

Chameleon Fields: Awaiting Surprises for Tests of Gravity in Space

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We present a novel scenario where a scalar field acquires a mass which depends on the local matter density: the field is massive on Earth, where the density is high, but is essentially free in the solar system, where the density is low. All existing tests of gravity are satisfied. We predict that near-future satellite experiments could measure an effective Newton's constant in space different from that on Earth, as well as violations of the equivalence principle stronger than currently allowed by laboratory experiments.

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Recent observations suggest the existence of a scalar field which is presently evolving on cosmological time scales. Indeed, the Universe is undergoing a period of accelerated expansion as a result of a dark energy component with negative pressure. Although the current data are consistent with this being a cosmological constant, the dark energy is more generally modeled as quintessence [1]: a scalar field rolling down a flat potential. In order for such a scalar field to evolve cosmologically today, its mass must be of order H_0 , the present Hubble parameter. Thus, one would naively expect it to be essentially massless on solar system scales, in which case tests of the equivalence principle (EP) [2] would constrain its coupling to matter to be unnaturally small.

In this Letter, we propose a novel scenario which allows scalar fields to evolve on cosmological time scales today while having couplings of order unity to matter, as expected from string theory. The idea is that the mass of the scalar field is not constant in space and time, but rather depends on the environment, in particular, on the local matter density. Thus, in regions of high density, such as on Earth, the mass of the field can be sufficiently large to satisfy constraints on EP violations and fifth force. Meanwhile, on cosmological scales where the matter density is 10^{30} times smaller, the mass of the field can be of order H_0 , thus allowing the field to evolve cosmologically today. The philosophy, therefore, is that cosmological scalar fields, such as quintessence, have not yet been detected in local tests of the EP because we happen to live in a dense environment. Since their physical characteristics depend sensitively on their environment, we dub such scalar fields chameleons.

In our model, the strength of EP violations and the magnitude of the fifth force mediated by the chameleon can be drastically different in space than in the laboratory. In particular, we find exciting new predictions for near-future satellite experiments, such as Satellite Energy Exchange (SEE), Microscope (μ SCOPE), Galileo Galilei (GG), and Satellite Test of Equivalence Principle (STEP) [3], that will test gravity in space. We find that it is possible for SEE to measure an effective Newton's con-

stant that differs by order unity from the value measured on Earth. Moreover, the μ SCOPE, GG, and STEP satellites could detect violations of the EP larger than currently allowed by laboratory experiments. Such outcomes would strongly suggest that a chameleonlike model is realized in Nature. Furthermore, they strengthen the scientific case for these missions. Some of the results below require lengthy calculations that are presented in a companion paper [4]. The cosmology is studied elsewhere [5].

Consider the general Lagrangian

$$\mathcal{L} = \sqrt{-g} \left\{ -\frac{M_{\text{Pl}}^2 \mathcal{R}}{2} + \frac{(\partial\phi)^2}{2} + V(\phi) \right\} + \mathcal{L}_m(\psi^{(i)}, g_{\mu\nu}^{(i)}), \quad (1)$$

where $M_{\text{Pl}} \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass. Each matter field $\psi^{(i)}$, labeled by i , couples to a metric $g_{\mu\nu}^{(i)}$ related to the Einstein-frame metric $g_{\mu\nu}$ by a conformal transformation: $g_{\mu\nu}^{(i)} = \exp(2\beta_i\phi/M_{\text{Pl}})g_{\mu\nu}$, where β_i are dimensionless constants. In harmony with string theory, we allow the β_i 's to be of order unity and to assume different values for different matter species.

The potential $V(\phi)$ is assumed to be of the runaway form. That is, it is monotonically decreasing and satisfies $V, V_{,\phi}/V, V_{,\phi\phi}/V_{,\phi} \dots \rightarrow 0$ as $\phi \rightarrow \infty$, as well as $V, V_{,\phi}/V, V_{,\phi\phi}/V_{,\phi} \dots \rightarrow \infty$ as $\phi \rightarrow 0$. See the dashed curve in Fig. 1. A prototypical example is the inverse power-law potential: $V(\phi) = M^{4+n}\phi^{-n}$, where n is positive and M has units of mass. The runaway form is generic to nonperturbative potentials in string theory and is also desirable for quintessence models [6].

For simplicity, we focus on a single matter component coupled to a metric $\tilde{g}_{\mu\nu} = \exp(2\beta\phi/M_{\text{Pl}})g_{\mu\nu}$. For non-relativistic matter, one has $\tilde{g}^{\mu\nu}T_{\mu\nu} \approx -\tilde{\rho}$, where $T_{\mu\nu} = (2/\sqrt{-\tilde{g}})\delta\mathcal{L}_m/\delta\tilde{g}^{\mu\nu}$ and $\tilde{\rho}$ are the stress tensor and corresponding energy density, respectively. For convenience, however, we express our equations in terms of $\rho \equiv \tilde{\rho}e^{3\beta\phi/M_{\text{Pl}}}$, which is conserved in the Einstein frame and

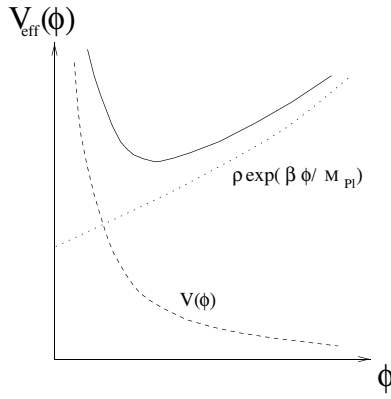


FIG. 1. The chameleon effective potential V_{eff} (solid curve) is the sum of a scalar potential $V(\phi)$ (dashed curve) and a density-dependent term (dotted curve).

hence is independent of ϕ . Equation (1) then gives

$$\nabla^2 \phi = V_{,\phi} + \frac{\beta}{M_{\text{Pl}}} \rho e^{\beta \phi / M_{\text{Pl}}}. \quad (2)$$

The key realization from Eq. (2) is that the dynamics of the chameleon are not governed by $V(\phi)$, but rather by an *effective* potential

$$V_{\text{eff}}(\phi) \equiv V(\phi) + \rho e^{\beta \phi / M_{\text{Pl}}}, \quad (3)$$

which is an explicit function of ρ_i . Moreover, although $V(\phi)$ is monotonically decreasing, V_{eff} has a minimum if $\beta > 0$. This is shown in Fig. 1. In particular, the value of ϕ at the minimum, ϕ_{min} , and the mass of small fluctuations about the minimum, m_{min} , both depend on ρ . More precisely, ϕ_{min} and m_{min} are decreasing and increasing functions of ρ , respectively. That is, the larger the density of the environment, the larger the mass of the chameleon.

Solution for a compact object.—We derive an approximate solution for ϕ for a compact object. For simplicity, we restrict our analysis to the static case and consider a spherically symmetric body of radius R_c , homogeneous density ρ_c , and total mass $M_c = 4\pi\rho_c R_c^3/3$. Ignoring the backreaction on the metric, Eq. (2) reduces to

$$\frac{d^2 \phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} = V_{,\phi} + \frac{\beta}{M_{\text{Pl}}} \rho(r) e^{\beta \phi / M_{\text{Pl}}}. \quad (4)$$

The density, $\rho(r)$, is equal to ρ_c for $r < R_c$ and to ρ_∞ for $r > R_c$, where ρ_∞ denotes the surrounding homogeneous matter density.

We denote by ϕ_c and ϕ_∞ the value of ϕ that minimizes V_{eff} with $\rho = \rho_c$ and ρ_∞ , respectively. The respective masses of small fluctuations are m_c and m_∞ . The boundary conditions specify that the solution be nonsingular at the origin ($d\phi/dr = 0$ at $r = 0$), and that the force on a test particle vanishes at infinity ($\phi \rightarrow \phi_\infty$ as $r \rightarrow \infty$).

For sufficiently large objects, the solution can be described as follows. Within the object, $r < R_c$, the field minimizes V_{eff} , and thus $\phi \approx \phi_c$. This holds true every-

where inside the object except within a thin shell of thickness ΔR_c below the surface where the field grows. Outside the object, $r > R_c$, the profile for ϕ is essentially that of a massive scalar, $\phi \sim \exp(-m_\infty r)/r$, and tends to ϕ_∞ for $r \gg R_c$, as required by the second boundary condition.

A detailed calculation [4] shows that the thickness of the thin shell, ΔR_c , is related to ϕ_∞ , ϕ_c , and the Newtonian potential of the object, $\Phi_c = M_c/8\pi M_{\text{Pl}}^2 R_c$, by

$$\frac{\Delta R_c}{R_c} \approx \frac{\phi_\infty - \phi_c}{6\beta M_{\text{Pl}} \Phi_c}. \quad (5)$$

Moreover, the exterior solution ($r > R_c$) is given by [4]

$$\phi(r) \approx -\phi_\infty \left(1 - \frac{\phi_\infty - \phi_c}{6\beta M_{\text{Pl}} \Phi_c} \right) \frac{R_c e^{-m_\infty(r-R_c)}}{r} + \phi_\infty. \quad (6)$$

Assuming that the density contrast is high, $\phi_c \ll \phi_\infty$, and in the limit that the shell is thin, $\Delta R_c/R_c \ll 1$, Eqs. (5) and (6) combine to give, for $r > R_c$,

$$\phi(r) \approx -\left(\frac{\beta}{4\pi M_{\text{Pl}}} \right) \left(\frac{3\Delta R_c}{R_c} \right) \frac{M_c e^{-m_\infty(r-R_c)}}{r} + \phi_\infty. \quad (7)$$

The above applies only to objects satisfying the thin-shell condition: $\Delta R_c/R_c \ll 1$. From Eq. (5), whether or not this condition is satisfied depends on the ratio of the difference in ϕ potential, $\phi_\infty - \phi_c$, to the Newtonian potential of the object, Φ_c . In particular, for fixed $\phi_\infty - \phi_c$ (i.e., fixed density contrast), then the more massive the object, the easier it is to satisfy this condition.

Objects with $\Delta R_c/R_c \geq 1$, however, do not satisfy the thin-shell condition. Instead, one has $\phi \sim \phi_\infty$ everywhere in this case, and the exterior solution is

$$\phi(r) \approx -\left(\frac{\beta}{4\pi M_{\text{Pl}}} \right) \frac{M_c e^{-m_\infty(r-R_c)}}{r} + \phi_\infty. \quad (8)$$

Comparison with Eq. (7) shows that the ϕ profile outside large objects is suppressed by a factor of $\Delta R_c/R_c \ll 1$.

The thin-shell effect is a consequence of the nonlinearity of Eq. (4). It follows from requiring the above boundary conditions as well as continuity of ϕ and $d\phi/dr$ at $r = R_c$. Satisfying these conditions for sufficiently large objects inevitably leads to a thin shell.

This is confirmed by numerical calculations. Consider, e.g., $V(\phi) = M^5/\phi$ with $M \approx 6 \text{ mm}^{-1}$ and $\beta = 1$. (We will find that $M \lesssim 1 \text{ mm}^{-1}$, so this is a realistic choice.) The densities are $\rho_c = 1 \text{ g/cm}^3$ and $\rho_\infty = 10^{-4} \text{ g/cm}^3$, mimicking a ball of Be in air, corresponding to $\phi_c/M = 1$ and $\phi_\infty/M = 100$, respectively. Figure 2 shows the numerical solution for an object of radius $R_c = 40M^{-1}$. Equation (5) predicts a thin shell of thickness $\Delta R_c/R_c \approx 0.0625$ in this case. This is confirmed by the numerics. Indeed, we see from Fig. 2 that $\phi \approx \phi_c$ everywhere inside the object, except within $38 \leq rM \leq 40$. Moreover, the dotted line is a plot of Eq. (6) and agrees to within 2% with the numerics.

Figure 3 is the solution for an object with $R_c = M^{-1}$. The numerics confirm that there is no thin shell in this case. Indeed, $\phi \sim \phi_\infty$ everywhere. Moreover, the dotted line is a plot of Eq. (8) and is barely distinguishable from the numerical solution. This proves unambiguously that the thin-shell effect is real, and that the above expressions provide very good approximations to the actual solution. We should stress that these conclusions are not specific to the particular values of n , β , and M chosen here.

Let us apply these results to the Earth, crudely modeled as a sphere of radius R_\oplus and density $\rho_\oplus = 10 \text{ g/cm}^3$, with an atmosphere 10 km thick with $\rho_{\text{atm}} \approx 10^{-3} \text{ g/cm}^3$. Far away the matter density is approximately that of baryonic gas and dark matter in our neighborhood of the Milky Way: $\rho_G \approx 10^{-24} \text{ g/cm}^3$. Henceforth, ϕ_\oplus , ϕ_{atm} , and ϕ_G denote the value of ϕ which minimizes V_{eff} for $\rho = \rho_\oplus, \rho_{\text{atm}}$, and ρ_G , respectively. The respective masses are m_\oplus, m_{atm} , and m_G .

The Earth must have a thin shell, for otherwise unacceptably large violations of the EP will ensue [4]. Thus the ϕ field outside the Earth is given by Eq. (7) with $\phi_\infty = \phi_G$ and $m_\infty = m_G$:

$$\phi(r) \approx -\left(\frac{\beta}{4\pi M_{\text{Pl}}}\right)\left(\frac{3\Delta R_\oplus}{R_\oplus}\right)\frac{M_\oplus e^{-m_G r}}{r} + \phi_G, \quad (9)$$

where $\Delta R_\oplus/R_\oplus = (\phi_G - \phi_\oplus)/6\beta M_{\text{Pl}}\Phi_\oplus \ll 1$. In fact, not only must the Earth have a thin shell, but so must the atmosphere. This results in a more stringent condition [4]

$$\frac{\Delta R_\oplus}{R_\oplus} = \frac{\phi_G - \phi_\oplus}{6\beta M_{\text{Pl}}\Phi_\oplus} < 10^{-7}. \quad (10)$$

It then follows that $\phi \approx \phi_{\text{atm}}$ in the atmosphere.

For an inverse power-law potential, $V(\phi) = M^{4+n}\phi^{-n}$, Eq. (10) can be translated into a constraint on the scale M which, for n and β of order unity, is given by [4]

$$M \lesssim 10^{-3} \text{ eV} \approx (1 \text{ mm})^{-1}. \quad (11)$$

Remarkably, this coincides with the energy scale associated with the dark energy causing cosmic acceleration [5].

Equation (11) can also be expressed as a bound on the chameleon interaction range in the atmosphere (m_{atm}^{-1}), in the solar system (m_G^{-1}), and on cosmological scales today

(m_0^{-1}). For $n \leq 2$ and β of order unity, we find

$$\begin{aligned} m_{\text{atm}}^{-1} &\lesssim 1 \text{ mm} - 1 \text{ cm}, & m_G^{-1} &\lesssim 10 - 10^4 \text{ AU}, \\ m_0^{-1} &\lesssim 0.1 - 10^3 \text{ pc}. \end{aligned} \quad (12)$$

While ϕ -mediated interactions are short range in the atmosphere, ϕ is essentially free on solar system scales.

Laboratory tests.—We now argue that Eq. (11) ensures that laboratory tests of gravity are satisfied. Since these are usually performed in vacuum, we need an approximate solution for ϕ inside a vacuum chamber, which we model as a spherical cavity of radius R_{vac} . As a boundary condition, we impose $\phi \rightarrow \phi_{\text{atm}}$ far from the chamber. [Note that we use ϕ_{atm} rather than ϕ_\oplus in the laboratory since the field goes from ϕ_\oplus to ϕ_{atm} within $m_{\text{atm}}^{-1} \sim 1 \text{ mm}$ from the Earth's surface. See Eq. (12).] Numerical calculations then reveal that, inside the chamber, one has $\phi \approx \phi_{\text{vac}}$, where ϕ_{vac} satisfies $m^{-1}(\phi_{\text{vac}}) \equiv V_{,\phi\phi}^{-1/2}(\phi_{\text{vac}}) = R_{\text{vac}}$. That is, ϕ_{vac} is the field value about which the interaction range is of order R_{vac} . Intuitively, this is because $\rho \approx 0$ inside the chamber, and thus the only scale is R_{vac} , the size of the chamber.

Hence ϕ is essentially free within the vacuum chamber and generates a fifth-force correction to Newton's constant. If the two test masses used to measure G have no thin shell, then the correction will be of order unity for $\beta \sim \mathcal{O}(1)$, which is clearly ruled out. Thus the test masses must have a thin shell. If this is the case, then the ϕ field they generate is given by Eq. (7) with $\phi_\infty = \phi_{\text{vac}}$ and $m_\infty = R_{\text{vac}}^{-1}$, and the correction to G is of order $(\Delta R_c/R_c)^2$, where R_c is the radius of the test mass. Therefore, given the current accuracy of 10^{-3} on the value of G [2], this requires

$$\frac{\Delta R_c}{R_c} = \frac{\phi_{\text{vac}} - \phi_c}{6\beta M_{\text{Pl}}\Phi_c} \lesssim 10^{-3/2}. \quad (13)$$

This ensures that bounds from laboratory searches of a fifth force and EP violations are satisfied. For $V = M^{4+n}/\phi^n$ with β and n of order unity, and for typical test bodies inside a vacuum chamber of $R_{\text{vac}} \lesssim 1 \text{ m}$, it is easy to show that this condition follows from Eq. (11).

Solar system tests.—Tests of general relativity from solar system data are easily satisfied in our model because

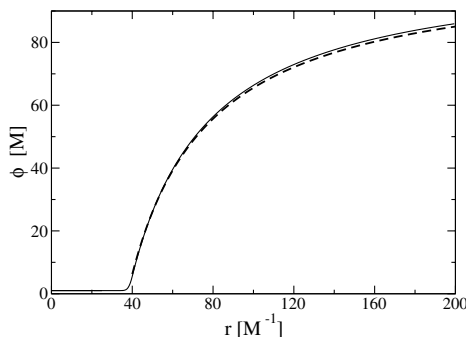


FIG. 2. Example of solution with thin shell.

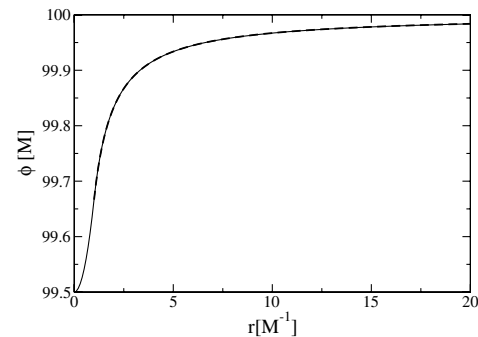


FIG. 3. Example of solution without thin shell.

of the thin-shell effect which suppresses the ϕ force between large objects. To see this, consider the profile generated by the Earth given in Eq. (9). Comparing with Eq. (8), we see that it can be thought of as the profile for a nearly massless scalar field with effective coupling

$$\beta_{\text{eff}} = 3\beta \cdot \frac{\Delta R_{\oplus}}{R_{\oplus}} < 3\beta \times 10^{-7}, \quad (14)$$

where we have used Eq. (10) in the last step. Hence, to describe the effects on planetary motion, we may think of the chameleon as a free scalar field with coupling β_{eff} . Since β_{eff} is so small, however, all bounds from solar system tests of gravity are easily satisfied [4]. For instance, consider light-deflection measurements [2]. Treating ϕ as a Brans-Dicke field with an effective Brans-Dicke parameter given by $3 + 2\omega_{\text{BD}} = (2\beta_{\text{eff}}^2)^{-1}$, we see from Eq. (14) that the constraint from light deflection, $\omega_{\text{BD}} > 3500$, is trivially satisfied. Similar arguments show that all current constraints from tests of gravity are satisfied [4].

Predictions for near-future tests of gravity in space.— Although ϕ mediates short-range interactions on Earth, we have seen that it is essentially a free field in the solar system. Thus the magnitude of EP violations and fifth force in our model are drastically different in space than on Earth. This opens the door to new and unexpected outcomes for near-future satellite experiments that will test the EP and search for a fifth force in space [3].

Consider the SEE project which will measure G in space by accurately determining the orbit of two test masses at an altitude of ≈ 1000 km. For a wide range of parameters, we predict that SEE will find a value for G different by $\mathcal{O}(1)$ corrections from that measured on Earth, due to fifth-force contributions that are significant in orbit but exponentially suppressed in the laboratory.

The *sine qua non* for this result is for the satellite *not* to have a thin shell, i.e., $\Delta R_{\text{SEE}}/R_{\text{SEE}} > 1$. The current design for the capsule has $\Phi_{\text{SEE}} \approx 10^{-24} \approx 10^{-15}\Phi_{\oplus}$. Combining with Eq. (10), it follows that the satellite will fail to have a thin shell if

$$10^{-15} < \frac{\Delta R_{\oplus}}{R_{\oplus}} < 10^{-7}. \quad (15)$$

Equation (15) ensures that the background value of the chameleon is essentially unperturbed by the satellite and thus that the ϕ -mediated force is long range within the capsule. Hence, the total force, gravitational plus chameleon mediated, between two bodies of mass M_i and coupling β_i , $i = 1, 2$, is

$$|\vec{F}| = \frac{GM_1M_2}{r^2}(1 + 2\beta_1\beta_2). \quad (16)$$

It follows, therefore, that SEE will measure an effective Newton's constant, $G_{\text{eff}} = G(1 + 2\beta_1\beta_2)$, which differs by order unity from the measured value on Earth.

Similarly, consider the resulting EP violations. The ϕ profile within the capsule [see Eq. (9)] results in an extra acceleration component a_{ϕ} for a test body with coupling β of $(a_{\phi}/a_N) \sim \beta^2 \Delta R_{\oplus}/R_{\oplus}$, where a_N is the Newtonian acceleration. For two bodies of different composition, this yields a relative difference in free-fall acceleration of

$$\eta \equiv \frac{\Delta a}{a} \approx 10^{-4} \beta^2 \frac{\Delta R_{\oplus}}{R_{\oplus}}, \quad (17)$$

where the numerical factor is appropriate for Be and Nb test masses [7] as used in STEP. Combining with Eq. (15) yields an allowed range of

$$\beta^2 \times 10^{-19} < \eta < \beta^2 \times 10^{-11}, \quad (18)$$

which overlaps with the sensitivity range of STEP ($\eta \gtrsim 10^{-18}$), GG ($\eta \gtrsim 10^{-17}$), and μ SCOPE ($\eta \gtrsim 10^{-15}$). Amazingly, η can be larger than 10^{-13} , the current bound from the (ground-based) Eöt-Wash experiment [8].

If the SEE Project measures a value for G different from on Earth, or if the STEP satellite finds an EP-violating signal stronger than allowed by laboratory experiments, this will constitute a signature for our model, for it would otherwise be difficult to reconcile the results in space with those on Earth.

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