuous Variables*, edited by S. L. Braunstein and A. K. Pati (Kluwer Academic, Dordrecht, 2003), pp. 231–265.
[4] B. Julsgaard, A. Kozhekin, and E. S. Polzik, Nature (London) **413**, 400 (2001).
[5] J. Hald *et al.*, Phys. Rev. Lett. **83**, 1319 (1999).
[6] A. Kuzmich and E. S. Polzik, Phys. Rev. Lett. **85**, 5639 (2000).
[7] C. Schori *et al.*, Phys. Rev. Lett. **89**, 057903 (2002);B. Julsgaard *et al.*, Quantum Inf. Comput. **3**, 518 (2003).
[8] A. E. Kozhekin, K. Mølmer, and E. S. Polzik, Phys. Rev. A **62**, 033809 (2000).
[9] N. J. Cerf, A. Ipe, and X. Rottenberg, Phys. Rev. Lett. **85**, 1754 (2000).
[10] N. J. Cerf, in *Quantum Information with Continuous Variables*, edited by S. L. Braunstein and A. K. Pati (Kluwer Academic, Dordrecht, 2003), pp. 277–293.
[11] S. L. Braunstein *et al.*, Phys. Rev. Lett. **86**, 4938 (2001).
[12] J. Fiurášek, Phys. Rev. Lett. **86**, 4942 (2001).
[13] F. Grosshans and N. J. Cerf, Phys. Rev. Lett. **92**, 047905 (2004).
[14] A. Kuzmich, N. P. Bigelow, and L. Mandel, Europhys. Lett. **42**, 481 (1998).
[15] L. M. Duan *et al.*, Phys. Rev. Lett. **85**, 5643 (2000).
[16] S. L. Braunstein, Phys. Rev. Lett. **80**, 4084 (1998).
[17] N. J. Cerf and S. Iblisdir, Phys. Rev. A **62**, 040301(R) (2000).
[18] J. Fiurášek, Phys. Rev. A **68**, 022304 (2003).
[19] S. F. Pereira, Z. Y. Ou, and H. J. Kimble, Phys. Rev. Lett. **72**, 214 (1994); K. Bencheikh, J. A. Levenson, Ph. Grangier, and O. Lopez, Phys. Rev. Lett. **75**, 3422 (1995).
[20] K. Hammerer *et al.*, quant-ph/0312156.
[21] B. Julsgaard *et al.*, quant-ph/0410072 [Nature (London) (to be published)].