Noise at the Crossover from Wigner Liquid to Wigner Glass

C. Reichhardt and C. J. Olson Reichhardt

Center for Nonlinear Studies and Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

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Using a simple classical model for interacting electrons in two dimensions with random disorder, we show that a crossover from a Wigner liquid to a Wigner glass occurs as a function of charge density. The noise power increases strongly at the crossover and the characteristics of the $1/f^{\alpha}$ noise change. When the temperature is increased, the noise power decreases. We compare these results with recent noise measurements in systems with two-dimensional metal-insulator transitions.

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In the two-dimensional (2D) metal-insulator transition (MIT) regime, both the Coulomb interactions between electrons and the disorder are expected to be strong, leading to the formation of an electron glass [1,2]. Recent experiments in 2D electron systems have revealed changes in the characteristics and the amplitude of the conduction noise as the charge density of the system is varied from a high charge density metallic phase to a lower charge density insulating phase [3–5]. This has been interpreted as evidence for an onset of glassy dynamics near the insulating phase. These studies also find that the noise has a $1/f^{\alpha}$ characteristic, with $\alpha = 1.0$ in the metallic phase changing over to $\alpha \approx 1.8$ in the glassy phase. Other experiments have found similar results with $\alpha = 0.75$ in the metallic phase and $\alpha = 1.3$ near the insulating phase [6]. The glassy phase is signified by a large increase in the noise power along with the change in α [3,4]. In addition, the noise power was observed to *decrease* with temperature [4,5], in contrast to single electron models with thermally activated trapping [7] and other models [8] that predict an increase in the noise power with T. This suggests the importance of electron-electron interactions at the MIT. Other recent experiments near the 2D MIT have also found $1/f^{\alpha}$ noise, strong increases in noise power with decreasing charge density, and decreasing noise with increasing T [6,9]. Additionally, $1/f^{\alpha}$ noise fluctuations in thin granular films have been interpreted as evidence for a glassy electron state [10]. More recent experiments have provided evidence that the large noise is due to charge, rather than spin, fluctuations [11].

In theoretical studies, it was proposed that at the 2D MIT a freezing from an electron liquid to a partially ordered Wigner glass [12] or a more strongly disordered electron glass [13] may occur. Other theories suggest that an intermediate metallic glass phase appears between the liquid and insulating phases [14]. It is also possible that the metallic glass phase may consist of solid phase insulating regions coexisting with stringlike liquid regions. Studies of glassy systems often find cooperative stringlike motions or dynamical heterogeneities [15]. Such motions can give rise to correlated dynamics and large fluctuations near a glass transition. It is, however, unclear what the origin of such cooperativity would be in the electron glass systems.

The noise studies suggest that in the 2D electron systems there is a crossover from a weakly pinned liquidlike region with low noise power to a more strongly partially pinned state with high noise power. This is also consistent with the theoretical prediction that the electron liquid freezes into a 2D disordered solid. In this work we propose a simple model for a classical 2D electron system consisting of interacting electrons with random disorder and temperature. We monitor the fluctuations and noise characteristics of the current as a function of electron density or temperature. The advantage of our model is that a large number of interacting electrons can be conveniently simulated, while a full quantum mechanical model of similar size would be computationally prohibitive. Despite the limitations of this model, we show that this approach captures many of the key experimental observations. Additionally, although our primary focus is to gain insight into the physics near the 2D MIT, our model is also relevant for other classical charge systems undergoing crossovers from glass to liquid states, such as charged colloids interacting with random disorder. Our model is similar to previous studies of 2D classical electron systems with disorder [16,17]; however, these previous studies focused on the microscopics of the defects in the lattice [16] or the sliding dynamics [17]. In the present work we focus on the noise fluctuations in the strongly disordered phase as it changes from a liquid to a frozen state as a function of electron density for a fixed amount of disorder. Our model applies to the region of the 2D metal-insulator transition where the system starts to become insulating. In this case, the electrons are becoming occasionally trapped and act more classically.

Our model consists of a 2D system of N_s interacting electrons with periodic boundary conditions in the x and y directions. There are also N_p defect sites which attract the electrons. We assume the electron motion is at finite temperature and the time evolution occurs through Langevin dynamics. The damping on the electrons comes from their interactions with phonons or small scattering sites. The equation of motion for an electron i is

$$\eta \mathbf{v} = \mathbf{f}_i = -\sum_j^{N_s} \nabla U(r_{ij}) + \mathbf{f}_i^s + \mathbf{f}_i^T + \mathbf{f}_d. \quad (1)$$

Here $\eta = 1$ is the damping constant and $U_i(r) = -q^2/r$, with $q = 1$, is the electron-electron interaction potential, treated as in [18]. The term $\mathbf{f}_i^s$ comes from the N_p randomly spaced defect sites modeled as parabolic traps of radius $r_p = 0.2$ and strength $f_p = 1.0$. The thermal noise $\mathbf{f}_i^T$ arises from random Langevin kicks with $\langle f^T(t)\rangle = 0$ and $\langle f_i^T(t) f_j^T(t')\rangle = 2\eta k_B T \delta_{ij}\delta(t - t')$. The driving term $\mathbf{f}_d = f_d \hat{x}$ comes from an applied voltage, and we take $f_d = 0.1$. We start at a high temperature where the charges are diffusing rapidly and cool to a lower temperature. We then wait for 10^4 simulation time steps to reduce transient effects before applying the drive and measuring the average velocity v of the electrons, which is proportional to the conductance or inversely proportional to the resistance. We do this for a series of electron densities at fixed disorder strength. We have considered samples with constant pin densities n_p for different system sizes such that N_p ranges from 317 to 1200.

We first consider the average electron velocity as a function of temperature and charge density for a system with a fixed $N_p = 1200$. In Fig. 1 we show a series of conductance curves vs T for charge density varied over nearly an order of magnitude, $0.4 \leq N_s/N_p \leq 2.67$. For high $N_s/N_p > 0.6$ the conductance is finite down to $T = 0$, while for $N_s/N_p \leq 0.6$ the electron velocity drops to zero within our resolution, indicating that all the electrons are strongly pinned in an insulating phase. As N_s/N_p increases above 1.0, the downward curvature of v at low T decreases. These curves appear very similar to those typically observed in 2D MIT studies [3]. One difference is that we do not find a charge density $n_s^{\rm up}$ above which the slope of the velocities turns up slightly at low T, as in the experiments. This may be due to the fact that in our model we do not directly include phonons. In the experimental regime of interest to us here, the large noise increases occur at charge densities $n_s < n_s^{\rm up}$, where the velocity curves bend down at low T.

We next consider the relative fluctuations in the velocities, $\delta v(t) = [v(t) - \langle v\rangle]$ for varied N_s/N_p at a fixed $T = 0.09$ for the system in Fig. 1. This analysis is similar to that performed in experiments [3–5]. In Fig. 2 we show the time traces of the relative velocity fluctuations for $N_s/N_p = 0.5, 0.7, 1.05, 1.64$, and 2.67. Here the fluctuations increase as N_s drops, in agreement with the experiments [3]. For $N_s/N_p < 0.44$, the system is pinned and there are no fluctuations. It is possible that, over a longer time interval such as that accessible experimentally, there would be even larger fluctuations at these small N_s values; however, this is beyond the time scale we can access with simulations. A similar series of time traces can be obtained for δv at fixed N_s/N_p for increasing temperature (not shown). Here the fluctuations are reduced at higher T, in agreement with experiments [3].

From the fluctuations δv we measure the power spectrum

$$S(\nu) = \left| \int \delta v(t) e^{-2\pi i \nu t} dt \right|^2. \quad (2)$$

In Fig. 3 we plot $S(\nu)$ for two different charge densities. At $N_s/N_p = 0.5$ (upper curve), the spectrum shows a $1/f^\alpha$ characteristic with $\alpha = 1.37$ over a few orders of magnitude in the frequency. In contrast, for $N_s/N_p = 1.67$ (lower curve), the noise power at lower frequencies is considerably reduced and the spectra is white with $\alpha \approx 0$. We note that it is the lower frequencies which will be

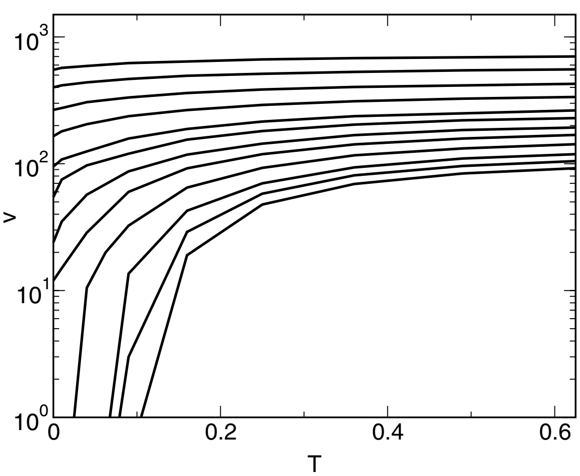

FIG. 1. The average electron velocity v vs T for $N_s/N_p = 2.67, 2.13, 1.67, 1.33, 1.07, 0.94, 0.8, 0.7, 0.6, 0.5, 0.45$, and 0.4, from top to bottom.

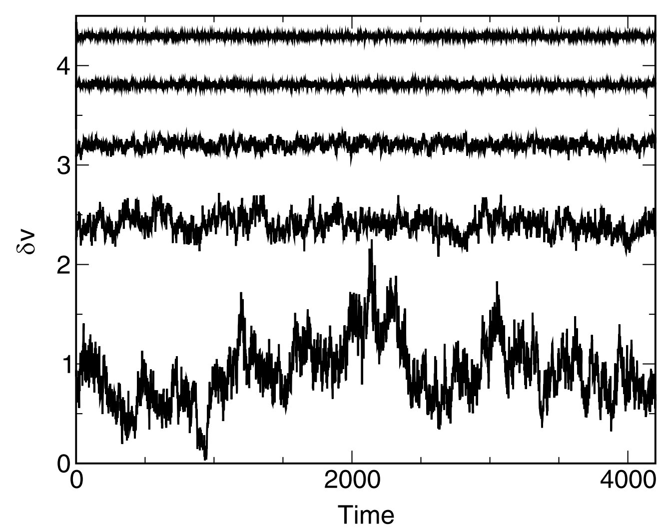

FIG. 2. The relative velocity fluctuations δv vs time for the system in Fig. 1 for $T = 0.09$ at $N_s/N_p = 0.5, 0.7, 1.05, 1.64$, and 2.67, from bottom to top. The curves have been shifted up for clarity.

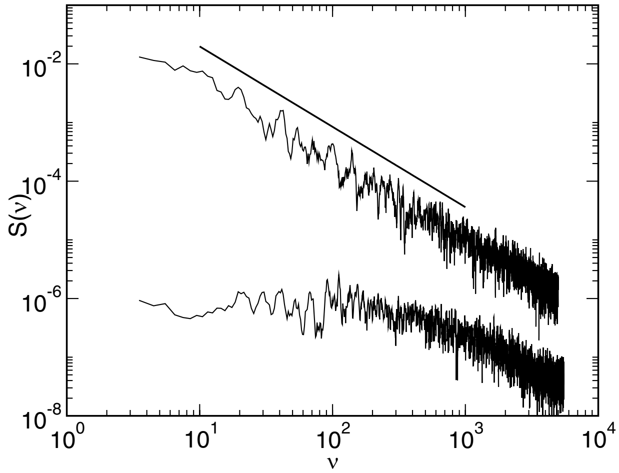

FIG. 3. The power spectra $S(\nu)$ for the velocity fluctuations $\delta v(t)$ at $N_s/N_p = 0.5$ (upper curve) and $N_s/N_p = 1.67$ (lower curve). The solid line has slope $\alpha = 1.37$.

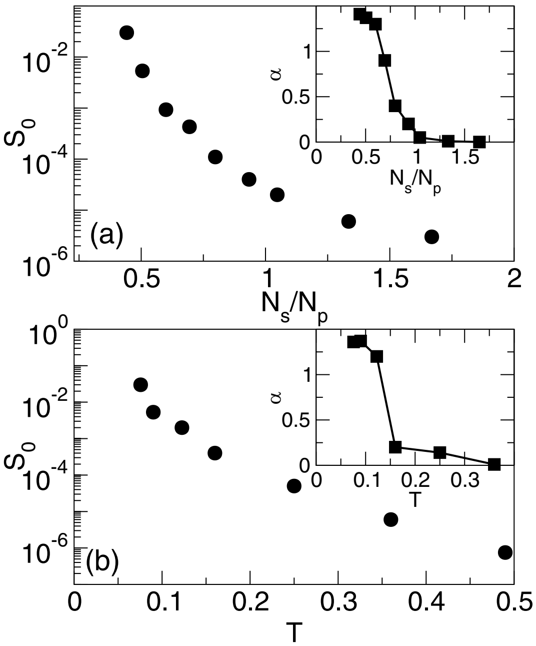

FIG. 4. (a) The integrated noise power S_0 vs N_s/N_p for $T = 0.09$. Inset: The power spectrum exponent α vs N_s/N_p for $T = 0.09$. (b) S_0 vs T for $N_s/N_p = 0.5$. Inset: α vs T for fixed $N_s/N_p = 0.5$.

most readily accessible in experiment. For fixed $N_s/N_p = 0.5$, we find that the power spectrum becomes white upon increasing T. We note that our results differ quantitatively from the experimental noise measurements [3] which find $1/f^\alpha$ noise with $\alpha = 1.0$ near the metallic phase and $\alpha = 1.8$ in the glassy regime. Our exponent $\alpha = 1.37$ is close to the $\alpha = 1.3$ found in the glassy regime in other experiments [6], where $\alpha = 0.75$ in the metallic regime. It is possible that the exponents are not universal but depend on the details of the disorder strength; nevertheless, our results are in qualitative agreement with the experiments.

In Fig. 4(a) we show the noise power S_0 integrated over the first octave vs N_s/N_p for a fixed $T = 0.09$. The noise power increases by 4 orders of magnitude as N_s/N_p is reduced. At low N_s/N_p, the noise power decreases almost exponentially with charge density and begins to saturate at high N_s/N_p. Both of these observations are in agreement with the experimental results [4,5]. Here our definition differs by a factor of $1/\langle v\rangle^2$ from the noise measured in the experiment; this does not affect α but would further enhance the increase in noise power at low density and temperature where $\langle v\rangle$ approaches zero. In the inset of Fig. 4(a) we plot the noise spectrum exponent α vs N_s/N_p. A large increase in α occurs near $N_s/N_p = 0.7$. A similar sharp increase in the exponent is also observed in experiments [4–6] as a function of charge density and has been interpreted as the glassy freezing of electrons. In Fig. 4(b) we plot S_0 vs T for fixed $N_s/N_p = 0.5$. Here the noise power drops exponentially over 4 orders of magnitude with increasing T, which is in agreement with the experiments [4,5]. We note that most single electron models predict an *increase* in the noise power with temperature [7,8]. In the hopping regime [19], another model predicts noise power that increases with T [20], although a more recent variable range hopping model predicts a decrease in the noise power as a function of T [21]. These discrepancies suggest that the noise in the experiment is not due to single electron hopping events but is instead caused by correlated electron motions. In the inset of Fig. 4(b) we plot α vs T, where a sharp increase in α occurs near $T = 0.125$ at the onset of the glassy freezing.

We have also measured the non-Gaussian nature of the noise. At high N_s and T the noise fluctuations are Gaussian; however, in the regions of high noise power we find non-Gaussian noise fluctuations with a skewed distribution. Experiments have also found evidence for non-Gaussian fluctuations in the glassy regimes [5].

Next we show evidence that the large noise is due to correlated regions of stringlike electron flow, and that within these regions the electrons move in 1D or quasi-1D channels. Because of the reduced dimensionality, the electron motion is more correlated. In Fig. 5(a) we show the trajectories of the electrons for a fixed period of time for a system with $T = 0.09$ at $N_s/N_p = 1.67$. Here the electrons can flow freely throughout the sample, although there are some areas where electrons become temporarily trapped by a defect site. In Fig. 5(b) at $N_s/N_p = 1.37$, where the noise power is larger than in the system shown in Fig. 5(a), larger pinned regions appear and the electron motion consists of a mixture of 2D and 1D regions. If the trajectories are followed over longer times, motion occurs throughout the entire sample. In Fig. 5(c) at $N_s/N_p = 0.5$, where the noise power and α are both maximum, the electron motion occurs mostly in the form of 1D channels that percolate through the sample. There are also regions where the electron motion occurs in small rings. The

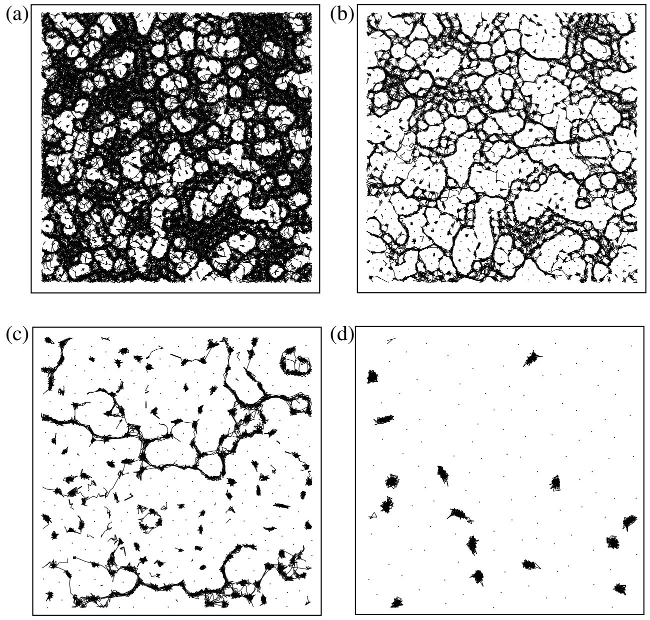

FIG. 5. Electron trajectories for a fixed period of time for fixed $T = 0.09$ at (a) $N_s/N_p = 1.67$, (b) 1.37, (c) 0.5, and (d) 0.3.

channel structures change very slowly with time, with a channel occasionally shutting off while another emerges elsewhere. It is the intermittent opening of the 1D channels which gives rise to the large noise fluctuations in this regime. When a percolating 1D channel opens, all the electrons in that channel move in a correlated fashion leading to a large increase in the conduction. Conversely, if a percolating channel closes, all the electrons in that channel cease to move. It is well known that fluctuations in 1D are much more strongly enhanced than in 2D. As T or N_s is increased, the motion becomes increasingly 2D in nature and the strong correlations of the electron motion are lost. We also note that the appearance of stringlike motions in the large noise regions is consistent with studies in glassy systems, where dynamical heterogeneities in the form of 1D stringlike motions of particles have been observed in conjunction with large noise [15]. In Fig. 5(d) at $N_s/N_p = 0.3$, deep in the insulating regime, there are no channels. Instead, the infrequent motion of electrons occurs only by small jumps from defect to defect.

It is beyond the scope of this Letter to determine whether there is a true phase transition associated with the onset of the glassy behavior or simply a crossover which could be kinetic in nature. This is an open question within the glassy systems in general. However, since our system is 2D and no power law divergences occur, it is more likely that the onset of the large noise is associated with a crossover in the dynamics of the channels.

In conclusion, we have presented a simple model for the glassy freezing of interacting electrons in 2D with random disorder. For high electron density or high temperatures, the electrons form a 2D liquid state and we find low conduction noise power with a white spectra. As the density of the electrons is lowered for fixed temperature, or, conversely, as the temperature is lowered for fixed low electron density, there is a crossover to a $1/f^{\alpha}$ noise with large low frequency power and $\alpha = 1.37$. In this glassy regime, the electrons move in 1D intermittent stringlike paths which percolate throughout the sample. Similar stringlike motions are also observed in other glass forming systems. For low electron density, all the electrons are frozen by the defect sites and the motion occurs only by single electron hopping events. We find that the noise power decreases exponentially with temperature, in agreement with experiment. Many of our results are in qualitative agreement with recent experiments on 2D electron systems near the metal-insulator transition.

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