Limit to Minimally Destructive Optical Detection of Atoms

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All optical techniques used to probe the properties of Bose-Einstein condensates have been based on dispersion and absorption that can be described by a two-level atom. Both phenomena lead to spontaneous emission that is destructive at the low energies involved with ultracold atomic systems. Recently, both were shown to lead to the same limit to the signal to noise ratio for a given destruction. We develop a new method for calculating the phase shift of a laser beam and show that no single-pass optical technique using classical light and a three-level atom can exceed this limit. This puts significant restrictions on potential nondestructive measurement schemes.

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Introduction.—The advent of modern cooling techniques has led to the creation of ultracold atomic samples in which the recoil of a single photon has a significant effect on the motional state of the system. Laser cooling, and more recently evaporative cooling, have allowed the creation of a Bose-Einstein condensate (BEC) of weakly interacting gases in which a large number of atoms enter the ground state of the system, forming a large, coherent matter wave [1]. Observation and control of the motional states of these atoms requires a detection method that does not involve spontaneous photon recoil.

Previously all ground state Bose-Einstein condensates have been detected via optical methods, with photon absorption providing a simple, though clearly destructive, measure of the atomic density and the phase shift of a laser beam providing a less destructive measure under some circumstances [2]. Both methods are based on physics that can be described by the two-level atom. It was recently shown in the limit of optically thin samples, that absorption and phase shift measurements have equal sensitivity for a given level of destruction, and that the signal to noise ratio (SNR) in this limit is a function of destruction (spontaneous emission rate) and bandwidth only [3,4]. This defines a hard limit on the SNR achievable from any single-pass technique based on the two-level atom and classical light beams. Using resonant interferometry improves the SNR by a factor of the square root of the finesse of the system [3]. Using squeezed light improves the SNR by a factor of the squeezing. These methods enhance the SNR of the measurement [5], but are so technically challenging that they would only be worth pursuing if there were no better method. This Letter addresses the question: *Can a large phase shift be generated without a correspondingly high spontaneous emission rate?* If so, the detection limit for a two-level atom can be avoided.

It is known that correctly prepared three-level systems in the lambda configuration allow a weak probe beam to experience a nonzero phase shift without any absorption, suggesting that manipulation of coherences in a three-level system might provide a less destructive detection method [6,7]. In this Letter, we show that this is not the case, and that no other arbitrarily nondestructive detection can be generated with a three-level system.

The two-level limit.—The SNR for a purely shot-noise limited measurement of a phase $\Delta\phi$ is limited by the temporal and spatial bandwidth, detector efficiency η and the strength of the electric field in the interferometer:

$$\mathrm{SNR} = \sqrt{\frac{\eta P}{B\hbar\omega}}|\Delta\phi| \tag{1}$$

where P is the power in the laser, ω is the frequency of the laser, and B is the temporal bandwidth of the measurement [3]. Even reaching this shot-noise limit can be experimentally challenging, and it can only be improved by using nonclassical light sources or resonant interferometry. In the far off-resonant limit the phase shift imparted on a laser beam by an atomic cloud is equal to

$$\Delta\phi = -\frac{\tilde{n}\sigma\gamma}{4\Delta} \tag{2}$$

where $\tilde{n}$ is the column density of the atomic cloud, $\sigma = 6\pi/k^2$ is the single atom cross section, γ is the spontaneous emission rate of the excited state and Δ is the detuning from resonance [2]. In this limit the off-diagonal element of the atomic density matrix ρ_{eg} is given by $\mathrm{Re}\{\rho_{eg}\} = -\Omega/(2\Delta)$, giving:

$$\Delta\phi = \frac{\tilde{n}\sigma\gamma}{2\Omega}\mathrm{Re}\{\rho_{eg}\} \tag{3}$$

where Ω is the Rabi frequency of the laser field. This second version is in fact true in general, as we will show in Eq. (17) later in this Letter. In this off-resonance limit, Eqs. (1) and (3) combine to give a SNR that contains only atomic parameters, and the square root of the excited state population, $P_e = \frac{\Omega^2}{4\Delta^2}$:

$$\mathrm{SNR} = \frac{\tilde{n}}{2}\sqrt{\frac{\eta A\sigma\gamma P_e}{B}} \tag{4}$$

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where A is the area of the atomic cloud that was sampled–essentially the spatial bandwidth of the measurement. As the excited state population drives the heating due to spontaneous emission, this unavoidable link between the sensitivity of the measurement and the excited state population produces a fundamental limit to nondestructive detection using this technique. The difficulty of gaining spatial information from cavity-based interferometers and using squeezed beams to improve interferometric measurements means that the most attractive way of beating this limit would be to find a way to obtain a greater phase shift for a given excited state population.

Detection with three levels.—When an excited state is coupled to two ground states by a single laser, and a coherence between those states can be generated, then it is possible to generate instantaneously a high phase shift with zero absorption of a probe beam (see, for example, Sec. 7.5 in Ref. [7]). The lack of absorption occurs due to a destructive interference between the two allowable transitions. Unfortunately, a finite phase shift on a laser with zero absorption of that laser does not necessarily mean that there is no excited state population. It only means that to first order the excited state population is not changed by the probe beam. In more complicated systems, a laser can have zero absorption while the atom has nonzero spontaneous emission by absorbing other beams. In terms of nondestructive optical detection, the relevant question to ask is not whether a finite phase shift can exist without absorption, but whether it can coexist without *excited state population.*

It is certainly possible to obtain any required coherence between the ground states in a three-level system without excited state population. One eigenstate of the system in the presence of two laser beams is a linear combination of only the two ground states. Atoms initially in one of the ground states can be moved into this dark state and back again by having the two lasers sweeping spatially or temporally across the atoms such that the initially unoccupied ground state and the excited state are coupled first and last. If the laser intensities change slowly enough, then by the adiabatic theorem, the system will remain in the dark state, and the atoms will be left undisturbed. This proposal certainly satisfies the nondestructive criterion of detection, but as a detection scheme it has one critical flaw. The phase shift on both laser beams can be shown to be zero.

We will first examine the phase shift using standard methods. When the response of the atoms is linear in the electric field, the phase shift and absorption of the field is directly related to the real and imaginary parts of the susceptibility χ, as the refractive index is given by $n^2 = 1 + \chi$. Here we use the refractive index and susceptibility normalized to the effect of a single atom. The real part of the refractive index is in turn simply related to the phase shift experienced by the probe laser:

$$\Delta\phi = [\mathrm{Re}(n) - 1]\omega\tilde{n}/c \tag{5}$$

where $\tilde{n}$ is the column density of the sample, and is required here to include the effect of all the atoms in the laser beam. The susceptibility can be written in terms of the field strength and the atomic coherences [7]:

$$\chi = \frac{2}{\epsilon_0 \mathcal{E}}(d_{ba}\rho_{ab} + d_{ca}\rho_{ac}) \tag{6}$$

where a is the excited state, b and c are the ground states, d_{ij} is the dipole moment between states i and j, $\mathcal{E}$ is the electric field, and ρ is the atomic density matrix in the rotating frame. This equation can be combined with Eq. (5) to show that

$$\begin{aligned}|\Delta\phi| &\le \frac{|\chi|\omega\tilde{n}}{2c} \le \frac{\tilde{n}\sigma\gamma_{ab}}{2\Omega_{ba}}|\rho_{ab}| + \frac{\tilde{n}\sigma\gamma_{ac}}{2\Omega_{ca}}|\rho_{ac}| \\ &\le \frac{\tilde{n}\sigma\gamma_{ab}}{2\Omega_{ba}}\sqrt{\rho_{aa}}\sqrt{\rho_{bb}} + \frac{\tilde{n}\sigma\gamma_{ac}}{2\Omega_{ca}}\sqrt{\rho_{aa}}\sqrt{\rho_{cc}} \\ &\le \frac{\tilde{n}\sigma\gamma_{ab}\sqrt{P_e}}{2\Omega_{ba}} + \frac{\tilde{n}\sigma\gamma_{ac}\sqrt{P_e}}{2\Omega_{ca}}\end{aligned} \tag{7}$$

where in successive lines we have used Cauchy's inequality, the property of density matrices that $|\rho_{ij}|^2 \le \rho_{ii}\rho_{jj}$, and identified ρ_{aa} with the excited state population P_e for this system. We have used the normal notation for the Rabi frequencies of the laser. This relationship between an upper bound for the phase shift and the excited state population is the reason that this system cannot be used for nondestructive detection. The shot-noise limited measurement of this phase shift using Eq. (1) is

$$\mathrm{SNR} \le \frac{\tilde{n}}{2}\sqrt{\frac{\eta A\sigma P_e}{B}}(\sqrt{\gamma_{ab}} + \sqrt{\gamma_{ac}}) \tag{8}$$

This shows that the measurement cannot be made arbitrarily sensitive for a given spontaneous emission rate. The similarity of this equation to the two-level limit (4) is due to the fact that the phase shift is coming from the coherences between the excited state and the ground states, as seen in Eq. (6). When these two terms have equal and opposite imaginary part then there is no absorption of the beam, but in order to have no excited state they must each be identically zero, as ρ_{aj} must be zero for any state j if $\rho_{aa} = 0$. Thus, if there is no excited state the susceptibility must be zero, and there is no phase shift on the laser beam. It is easy to generalize this result to produce an equivalent limit for any system where the response of the system to the optical field can be written in terms of a linear susceptibility.

Nonperturbative phase shift in multilevel systems.—The above analysis is not the last word on nondestructive detection with these systems, as it requires the system response to be described by a linear susceptibility. This leaves the possibility that nondestructive detection can be achieved when there is a highly nonlinear susceptibility. It is possible to use a higher order of perturbation theory, for example, assuming that the polarization responds quadratically to the electric field, $\mathcal{P} = \epsilon_0(\chi\mathcal{E} + \chi_2\mathcal{E}^2)$,

but this approach leads to very complicated models, and is still not a fully general result.

By identifying the physical origin of the phase shift on a laser beam, we show that it is possible to calculate it for a nonperturbative system. Rather than using Maxwell's equations and assumptions about the polarizability of the atoms, we note that the phase shift of the laser is intimately related to the light shifts experienced by the atomic levels during their interaction [8]. This relationship is greatly simplified where we are only interested in atomic dynamics with low rates of spontaneous emission, as required for nondestructive detection. In this case, the evolution is trivial in terms of the eigenstates of the coherent part of the evolution. Although this is almost identical to a perturbative approximation for two-level systems, it is conceivable that multilevel systems may have nonlinear effects with negligible spontaneous emission. Our method would be less suitable for dealing with nonlinear effects that were accompanied by significant spontaneous emission, but these situations are clearly not candidates for nondestructive detection.

The interaction picture Hamiltonian for a laser coupled to a two-level atom, in the rotating-wave approximation, is

$$H_2 = \hbar\Delta|e\rangle\langle e| + g(\hat{a}^\dagger|g\rangle\langle e| + \text{adj.}) \tag{9}$$

where Δ is the detuning from resonance, $\hat{a}$ is the annihilation operator for the laser mode, $g = d\sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}$ is the coupling constant for the transition, ω is the laser frequency, d is the dipole moment, and V is the quantisation volume. This system couples the excited state with $n-1$ photons in the optical field to the ground state with n photons, forming a series of isolated submanifolds $\{|g, n\rangle, |e, n-1\rangle\}$. The laser must be far off-resonance to avoid high spontaneous emission rates. In this case the two eigenstates are a nearly pure excited state $|+_n\rangle \approx |e, n-1\rangle$ and a nearly pure ground state $|-_n\rangle \approx |g, n\rangle$. Thus, if an atom starts in the ground state $|\Psi_{\text{init}}\rangle = \sum_n c_n|g, n\rangle$, where c_n are the coefficients of the photon number modes, then a good approximation to the state after the atom has interacted with the laser is

$$\begin{aligned}|\Psi_{\text{final}}\rangle &= \sum_n c_n|g, n\rangle e^{-(i/\hbar)\Delta E_n l/c} \\ &= |g\rangle \otimes \left(\sum_n c_n|n\rangle e^{-(i/\hbar)\Delta E_n l/c}\right)\end{aligned} \tag{10}$$

where ΔE_n is the light shift of $|-_n\rangle$ and l/c is the interaction time. To compare with (2), we note that for large detuning the light shift is $\Delta E_n = -\frac{|g|^2 n}{\hbar\Delta}$. Substituting this into Eq. (10) and multiplying by the number of atoms in the quantisation volume, we find that:

$$\begin{aligned}|\Psi_{\text{total}}\rangle &= \sum_n c_n|g, n\rangle e^{[i(\tilde{n}\sigma\gamma/4\Delta)n]} \\ &= |g\rangle \otimes \left(\sum_n c_n|n\rangle e^{i\tilde{n}\sigma\gamma n/4\Delta}\right).\end{aligned} \tag{11}$$

This is exactly the form for a light field that has experienced a phase shift of

$$\Delta\phi = -\frac{\tilde{n}\sigma\gamma}{4\Delta}, \tag{12}$$

which is easily seen when the coefficients c_n correspond to those of a coherent state $|\alpha\rangle$, and the total state can be rewritten in the form $|\Psi_{\text{total}}\rangle = |g\rangle \otimes |\alpha e^{-i\Delta\phi}\rangle$. This result is identical to Eq. (2), but does not involve assumptions about the polarization of the atoms. The large detuning limit was used to compare the results with the previous method, but the only necessary assumption was that of negligible spontaneous emission, which is already required for nondestructive detection.

The interaction picture Hamiltonian for an excited atomic state $|e\rangle$ coupled by two lasers to two ground states $|g_1\rangle$ and $|g_2\rangle$, in the rotating-wave approximation, is

$$\begin{aligned}H_3 = \hbar\Delta|e\rangle\langle e| + \hbar\delta|g_2\rangle\langle g_2| + (g_1\hat{a}_1{}^\dagger|g_1\rangle\langle e| \\ + g_2\hat{a}_2{}^\dagger|g_2\rangle\langle e| + \text{adj.})\end{aligned} \tag{13}$$

where Δ is the detuning of the first laser from resonance, δ is the two-photon detuning, $\hat{a}_j$ is the annihilation operator for the jth laser mode, $g_j = d_j\sqrt{\frac{\hbar\omega_j}{2\epsilon_0 V}}$ is the coupling constant for the transition from $|e\rangle$ to $|g_j\rangle$, and d_j is the dipole moment of that transition. This system couples the states $|g_1, n, m\rangle$, $|e, n-1, m\rangle$ and $|g_2, n-1, m+1\rangle$, where the three labels are the atomic state and number of photons in each laser mode, respectively.

In the nontrivial case where both laser fields are nonzero, neither of the two ground states are approximately eigenstates. There is an eigenstate $|QD\rangle$ of the system that is close to a linear combination of the two ground states, and in principle the atoms can be moved from either ground state into this quasidark state in a reversible manner, as described above. The excited state component of this state is zero for two-photon resonance ($\delta = 0$), where it is truly a dark state, but the phase shift on each laser beam is also zero in this instance. We must therefore find the phase shift for finite values of δ in order to determine whether this system can produce a large phase shift for a given excited state population, and therefore be used to provide a more sensitive nondestructive detection method. As we will eventually show that this is impossible, it is not necessary to consider possible imperfections in the state preparation process that would only make our conclusion stronger.

The initial state of the system must be $|\Psi_{\text{init}}\rangle = \sum_{n,m} c_n d_m|QD, n, m\rangle$ if the spontaneous emission rate is going to be negligible, so the interaction of the light and the atoms must produce the state

$$|\Psi_{\text{final}}\rangle = \sum_{n,m} c_n d_m|QD, n, m\rangle e^{-(i/\hbar)\langle QD,n,m|H_3|QD,n,m\rangle l/c}. \tag{14}$$

This does not necessarily correspond to a phase shift of

the two laser beams and an overall phase rotation of the entire state, which would only be the case for $\frac{\langle QD,n,m|H_3|QD,n,m\rangle l}{\hbar c} = \phi_0 + \phi_1 n + \phi_2 m$. Any higher order function of the two-photon numbers would imply the existence of correlations between the beams. To the extent that a phase shift $\Delta\phi_j$ exists for the jth laser beam, it is therefore given by

$$\Delta\phi_j|_{\text{per atom}} = \frac{\partial\langle H_3\rangle}{\partial n_j}\frac{l}{\hbar c} \tag{15}$$

where n_j is the photon number of the jth beam, and the derivative is evaluated at the mean photon numbers for all the beams. The derivative of the dressed state eigenvalue can be found from first order perturbation theory using the Hellman-Feynman theorem [9]:

$$\begin{aligned}\Delta\phi_j|_{\text{per atom}} &= \frac{\partial\langle H_3\rangle}{\partial(g_j\sqrt{n_j})}\frac{g_j}{2\sqrt{n_j}}\frac{l}{\hbar c} \\ &= \langle(|g_j\rangle\langle e| + |e\rangle\langle g_j|)\rangle\frac{g_j l}{2\hbar c\sqrt{n_j}} \\ &= \frac{l\sigma_j\gamma_j}{2V\Omega_j}\mathrm{Re}\{\rho_{eg_j}\}\end{aligned} \tag{16}$$

where σ_j and γ_j are the cross section and spontaneous emission rate for the transition, and $\Omega_j = 2g_j\sqrt{n_j}/\hbar$ is the usual definition of the Rabi frequency. Multiplying by the total number of atoms in the quantisation volume, we find the total phase shift on the laser is given by

$$\Delta\phi_j = \frac{\tilde{n}\sigma_j\gamma_j}{2\Omega_j}\mathrm{Re}\{\rho_{eg_j}\} \tag{17}$$

This agrees completely with Eq. (3) for the two-level case, and generates the same sensitivity limit as shown in (4).

If only one laser is used on both transitions, as we described above using the susceptibility method, then we can follow this argument precisely, identifying the two field operators $\hat{a}_1$ and $\hat{a}_2$ in Eq. (13). By an identical argument, we find that

$$\begin{aligned}\Delta\phi|_{\text{per atom}} &= \frac{\partial\langle H_3\rangle}{\partial n}\frac{l}{\hbar c} \\ &= \sum_{j=1,2}\langle(|g_j\rangle\langle e| + |e\rangle\langle g_j|)\rangle\frac{g_j l}{2\hbar c\sqrt{n}} \\ &= \sum_{j=1,2}\frac{l\sigma\gamma_j}{2V\Omega_j}\mathrm{Re}\{\rho_{eg_j}\}.\end{aligned} \tag{18}$$

This leads directly to the upper bound for the phase shift shown in (7) and therefore the sensitivity limit shown in Eq. (8). The difference is that this result requires no approximations apart from the requirement that there is minimal spontaneous emission. Together, the limits (4) and (8) cover all possibilities for using a three-level scheme for nondestructive detection of atoms, so we can conclude that there are no advantages to such schemes over two-level detection techniques.

Conclusions.—For a two-level atom, the SNR for a quantum noise limited measurement of the column density, either via absorption or phase shift in the thin cloud limit, depends only on destruction (spontaneous emission rate) and bandwidth. We have shown that the use of any combination of coherences between levels in a three-level atom will not improve the SNR for such a measurement. Quasidark states can exhibit phase shifts that change very quickly with detuning, but any large phase shift on either laser is always associated with a large total excited state population and accompanying spontaneous emission. Although this result is restricted to a three-level atom, the proof can be generalized to any minimally destructive detection scheme based on any number of atomic levels interacting with any number of lasers, and this proof will appear in a forthcoming paper.

Although squeezed states of light or multipass interferometry are experimentally challenging for (at present) moderate gains in the SNR, they are the only ways we have found to improve on the single-pass limit imposed by the two-level atom using classical light. As a consequence, it is important that both techniques be developed. The only alternative is to investigate nonoptical detection. Sensitive cryogenic detectors such as SQUIDS make this an interesting possibility for any atomic species with nonzero spin in a cryogenic environment, such as atomic hydrogen.

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