Hall Magnetic Reconnection Rate

J. D. Huba
Plasma Physics Division, Naval Research Laboratory, Washington, D.C., USA

L. I. Rudakov
Berkeley Scholars, Inc., Beltsville, Maryland, USA
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Two-dimensional Hall magnetohydrodynamic simulations are used to determine the magnetic reconnection rate in the Hall limit. The simulations are run until a steady state is achieved for four initial current sheet thicknesses: $L = 1, 5, 10$, and $20c/\omega_{pi}$, where c/ω_{pi} is the ion inertial length. It is found that the asymptotic (i.e., time independent) state of the system is nearly independent of the initial current sheet width. Specifically, the Hall reconnection rate is weakly dependent on the initial current layer width and is $\partial\Phi/\partial t \lesssim 0.1V_{A0}B_0$, where Φ the reconnected flux, and V_{A0} and B_0 are the Alfvén velocity and magnetic field strength in the upstream region. Moreover, this rate appears to be independent of the scale length on which the electron "frozen-in" condition is broken (as long as it is $<c/\omega_{pi}$) and of the system size.

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It is recognized that Hall physics plays a critical role in the dynamics of magnetic reconnection. The National Science Foundation Global Environment Modeling (GEM) challenge on reconnection physics concluded that Hall physics was the minimum physics needed to achieve fast reconnection, regardless of the mechanism that decouples the electrons from the magnetic field [1–8]. This conclusion was reached because fast reconnection rates were obtained using Hall MHD, hybrid, and particle simulation codes, whereas a slow reconnection rate was obtained using a resistive MHD code. Shay *et al.* [9] suggested that the magnetic reconnection rate in the Hall limit depended only upon the upstream Alfvén velocity V_{A0}, independent of system size L. Subsequent studies disputed this finding and found that the maximum reconnection rate scales as $(c/\omega_{pi}L)^{3/2}$ [10] and $(c/\omega_{pi}L)^{1/2}$ [11]. Very recently, Shay *et al.* [12] reported that the reconnection process consisted of two phases: a "developmental phase" that depends on system size, and an "asymptotic phase" that is associated with fast reconnection and is independent of system size. The reconnection rate in the "asymptotic phase" scales as the upstream Alfvén velocity on B_d, the magnetic field just upstream of the Hall-dominated region.

However, the aforementioned simulations did not achieve an asymptotic state in the sense of a time independent final state. This is probably due to the boundary conditions used. Shay *et al.* [9,12] used doubly periodic boundary conditions, while Wang *et al.* [10] and Fitzpatrick [11] used a combination of conducting wall and periodic boundary conditions. In this Letter, we present results for a steady state reconnection system using the NRL Hall MHD code VOODOO [13]. Simulations are run for initial current sheet thicknesses of $L = 1, 5, 10$, and $20\ c/\omega_{pi}$, where c/ω_{pi} is the ion inertial length. Zero-gradient boundary conditions are used to allow for inflow and outflow at the boundaries and the system can evolve to a steady state. We find that the asymptotic (i.e., time independent) state of the system is nearly independent on the initial current sheet width and system size. In particular, the Hall reconnection rate is weakly dependent on the initial current layer width and is $\partial\Phi/\partial t \lesssim 0.1V_{A0}B_0$, where Φ is the reconnected flux, and V_{A0} and B_0 are the Alfvén velocity and magnetic field strength in the upstream region.

The Hall MHD equations used in our analysis are based on Ohm's law $\mathbf{E} = \mathbf{V}_e \times \mathbf{B}/c$; we neglect electron inertia and pressure. The equations are

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\rho\mathbf{V} = 0, \tag{1}$$

$$\frac{\partial\rho\mathbf{V}}{\partial t} + \nabla\cdot[\rho\mathbf{V}\mathbf{V} + (P + B^2/8\pi)\underline{\mathbf{I}} - \mathbf{B}\mathbf{B}/4\pi] = 0, \tag{2}$$

$$\frac{\partial P}{\partial t} + \nabla\cdot P\mathbf{V} = -(\gamma - 1)P\nabla\cdot\mathbf{V}, \tag{3}$$

$$\frac{\partial\mathbf{B}}{\partial t} = -c\nabla\times\mathbf{E} = \nabla\times[(\mathbf{V} + \mathbf{V}_H)\times\mathbf{B}], \tag{4}$$

where $\mathbf{V}_H = -\mathbf{J}/ne$ and we take $\gamma = 5/3$. The variable $\mathbf{V}_H$ is defined as a "Hall velocity" to explicitly show where the Hall term enters the equation: it is the electron drift velocity in the ion rest frame. The Hall electric field is defined as $\mathbf{E}_H = \mathbf{V}_H \times \mathbf{B}/c$. We use the NRL Hall MHD code VOODOO [13] to solve these equations. VOODOO is a high-order, finite-volume code that uses a distribution function scheme to calculate the fluxes of mass, momentum, and energy at cell interfaces, as well as the $\mathbf{V} \times \mathbf{B}$ electric field [14]. The Hall electric field is updated based on an upwind scheme using high-order magnetic field values and is subcycled on the ideal MHD time scale. This scheme substantially decreases the computation time by separating the Alfvén and whistler time scales. The partial donor cell method is used to limit fluxes at sharp discontinuities [15]. Numerical resistivity, which depends on grid size, provides the mechanism to break the electron "frozen-in" condition. The code uses an eighth order spatial interpolation scheme and a second order temporal scheme.

The simulation parameters are as follows. The equilibrium magnetic field is $B_y(x) = B_0 \tanh(x/L)$. The temperature is defined to be $C_s = 0.41 V_{A0}$, where $C_s = (2T/m_i)^{1/2}$. The density is $n = n_0$ at $x = 0$ and is $n_1 = 0.2 n_0$ for $|x| \gg L$. The spatial scales are normalized to the ion inertial length (c/ω_{pi0}), the time scale to the ion gyrofrequency (Ω_{i0}), the velocity to the Alfvén velocity (V_{A0}), and the electric field is normalized to $E_0 = V_{A0} B_0 / c$ using $n = n_1$ and $B = B_0$ (i.e., the upstream region). The size of the simulation box is $L_x = 70$ and $L_y = 84$ with a mesh size 90×160. A nonuniform stretched mesh is used in both the x and y directions. There are $\gtrsim 32$ grid points within the current layer; the minimum grid width is $0.023 c/\omega_{pi}$, which is comparable to the electron inertial length. Zero-gradient boundary conditions are used for all variables in both the x and y directions ($\partial/\partial x = 0$ and $\partial/\partial y = 0$). Physically, this implies that there is no acceleration across the boundaries. The simulations are initialized with a magnetic perturbation $\delta B_x = \delta B \sin(2\pi y/L_y) \cos(\pi x/L_x)$ and $\delta B_y = (\delta B/2)(L_y/L_x) \cos(2\pi y/L_y) \sin(\pi x/L_x)$. We use $\delta B = 0.1 B_0$. The simulations are run until a steady state is achieved. Finally, we report results from four simulation studies using initial current sheet widths $L = 1, 5, 10$, and 20.

In Fig. 1 we plot gray-scale contours of the plasma density for the initial and final states of the system as a function of x and y for different values of the initial current sheet width L: $L = 1, 5, 10, 20$. The density of the initial states is normalized to $n_0/2$ and the density of the final states is normalized to $n_0/3$. The white contour denotes high density and the dark contour denotes low density. The important result is that the final states for all four cases are very similar despite the very different initial conditions. This strongly suggests that the asymptotic state of a Hall-dominated reconnection plasma is independent of initial conditions: specifically, the width of the current layer.

In Fig. 2 we plot the maximum values of V_x, V_y, V_z, and E_{Hz} as a function of time for $L = 5$. Here, E_{Hz} is the Hall component of the field electric field in the z direction. This figure is representative of the time evolution of these quantities for each of the simulations performed. The sequence of events is as follows. First, the magnetic perturbation at $t = 0$ initiates a slow reconnection process. Second, fast Hall reconnection is triggered when the current layer thins to a width comparable to the ion inertial length c/ω_{pi}. The onset time for fast Hall reconnection occurs at $t \simeq 100$ and is associated with the sharp increase in the Hall electric field E_{Hz} and plasma velocity V_z; the onset time for fast reconnection depends on the initial current layer width as well as the magnitude of the magnetic perturbation. Finally, the plasma evolves to a steady state when the plasma is fully energized, i.e., the maximum kinetic energy of the plasma is reached. The rapid growth ceases at $t \simeq 300$. The asymptotic values of the maximum velocities are $V_x \simeq 0.14$, $V_y \simeq 1.32$, and $V_z \simeq 0.56$.

In Fig. 3 we plot the maximum value of the Hall electric field in the z direction E_{Hz} as a function of time for $L = 1, 5, 10$, and 20. We find that the Hall reconnection electric field is weakly dependent on the

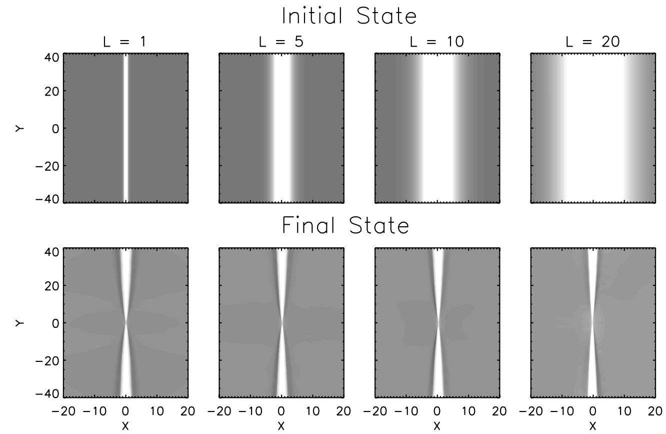

FIG. 1. Gray-scale contours of the plasma density for the initial and final states of the system as a function of x and y for different values of the initial current sheet width: $L = 1, 5, 10$, and 20.

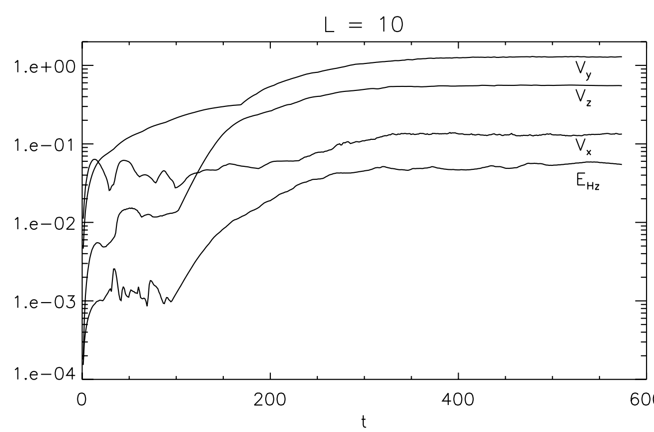

FIG. 2. Plot of the maximum values of V_x, V_y, V_z, and E_{Hz} as a function of time for $L = 5$. Here, E_{Hz} is the Hall component of the electric field in the z direction.

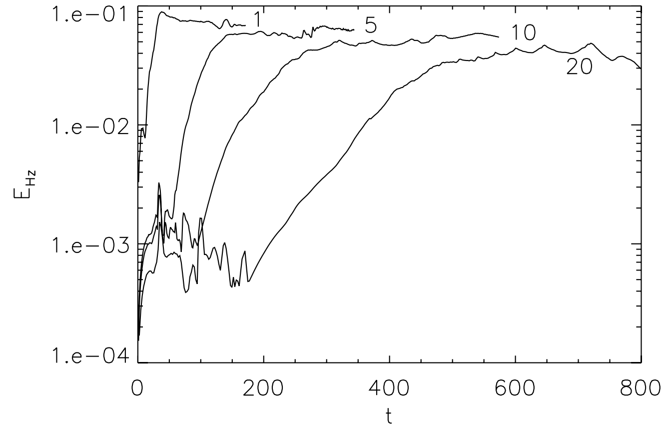

FIG. 3. Plot the maximum value of the Hall electric field in the z direction E_{Hz} as a function of time for $L = 1, 5, 10$, and 20.

initial width of the current layer: the field decreases by $\sim$33% as the current layer width increases by a factor of 20. However, the time to reach the maximum Hall electric field after the onset of fast reconnection is dependent on the initial current width. We use E_{Hz} as the parameter to determine the onset of fast magnetic reconnection and the time at which an asymptotic state is reached.

In Fig. 4(a) we plot the energization time $\tau_E = t_A - t_O$, where t_O is the onset time determined by the sudden increase in E_{Hz} and t_A is the asymptotic time when then E_{Hz} reaches 90% of its saturated value. The energization time is linearly proportional to the initial width of the current layer: $\tau_E \simeq 20L$. This relation can be rewritten as

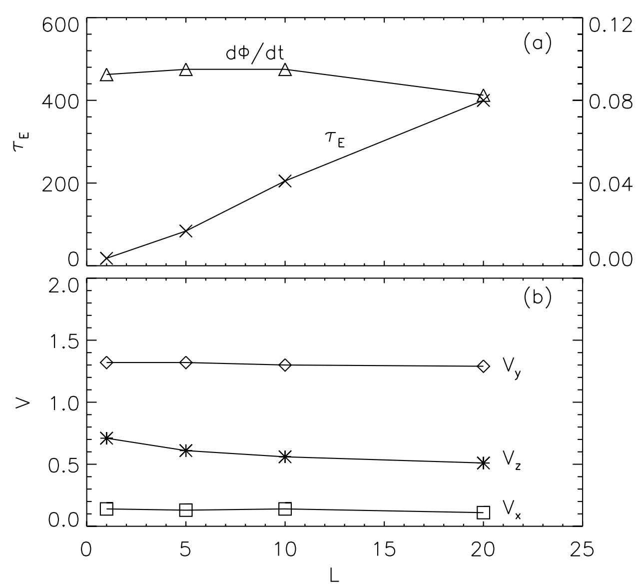

FIG. 4. (a) The energization time τ_E and of the Hall reconnection rate $\partial\Phi/\partial t$ as a function of the initial current sheet width L. (b) Asymptotic values of the plasma velocity as a function of the initial current sheet width L.

$t \simeq 20L/V_{A0}$. We explain this as follows. The system relaxes to a final state that is largely independent of the initial current sheet width as shown in Fig. 1. However, to achieve this final state the plasma must reconfigure itself in the x direction. The time scale for this is $\propto L/v_x$. Since v_x is weakly dependent on L, the time scale to energize the plasma (or to reach steady state values) is linearly proportional to L. If we assume that the inflow velocity v_x during the energization phase is half the maximum inflow velocity $V_x \simeq 0.1V_{A0}$ (consistent with the results shown in Fig. 2), then we obtain $L \simeq 20L/V_{A0}$, which agrees with the scaling obtained from the simulations. Shay *et al.* [12] also derived an expression for the energization time of the plasma based on several simplifying assumptions. Using the notation in this paper, they obtained the formula $\tau_E \simeq (L/\alpha V_{A0})\ln(B_0/B_{d0})$, where $\alpha = V_x/V_y$ and B_{d0} is the magnetic field at $t = 0$ just upstream of the Hall-dominated region. Taking $\alpha \simeq 0.1$ and $B_0/B_{d0} \simeq 8$, one obtains $\tau_E \simeq 20L/V_{A0}$, which is also consistent with our results. Last, we note that the values of t_A used to determine the energization time are approximate and that the scaling may not be linear for $L \gg 1$. We intend to examine this issue more closely in future work.

The reconnection rate is calculated as follows. The reconnected flux Φ is defined as

$$\begin{aligned}\Phi(t) &= \int_0^\infty B_x(0, y, t)dy \\ &= \int_0^{L_y/2} B_x(0, y, t)dy + \int_{L_y/2}^\infty B_x(0, y, t)dy. \quad (5)\end{aligned}$$

When the system reaches steady state, we find that B_x is independent of time in the simulation region $0 < y < L_y/2$. The flux that leaves the system, i.e., the final term in (5), can be approximated as

$$\begin{aligned}\int_{L_y/2}^\infty B_x(0, y, t)dy &\simeq \int_{t_A}^t B_x(0, L_y/2)V_y(0, L_y/2)dt \\ &= B_x(0, L_y/2)V_y(0, L_y/2)(t - t_A). \quad (6)\end{aligned}$$

The reconnected flux rate $\partial\Phi/\partial t$ is then

$$\frac{\partial\Phi(t)}{\partial t} \simeq B_x(0, L_y/2)V_y(0, L_y/2). \quad (7)$$

Based upon the simulation results shown in Fig. 4(a), we find that the magnetic reconnection rate is

$$\frac{\partial\Phi(t)}{\partial t} \lesssim 0.1V_{A0}B_0. \quad (8)$$

Last, in Fig. 4(b) the asymptotic values of the plasma velocity as a function of the initial current sheet width are shown. We find that the inflow velocity V_x and outflow velocity V_y are independent of the initial current sheet width. The asymptotic values are $V_x \simeq 0.13$ and $V_y \simeq 1.30$. The asymptotic velocity in the z direction is $V_z \simeq 0.6$.

We performed a simulation of the $L = 1$ case initiated with a smaller magnetic perturbation (i.e., $\delta B = 0.02B_0$). The results are the same as for the case with a larger initial perturbation except that the onset time for fast Hall reconnection is slightly later. We also performed a simulation for the case $L = 1$ with a significantly higher resolution: the minimum grid width was reduced by a factor of 4 (i.e., $\Delta x_{\min} = 6 \times 10^{-3} c/\omega_{pi}$). We obtained somewhat higher values of inflow and outflow speeds in the reconnection plane: $V_x \simeq 0.15V_{A0}$ and $V_y \simeq 1.4V_{A0}$. However, the reconnection rate is the same as for the case $L = 1$ shown in Fig. 4(a). This suggests that the reconnection rate is insensitive to the scale on which the electron frozen-in condition is broken (as long as it is $<c/\omega_{pi}$). Finally, we have performed simulation studies for $L = 2$ and $L = 5$ for a system size $L_x = 85$ and $L_y = 166$ with a mesh 94×214. This corresponds to a system much larger than that used in the aforementioned results. We find that the current sheet collapses to a final state as shown in Fig. 1 and that the reconnection rate is $\sim 0.09V_{A0}B_0$ in both cases, which is consistent with the results shown in Fig. 4. Thus, the results reported in this Letter do not appear to be sensitive to the initial perturbation, the grid scale, or the system size.

The energization time is estimated for several space plasma regions where magnetic reconnection is expected to be important: the Earth's magnetotail and the solar corona. The upstream magnetic field in the Earth's magnetotail is $B_0 \simeq 2 \times 10^{-4}$ G, so that $\Omega_{ci}^{-1} \simeq 0.5$ s. If we take an initial current sheet width $L \simeq 2\mathrm{R}_E \simeq 24c/\omega_{pi}$, then the energization time is $\tau_E \simeq 500\omega_{ci}^{-1} \simeq 250$ s; this is consistent with time scales of energetic events in the magnetotail. The magnetic field in an active solar region can be $B_0 \simeq 10^2$ G, so that $\Omega_{ci}^{-1} \simeq 10^{-6}$ s. The initial width of a coronal current layer can be $L \simeq 10^3 - 10^4$ km, while $c/\omega_{pi} \simeq 10^{-2}$ km, so that $L \simeq 10^5 - 10^6 c/\omega_{pi}$. Extrapolating the results in Fig. 4(a) we estimate $\tau_E \simeq 2 \times 10^6 - 2 \times 10^7 \Omega_{ci}^{-1} \simeq 2 - 200$ s; this is consistent with the time scale of impulsive flares. Although these are crude estimates, they do indicate that collisionless magnetic reconnection, initiated by the Hall term, can lead to very rapid energization of the plasma and is consistent with observations.

In conclusion, we have investigated the dependence of Hall magnetic reconnection dynamics on the initial width of a reversed-field current layer for a nonperiodic system. We find that the asymptotic (i.e., time independent) state of the system is almost independent of the initial current sheet width and the system size, consistent with the conclusions of Shay *et al.* [9,12]. This occurs because the current sheet is unstable to the imposed magnetic field perturbation and it collapses to the only physical scale in the system: the ion inertial length. As the system evolves it sets up a steady state configuration in which plasma flows in through the x boundaries and out through the y boundaries. We do not consider this a "forced reconnection" situation because the inward flows at the x boundaries are not prescribed: they develop self-consistently. The Hall magnetic reconnection rate is estimated to be $\partial\Phi/\partial t \lesssim 0.1V_{A0}B_0$, where Φ is the reconnected flux, and V_{A0} and B_0 are the Alfvén velocity and magnetic field strength in the upstream region. Furthermore, this rate appears to be independent of the scale length on which the electron frozen-in condition is broken (as long as it is $<c/\omega_{pi}$). However, the time to achieve maximum flow speeds after the onset of fast reconnection is linearly proportional to the current sheet width. It is not expected that 3D effects will impact these results [16,17]. However, the role of a guide field may have an affect and we are currently studying this issue.

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[1] J. Birn, J. F. Drake, M. A. Shay, B. N. Rogers, R. E. Denton, M. Hesse, M. Kuznetsova, Z.W. Ma, A. Bhattacharjee, A. Otto, and P. L. Pritchett, J. Geophys. Res. **106**, 3715 (2001).
[2] M. Hesse, J. Birn, and M. Kuznetsova, J. Geophys. Res. **106**, 3721 (2001).
[3] J. Birn and M. Hesse, J. Geophys. Res. **106**, 3737 (2001).
[4] A. Otto, J. Geophys. Res. **106**, 3751 (2001).
[5] M. A. Shay, J. F. Drake, B. N. Rogers, and R. E. Denton, J. Geophys. Res. **106**, 3759 (2001).
[6] Z.W. Ma and A. Bhattacharjee, J. Geophys. Res. **106**, 3773 (2001).
[7] P. Pritchett, J. Geophys. Res. **106**, 3783 (2001).
[8] L. Yin, D. Winske, S. P. Gary, and J. Birn, J. Geophys. Res. **106**, 10 761 (2001).
[9] M. A. Shay, J. F. Drake, B. N. Rogers, and R. E. Denton, Geophys. Res. Lett. **26**, 2163 (1999).
[10] X. Wang, A. Bhattacharjee, and Z.W. Ma, Phys. Rev. Lett. **87**, 265003 (2001).
[11] R. Fitzpatrick, Phys. Plasmas **11**, 937 (2004).
[12] M. A. Shay, J. F. Drake, M. Swisdak, and B. N. Rogers, Phys. Plasmas **11**, 2199 (2004).
[13] J. D. Huba, in *Space Plasma Simulation*, edited by J. Büchner, C. T. Dum, and M. Scholer (Springer, New York, 2003), p. 170.
[14] J. D. Huba and J. G. Lyon, J. Plasma Phys. **61**, 391 (1999).
[15] K. Hain, J. Comput. Phys. **73**, 131 (1987).
[16] J. D. Huba and L. I. Rudakov, Phys. Plasmas **9**, 4435 (2002).
[17] J. D. Huba and L. I. Rudakov, Phys. Plasmas **10**, 3139 (2003).