Electron Correlation in the Second Landau Level: A Competition Between Many Nearly Degenerate Quantum Phases

J. S. Xia,[1,2] W. Pan,[3,2,*] C. L. Vicente,[1,2] E. D. Adams,[1,2] N. S. Sullivan,[1,2] H. L. Stormer,[4,5] D. C. Tsui,[3] L. N. Pfeiffer,[5] K.W. Baldwin,[5] and K.W. West[5]

[1]*University of Florida, Gainesville, Florida 32611 USA*
[2]*National High Magnetic Field Laboratory (NHMFL), Tallahassee, Florida 32310 USA*
[3]*Princeton University, Princeton, New Jersey 08544 USA*
[4]*Columbia University, New York, New York 10027 USA*
[5]*Bell Labs, Lucent Technologies, Murray Hill, New Jersey 07954 USA*
(Received 23 June 2004; published 22 October 2004)

At a very low-temperature of 9 mK, electrons in the second Landau level of an extremely high-mobility two-dimensional electron system exhibit a very complex electronic behavior. With a varying filling factor, quantum liquids of different origins compete with several insulating phases leading to an irregular pattern in the transport parameters. We observe a fully developed $\nu = 2 + 2/5$ state separated from the even-denominator $\nu = 2 + 1/2$ state by an insulating phase and a $\nu = 2 + 2/7$ and $\nu = 2 + 1/5$ state surrounded by such phases. A developing plateau at $\nu = 2 + 3/8$ points to the existence of other even-denominator states.

DOI: 10.1103/PhysRevLett.93.176809 PACS numbers: 73.43.Qt, 73.20.Qt, 73.63.Hs

Low-temperature electron correlation in the lowest Landau level (LL) of a two-dimensional electron system (2DES) separates largely into two regions. At very low filling factor $\nu \leq 1/5$ an insulating phase exists, which has now quite convincingly been determined to be a pinned electron solid [1]. At higher filling factor $1 > \nu > \sim 1/5$ the multiple sequences of fractional quantum Hall effect (FQHE) liquids [2] dominate, which show the characteristic vanishing magnetoresistance R_{xx} and quantized Hall resistance R_{xy} at many odd-denominator rational fractional fillings $\nu = p/q$ [3]. Altogether about 50 such FQHE states have been observed in this region. Their multiple sequences can largely be described within the composite fermion (CF) model [4], with the exact origin of some higher order states still being argued. The electrical behavior between FQHE states carries no particularly strong transport signature, being thought of as arising largely from the conduction of excited quasiparticles of the neighboring FQHE states.

At high LL's a very different pattern seems to emerge. There charge density wave (CDW) or liquid crystal like states dominate, often referred to as electronic stripe and bubble phases [5]. Characteristically these states are pinned to the lattice, immobilizing the electrons of this LL, which leads to transport properties identical to those of the neighboring integer quantum Hall effect (IQHE) states. FQHE states are absent in these high LL's, except for the recent observation of two FQHE features in the third LL, at elevated temperatures [6]. Of course, high LL fillings typically occur at lower magnetic fields and, hence, at poorer resolution of potential FQHE features. However, very general theoretical arguments [7,8] based on an increasing extent of the wave function with increasing LL index, hence the increasing importance of exchange and the diminishing applicability of pointlike interactions, clearly support this trend.

It is in the second LL where electron liquids and electron solids collide. The larger extent of the wave function as compared to the lowest LL and its additional zero allows for a much broader range of electron correlations to be favorable, leading to an ever changing competition between multiple electronic phases as the filling factor is varied and as the temperature is lowered.

An early example of the variety of electron correlations encountered in the second LL is the even-denominator $\nu = 2 + 1/2$ FQHE state [9,10]. From numerical calculations [11,12], supported by several experimental facts [13], there is good evidence that this $\nu = 5/2$ state consists of pairs of CF [14]. As a function of tilt—equivalent to a compression of the wave function perpendicular to the 2D plane—the FQHE phase gives way to a pinned, electrically anisotropic state as it appears in higher LL, thereby revealing the close balance between FQHE liquids and pinned CDW states in the second LL.

This delicate balance has been further highlighted through the observation of a reentrant integer quantum Hall effect (RIQHE) in the second LL [15]. At temperatures below 50 mK the Hall trace between the $\nu = 3$ and the $\nu = 4$ IQHE plateau does not follow the usual, almost classical, Hall line. Instead, at four regions between $\nu = 3$ and 4, it reverts to the level of the integer Hall plateaus; two to its upper value of $h/3e^2$ and two to its lower value of $h/4e^2$. The equivalent effect is seen in the lower spin state of the second LL. These observations still lack a definitive explanation; however, an interpretation in terms of a pinned solid (CDW or liquid crystal) phase appears inescapable [15–18]. These abrupt returns from a smooth Hall trace at higher temperatures to the IQHE at very low temperatures underscores the fragility of these phases. In the "windows" between these pinned electronic states, along the stretches of a steadily rising classical Hall line, so far only the $\nu = 3 + 1/5$ and $4 - 1/5$ FQHE (equivalently $\nu = 2 + 1/5$ and $3 - 1/5$) appeared, apart from the prominent even-denominator states at half fillings. These observations indicate that the FQHE liquids in the second LL have largely given way to insulating electronic many particle states.

Our recent data, presented in this Letter, point to a yet more complex state of affairs in the second LL. At very low temperatures of 9 mK in a very high-mobility 2DES specimen, we observe several new FQHE liquid states competing with the RIQHE states. In fact, while present at higher temperatures, some FQHE states seem to be destroyed by the RIQHE at lower temperatures. We observe well quantized FQHE states at $\nu = 2 + 1/3$ and $2 + 2/3$ in coexistence with the RIQHE and, quite importantly, a new, clearly quantized FQHE state at $\nu = 2 + 2/5$, the nature of such a state being theoretically intensely debated. Finally, there is evidence for the presence of a second even-denominator FQHE at a filling of $\nu = 2 + 3/8$ in our data whose origin is also enigmatic.

In our experiments we used a 30 nm wide quantum well that is delta-doped on both sides of the well at a setback distance of 100 nm. The electron density is $n = 3 \times 10^{11}\ \mathrm{cm}^{-2}$ and the mobility is $\mu = 31 \times 10^6\ \mathrm{cm}^2/Vs$. These values are established after illumination of the specimen with a red light emitting diode at low temperatures. The ultra-low-temperature experiments were carried out in a demagnetization-dilution refrigerator combination described in Ref. [10].

Figure 1 shows the diagonal resistance and the Hall resistance between filling factor $\nu = 2$ and 3, taken at the base temperature of $T = 9$ mK. At the extremes of the horizontal axis the standard quantized Hall plateaus in R_{xy} and vanishing resistance in R_{xx} are just visible. Between these field values, R_{xx} exhibits the typical spiky behavior seen in low-temperature measurements on 2DES. The Hall trace, however, is very unusual. Instead of moving monotonically and in a stairlike fashion from $h/3e^2$ to $h/2e^2$, R_{xy} along several stretches of filling factor returns to the value of the neighboring IQHE plateaus. These are the features of the RIQHE. In between the RIQHE we observe now several plateaus with fractional quantum number: (1) Fully developed FQHE states are observed at $\nu = 2 + 1/2$, $2 + 1/3$, and $2 + 2/3$, all having wide Hall plateaus quantized to better than one part in 10^4. (2) For the first time, a fully developed FQHE state at $\nu = 2 + 2/5$ is observed, showing a quantized Hall plateau and vanishingly small R_{xx}. (3) A new even-denominator FQHE state seems to be developing at $\nu = 2 + 3/8$ as deduced from a deep minimum in R_{xx} and a Hall plateau value within 0.2% of $R_{xy} = h/e^2/(2 + 3/8)$. (4) One of the RIQHE plateaus is split into two plateaus around $B \sim 5.7\ T$, corresponding to filling factor $\nu = 2 + 2/7$, where simultaneously a minimum occurs in R_{xx}. These features signal a complex competition between the FQHE state and the RIQHE state. (5) Finally, a strong $\nu = 2 + 4/5$ FQHE state is seen at $B \sim 4.6\ T$. At 9 mK, there is no FQHE at $\nu = 2 + 1/5$. Rather R_{xx} shows a peak. The rising Hall resistance at the B field just below $\nu = 2 + 1/5$ seems to indicate that another RIQHE is developing.

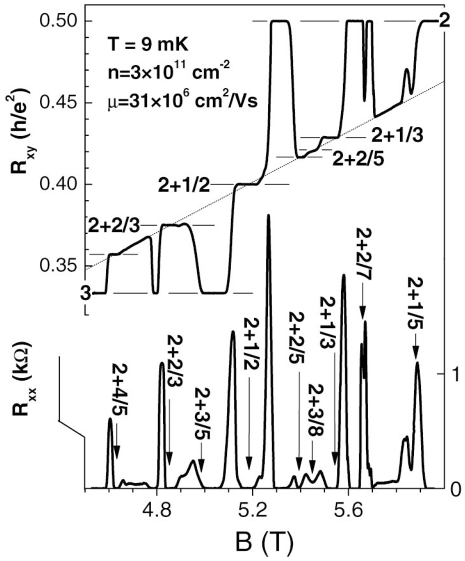

FIG. 1. R_{xx} and R_{xy} between $\nu = 2$ and $\nu = 3$ at 9 mK. Major FQHE states are marked by arrows. The horizontal lines show the expected Hall value of each QHE state. The dotted line is the calculated classical Hall resistance.

The fully developed $\nu = 2 + 2/5$ FQHE state is observed for the first time. At our lowest temperature of 9 mK the accuracy of the R_{xy} quantization is better than 0.02% (using R_{xy} at $\nu = 5/2$ as a reference) and R_{xx} reaches a low value of $\sim$5 ohm [Fig. 2(a)]. The formation of a true FQHE state is corroborated by the temperature dependence of dR_{xy}/dB which reaches zero at $\sim$10 mK, as shown in Fig. 2(c). The energy gap of this new state is $\sim$70 mK as determined from the T dependence of R_{xx} in Fig. 2(b).

The energy gap of the new $\nu = 2 + 2/5$ state is disproportionately small. In the lowest LL, the energy gap of the 2/5 state is about half of the gap at $\nu = 1/3$ [19], whereas in this second LL, the energy gap at 2/5 is only 1/10 of that at $\nu = 1/3$ ($\Delta_{7/3} \sim 0.6$ K). As to its origin, a first assumption would be the $\nu = 2 + 2/5$ state to be a hierarchical daughter state of the $\nu = 2 + 1/3$ state, as in the lowest LL [20]. However, Read and Rezayi [21] found, for coulombic interactions, vanishing overlap of such a hierarchical state with the exact ground state. This result is corroborated in a numerical study by Morf and d'Ambrumenil [22], which determined that no hierarchical state was stable between $\nu = 2 + 2/3$ and $2 + 1/3$. Also within the CF model the traditional $\nu = 2 + 2/5$ state is unstable as seen from the collapse of its neutral excitation in numerics [23]. Taken together, these studies

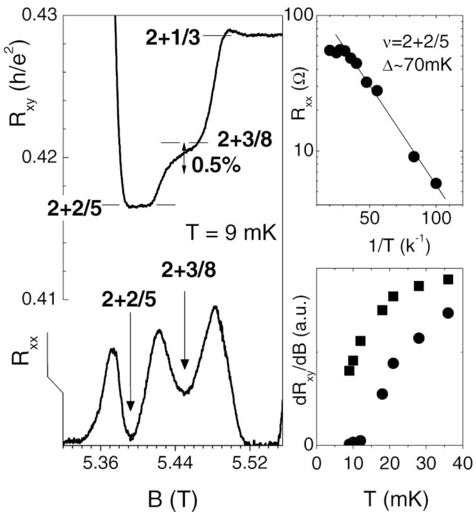

FIG. 2. (a) R_{xx} and R_{xy} around $\nu = 2 + 2/5$ and $\nu = 2 + 3/8$. The horizontal lines indicate expected values of the Hall plateaus of these two states. The vertical arrow shows the 0.5% deviation from the expected value for the $\nu = 2 + 3/8$ state. (b) Arrhenius plot for the R_{xx} minimum at $\nu = 2 + 2/5$. The line is a linear fit. (c) dR_{xy}/dB vs T for the $\nu = 2 + 2/5$ (solid circles) and $\nu = 2 + 3/8$ (solid squares) states.

strongly show that the physical origin of the $\nu = 2 + 2/5$ state remains unclear.

Several nonconventional ground state wave functions have been proposed for FQHE states in the second LL. Among them, the parafermioinc state of FQHE is the most exciting [21]. In this model, a number k (≤ 2) of electrons form clusters and it is the condensation of these clusters that gives rise to a FQHE state at $\nu = 2 + k/(k + 2)$. This model would explain the existence of the already observed FQHE states at $\nu = 2 + 2/3$ ($k = 4$) and $\nu = 2 + 1/3$, by particle-hole symmetry. It also predicts the next new FQHE state to occur at $\nu = 2 + 3/5$ and its particle-hole conjugate state at $\nu = 2 + 2/5$. Indeed, the $\nu = 2 + 2/5$ FQHE state is observed. However, there is no evidence of a FQHE state at $\nu = 2 + 3/5$. It is not clear whether this absence is due to a broken particle-hole symmetry, is related to the latter state residing at lower B field and, hence, being weaker, or is due to an asymmetry in the nearby RIQHE. Consistent with the parafermionic model, no other FQHE states occur between $\nu = 2 + 2/5$ and $2 + 1/2$. However, their absence could also be a result of the RIQHE being the true ground state in this regime.

In fact, the formation of k-electron clusters provides a compelling scenario to interpret the data in this filling factor range. Although the origin of RIQHE remains unclear, it is believed to arise from collective freezing of electron clusters [15–18]. Then the crossover from the $\nu = 2 + 2/5$ state to the RIQHE state may represent a quantum phase transition from an electron-cluster liquid to an electron-cluster solid, resembling the phase transition from electron liquid to electron solid in the lowest LL [1]. Numerical studies predict such a transition at $\nu \sim 2 + 0.37$ [17,18], which would be to the right of $\nu = 2 + 2/5$. Instead it occurs just to the left. This difference could well result from the finite thickness of 2DES's in real samples, which generally stabilizes the liquid states in the second LL [24], but is not included in the calculations.

Beyond the $\nu = 2 + 2/5$ FQHE state we clearly observe a new, developing even-denominator FQHE state at $\nu = 2 + 3/8$. As shown in Fig. 2(a), together with a deep minimum in R_{xx}, a Hall plateau is forming. It is centered at $R_{xy} = h/e^2/(2 + 3/8)$ within 0.2%. Moreover, dR_{xy}/dB at $\nu = 2 + 3/8$ decreases with decreasing temperature [Fig. 2(c)]. Extrapolating its temperature dependence, a flat Hall plateau is expected to form at an electron temperature of 2–3 mK.

At the present time, the physical origin of the $\nu = 2 + 3/8$ state remains unclear. The possibility of a FQHE state at $\nu = 3/8$ in the lowest LL was considered recently [25]. It will be interesting to see whether a similar mechanism could also be responsible for a $\nu = 2 + 3/8$ fraction in the second LL. On the other hand, the existence of the $\nu = 2 + 3/8$ state is not consistent with the parafermionic model, which would predict a FQHE at $\nu = 2 + 3/4$ ($k = 6$). However, the Hall resistance of Fig. 1 shows no such feature. Detailed numerical calculations will be required to resolve the origin of the $\nu = 2 + 3/8$ state.

In addition, the $\nu = 2 + 2/7$ and $\nu = 2 + 1/5$ states provide more striking evidence for the competition between solid and liquid phase in the second LL, see Fig. 3. At temperatures above $\sim$40 mK, the $\nu = 2 + 1/5$ state is well developed in R_{xx} and in R_{xy}. Yet on lowering T, the neighboring $\nu = 2$ IQHE is taking over, transforming the R_{xy} plateau into a local minimum, which lifts off from the quantized value and also shifts to higher ν. At the lowest $T = 9$ mK of Fig. 1, the remaining $\nu = 2 + 1/5$ features are clearly surrounded by the RIQHE whose Hall value tends toward $h/2e^2$. This behavior is yet more prominent around $\nu = 2 + 2/7$, where in Fig. 3 at 16 mK the quantized Hall value is approached in a very narrow downward spike. However, the state is increasingly destroyed on lowering T as is evident from the R_{xy} trace in Fig. 1. Furthermore, the T dependences of the RIQHE on both sides of $\nu = 2 + 2/7$ (as well as $\nu = 2 + 1/5$) are rather different, possibly indicating two different solid states on either side of the FQHE liquid. Overall, the observed features resemble the behavior at very low filling factor in the lowest LL [26]. There, the observed melting transition was attributed to the decrease of the free energy of the liquid as compared to the solid due to the increasing population of states in the roton minimum as T is raised. A similar mechanism may be responsible for the observed crossover in the second LL.

In summary, at the sample temperature of 9 mK, we observe a very complex electronic transport behavior in the second LL of a high quality 2DES, pointing to a close balance between liquid and solid ground state energies. A well quantized $\nu = 2 + 2/5$ state is observed in spite of its

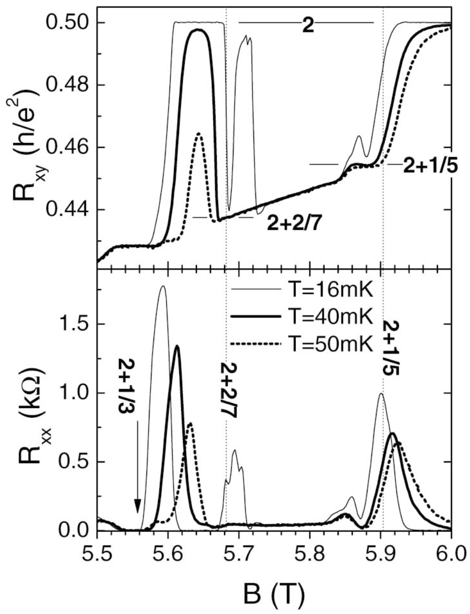

FIG. 3. Temperature dependence of R_{xx} and R_{xy} around the split Hall plateau and $\nu = 2 + 1/5$. The vertical lines mark the B field positions of the $\nu = 2 + 1/5$ and $2 + 2/7$ states. The horizontal lines mark the expected Hall resistance values for $\nu = 2 + 2/7$, $2 + 1/5$, and 2.

absence in numerical calculations based on traditional electron-electron (e-e) correlation. This may be evidence for the existence of parafermionic behavior. Some aspects of the competition between solid and liquid states are reminiscent of features at very low ν in the lowest LL and may have similar explanations. All together, e-e correlation in the second LL proves to be rather distinct from the lowest LL with many competing quantum phases closely spaced in energy.

We thank R. R. Du, F. D. M. Haldane, M. O. Goerbig, J. Jain, E. H. Rezayi, and S. H. Simon for useful discussions. This work was supported by NSF under DMR Grant No. 03-52738 and DOE under Grant No. DE-AIO2-04ER46133. The work at Princeton was supported by the AFOSR, the DOE, and the NSF. The work at Columbia was supported by DOE, NSF and by the W. M. Keck Foundation. Experiment was carried out at the high B/T facilities of the NHMFL, which is supported by NSF Cooperative Agreement No. DMR-0084173 and by the State of Florida.

*Present address: Sandia National Laboratories, Albuquerque, New Mexico 87185 USA.

[1] H. W. Jiang *et al.*, Phys. Rev. Lett. **65**, 633 (1990); P. D. Ye *et al.*, Phys. Rev. Lett. **89**, 176802 (2002); For a review, see, for example, the chapters by H. A. Fertig and M. Shayegan, in *Perspectives in Quantum Hall Effect*, edited by S. Das Sarma and A. Pinczuk (Wiley, New York, 1996).

[2] D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982); R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983); *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York (1990); T. Chakraborty and P. Pietiläinen, *The Quantum Hall Effects* (Springer, New York, 1995).

[3] An electron-hole symmetric solid of holes is expected for ν close to $\nu = 1$.

[4] J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989); B. I. Halperin, P. A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993); *Perspectives in Quantum Hall Effects*, edited by S. Das Sarma and A. Pinczuk (Wiley, New York, 1996); *Composite Fermion: A Unified View of the Quantum Hall Regime*, edited by O. Heinonen (World Scientific, Singapore, 1998).

[5] M. P. Lilly *et al.*, Phys. Rev. Lett. **82**, 394 (1999); R. R. Du *et al.*, Solid State Commun. **109**, 389 (1999); For a recent review, see, for example, M. M. Fogler, in *High Magnetic Fields: Applications in Condensed Matter Physics and Spectroscopy*, edited by C. Berthier, L.-P. Levy, and G. Martinez (Springer-Verlag, Berlin, 2002).

[6] G. Gervais *et al.*, cond-mat/0402169.

[7] H. Fukuyama, P. M. Platzman, and P.W. Anderson, Phys. Rev. B **19**, 5211 (1979).

[8] M. M. Fogler, A. A. Koulakov, and B. I. Shklovskii, Phys. Rev. B **54**, 1853 (1996).

[9] R. Willett *et al.*, Phys. Rev. Lett. **59**, 1776 (1987).

[10] W. Pan *et al.*, Phys. Rev. Lett. **83**, 3530 (1999).

[11] R. H. Morf, Phys. Rev. Lett. **80**, 1505 (1998).

[12] E. H. Rezayi and F. D. M. Haldane, Phys. Rev. Lett. **84**, 4685 (2000).

[13] W. Pan *et al.*, Phys. Rev. Lett. **83**, 820 (1999); M. P. Lilly *et al.*, Phys. Rev. Lett. **83**, 824 (1999).

[14] G. Moore and N. Read, Nucl. Phys. B **360**, 362 (1991); M. Greiter, X. G. Wen, and F. Wilczek, Phys. Rev. Lett. **66**, 3205 (1991); V.W. Scarola, Kwon Park, and J. K. Jain, Nature (London) **406**, 863 (2000).

[15] J. P. Eisenstein *et al.*, Phys. Rev. Lett. **88**, 076801 (2002).

[16] R. R. Du (private communication).

[17] N. Shibata and D. Yoshioka,J. Phys. Soc. Jpn. **72**, 664 (2003).

[18] M. O. Goerbig, P. Lederer, and C. M. Smith, Phys. Rev. B **69**, 115327 (2004).

[19] G. S. Boebinger *et al.*, Phys. Rev. B **36**, 7919 (1987); R. R. Du *et al.*, Phys. Rev. Lett. **70**, 2944 (1993).

[20] F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983); B. I. Halperin, Phys. Rev. Lett. **52**, 1583 (1984).

[21] N. Read and E. Rezayi, Phys. Rev. B **59**, 8084 (1999).

[22] R. Morf and N. d'Ambrumenil, Phys. Rev. Lett. **74**, 5116 (1995).

[23] V.W. Scarola, K. Park, and J. K. Jain, Phys. Rev. B **62**, 16259 (2000).

[24] L. Belkhir and J. K. Jain, Solid State Commun. **94**, 107 (1995).

[25] V.W. Scarola, J. K. Jain, and E. H. Rezayi, Phys. Rev. Lett. **88**, 216804 (2002); J. J. Quinn, A. Wojs, and K. S. Yi, Phys. Lett. A **318**, 152 (2003); A. Lopez and E. Fradkin, Phys. Rev. B **69**, 155322 (2004).

[26] W. Pan *et al.*, Phys. Rev. Lett. **88**, 176802 (2002).