Unified Derivation of Tunneling Times from Decoherence Functionals

Norifumi Yamada*

Department of Information Science, Fukui University, 3-9-1 Bunkyo, Fukui, Fukui 910-8507, Japan

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The four tunneling times, the Larmor time, the Büttiker-Landauer time, the Bohm-Wigner time, and the Pollak-Miller time, originally obtained from very different physical models, are derived in a unified manner from the Gell-Mann–Hartle decoherence functionals. The origin of the two types of derivatives in the expressions for these tunneling times is clarified at the level of Feynman paths.

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Tunneling is one of the most important quantum phenomena with a wide range of applications. The scanning tunneling microscope is a powerful tool in surface science, and various types of semiconductor devices have been developed that utilize tunneling effects. In spite of this remarkable success, there still remains a fundamental question about tunneling. That is, we still do not have a good understanding about tunneling time [1–5]. This should be compared to the case of tunneling probability. The concept of tunneling probability is clear; it is simply the ratio of the number of particles that are finally transmitted to the number of incident particles. By contrast, discussions are continuing at the conceptual level for tunneling time. This was, however, not a serious problem for decades, because (i) the stationary treatment of tunneling worked very well and (ii) the estimated tunneling times (e.g., 10^{-15}–10^{-14} sec [6]) were too short to relate them to experiment. Today, however, the importance of tunneling dynamics has been recognized in many fields and the progress of time-resolved spectroscopy has made it possible to observe the phenomena in solids whose time scales are comparable to the tunneling times.

In the literature, we find various tunneling *times* that are based on different ideas of characterizing the time spent by a particle under the barrier. "Which is the correct tunneling time?" used to be the central question in the tunneling time problem. Today, however, a certain consensus seems to exist that the various tunneling times should be understood in a broad sense that they characterize different or complementary aspects of tunneling process, rather than in the narrow sense as the time taken by the particle to tunnel through the barrier region. Among various tunneling times, of special importance seems to be the following four tunneling times [7–9] that are expressed in terms of the derivatives of the transmission amplitude $T = |T|e^{i\theta}$: the Larmor time $\tau_{\rm LM} = -\hbar\partial\theta/\partial V$ [10], the Büttiker-Landauer time $\tau_{\rm BL} = -\hbar\partial \ln|T|/\partial V$ [11], the Bohm-Wigner time (phase time) $\tau_{\rm BW} = \hbar\partial\theta/\partial E$ [12], and the Pollak-Miller time $\tau_{\rm PM} = \hbar\partial \ln|T|/\partial E$ [13], where V is the height of the potential and E the energy of the incident particles. These times have been extensively studied not only theoretically but also experimentally [14,15]. The physical models behind the four tunneling times are very different: $\tau_{\rm LM}$ was obtained by considering the spin precession by a small magnetic field confined in the barrier region, $\tau_{\rm BL}$ by considering the effect of an oscillating barrier on the tunneling particles, $\tau_{\rm BW}$ by following the peak motion of a broad wave packet, and $\tau_{\rm PM}$ by generalizing the classical collision time with the quantum mechanical flux-flux correlation function. It is not surprising that the four tunneling times can be expressed in terms of the transmission amplitude, since it describes the tunneling characteristics, but their similarity (they are V or E derivative of θ or $|T|$) is somewhat a surprise because the models are quite different.

In this Letter, we derive the four tunneling times in a unified manner without relying on specific models by using the Gell-Mann–Hartle decoherence functionals [16] for real time Feynman histories [17]. This derivation reveals that the two types of derivatives ($\partial/\partial V$ and $\partial/\partial E$) can be understood as resulting from two different definitions of the "time that a Feynman path takes to traverse the barrier region." We first review the decoherence functional for tunneling time. We then introduce two types of tunneling times, the resident (or dwell) time for transmission and the passage time, and calculate the decoherence functionals for both types of tunneling times. We derive $\tau_{\rm LM}$ and $\tau_{\rm BL}$ from the decoherence functional for the resident time, and $\tau_{\rm BW}$ and $\tau_{\rm PM}$ (with an additional time τ_0) from the decoherence functional for the passage time.

Consider the tunneling of a particle represented by a wave packet $\Psi(x, t)$. The particle is incident from the left on the rectangular barrier of height V_0 that extends from $x = 0$ to $x = d$. At $t = 0$, the packet is on the left side of the barrier. The tunneling probability P, the probability to find the particle on the right side of the barrier at sufficiently late times, is given by

$$P = \lim_{t\to\infty} \int_d^\infty dx|\Psi(x, t)|^2. \qquad (1)$$

We use $\Psi(x, t) = \int dx_0 K(x, t; x_0, 0)\Psi(x_0, 0)$, where K is the propagator. It is sufficient to consider $K(x, t; x_0, 0)$ only for $x_0 < 0$ and $x > d$ because the initial packet is localized in $x < 0$ and the integration range in Eq. (1) is $x > d$. Feynman's path integral [17] tells us that $K(x, t; x_0, 0)$ can be given as the sum of $e^{iS/\hbar}$ (S being the action) over all the paths $\{x(t)\}$ that connect $(x_0, 0)$ and (x, t). Let $\tau[x(\cdot)]$ be the "time taken by a Feynman path $x(t)$ to traverse the barrier region." Classifying all the paths according to the values of $\tau[x(\cdot)]$, we have

$$K(x, t; x_0, 0) = \int_0^t d\tau K(x, t; x_0, 0|\tau), \tag{2}$$

where $K(x, t; x_0, 0|\tau)$ is the sum of $e^{iS/\hbar}$ over those paths that satisfy $\tau[x(\cdot)] = \tau$. From Eq. (2), we have

$$\Psi(x, t) = \int_0^t d\tau \Psi(x, t|\tau), \tag{3}$$

where $\Psi(x, t|\tau) \equiv \int dx_0 K(x, t; x_0, 0|\tau)\Psi(x_0, 0)$. Using Eq. (3), we can write the tunneling probability as

$$P = \int_0^\infty d\tau' \int_0^\infty d\tau D(\tau', \tau), \tag{4}$$

$$D(\tau', \tau) \equiv \lim_{t\to\infty} \int_d^\infty dx \Psi(x, t|\tau')\Psi^*(x, t|\tau), \tag{5}$$

where $D(\tau', \tau)$ is the Gell-Mann–Hartle decoherence functional [16] for the case of tunneling time.

The general form of the decoherence functional is

$$D(\alpha', \alpha) = \int_{\alpha'} \delta q' \int_\alpha \delta q \delta(q'_f - q_f) \times \exp\{i(S[q'(\cdot)] - S[q(\cdot)])/\hbar\}\rho(q'_0, q_0), \tag{6}$$

where $\mathrm{Re}\, D(\alpha', \alpha)$ is a measure of the interference between the coarse-grained classes of Feynman histories α' and α both connecting (q_0, t_0) and (q_f, t_f), the integrals $\int_{\alpha'} \delta q'$ and $\int_\alpha \delta q$ represent the sums over paths with appropriate constraints consistent with the coarse grainings, and ρ is the initial density matrix. In our problem, α' (or α) corresponds to the coarse-grained class of paths that satisfy the constraint $\tau[x(\cdot)] = \tau'$ (or τ), $\int_{\alpha'} \delta q'$ and $\int_\alpha \delta q$ are the restricted path integrals subject to the constraints, and $\rho(x'_0, x_0) = \Psi(x'_0, 0)\Psi^*(x_0, 0)$. With these specializations, Eq. (6) reduces to Eq. (5) [18].

If $\mathrm{Re}\, D(\tau', \tau)$ is proportional to $\delta(\tau' - \tau)$, we have, by writing the proportionality coefficient as $P(\tau)$,

$$\mathrm{Re}\, D(\tau', \tau) = P(\tau)\delta(\tau' - \tau), \tag{7}$$

which is the *weak decoherence condition* [16] in the consistent history approach [16,19]. The approach claims that, if Eq. (7) holds, then a probability distribution can be defined for tunneling time; the probability distribution of tunneling times for transmitted particles is given by $\tilde{P}(\tau) \equiv P(\tau)/P$. To consider (the possibility of) a probability distribution of tunneling times has been a point of view adopted by many authors. Landauer and Martin [3] stated, "Whether traversal time is best viewed as such a distribution, or as a single time scale indicator, is still an open question in the opinion of the authors." Now, let us consider the following quantity:

$$I[F] \equiv \frac{1}{P} \int_0^\infty d\tau' \int_0^\infty d\tau F(\tau', \tau) D(\tau', \tau). \tag{8}$$

If we assume Eq. (7) to hold, we can show, by using the Hermiticity $D(\tau, \tau') = D^*(\tau', \tau)$, that $I[(\tau' + \tau)/2] = \int_0^\infty d\tau \tau \tilde{P}(\tau)$. Therefore, $I[(\tau' + \tau)/2]$ is the average tunneling time *if* Eq. (7) holds. Because of the interference between Feynman paths, however, Eq. (7) does not hold [18,20]. We formally call $I[(\tau' + \tau)/2]$ the "quasiaverage" in the sense of a quantity with the dimension of time that "fails to become" the true average due to the interference. The quasiaverage is real-valued due to the Hermiticity of D. Another real-valued quantity $\sigma^2 \equiv I[\tau'\tau] - I^2[(\tau' + \tau)/2]$ is the variance of tunneling times *if* Eq. (7) holds. Since it does not hold, we formally call σ^2 the "quasivariance."

Before calculating $D(\tau', \tau)$, note that $\tau[x(\cdot)]$ can be defined in several ways, since a Feynman path crosses $x = 0$ and $x = d$ many times in general. We consider two definitions (see Fig. 1): (i) $\tau[x(\cdot)]$ is defined as the sum of the times τ_n during which the path is in the barrier region [21], (ii) $\tau[x(\cdot)] \equiv \tau_{\mathrm{f}} - \tau_{\mathrm{i}}$, where τ_{f} is the last time the path leaves the barrier region and τ_{i} is the first time it enters the region [22]. Although these definitions are at the level of Feynman paths, it would be natural to expect that there exist two families of tunneling times, the resident (or dwell) time for transmission and the passage time, which correspond to (i) and (ii), respectively. We will see that τ_{LM} and τ_{BL} belong to the former family, while τ_{BW} and τ_{PM} belong to the latter. Although tentative, the resident time might be the time for the barrier, while the passage time might be the time for the particle, as Fig. 1 suggests. Existing approaches might then be understood as follows: the tunneling time for the barrier is measured by the particle in the Larmor clock approach [10,23], while it is measured by the barrier in the modulating barrier approach [11]; in the wave packet following approach [12], the tunneling time for the particle is measured by the particle. We attach subscript r to quantities for resident time, and p to those for passage time.

To calculate $D(\tau', \tau)$, we need to know $\Psi(x, t|\tau)$. For the case of resident time, we have, from [24],

$$\Psi_{\mathrm{r}}(x, t|\tau) = e^{-iV_0\tau/\hbar} \int \frac{dV}{2\pi\hbar} e^{iV\tau/\hbar}\Psi_V(x, t), \tag{9}$$

where $\Psi_V(x, t)$ is the transmitted packet when the poten-

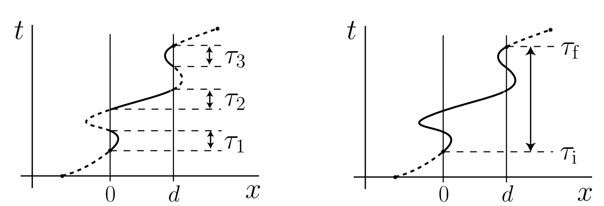

FIG. 1. Left: The solid parts of the Feynman path contribute to the resident (or dwell) time for transmission. Right: The solid part of the path contributes to the passage time.

tial height is V. Equation (9) leads us to [20]

$$D_{\rm r}(\tau', \tau) = \int dk |\psi(k)|^2 e^{-iV_0(\tau'-\tau)/\hbar} \mathcal{T}_k(\tau') \mathcal{T}_k^*(\tau), \quad (10)$$

$$\mathcal{T}_k(\tau) \equiv \int \frac{dV}{2\pi\hbar} e^{iV\tau/\hbar} T(k, V), \quad (11)$$

where $\psi(k)$ is the Fourier transform of $\Psi(x, 0)$, $T(k, V)$ is the transmission amplitude for the plane wave with wave number k incident on the square potential of height V, and $\int dV = \int_{-\infty}^{\infty} dV$ and $\int dk = \int_{-\infty}^{\infty} dk$ throughout the text. The V integrals arise from the constraint $\tau_{\rm r}[x(\cdot)] = \tau$ imposed on the restricted path integral for $K_{\rm r}(x, t; x_0, 0|\tau)$.

For the case of passage time, it was shown in [18] that

$$\Psi_{\rm p}(x, t|\tau) = K(d, \tau) \frac{\hbar}{im} \frac{\partial}{\partial x} \Psi_{\rm free}(x - d, t - \tau), \quad (12)$$

where $K(d, \tau) \equiv K(d, \tau; 0, 0)$ corresponds to the barrier penetration and $(\hbar/im)\partial\Psi_{\rm free}/\partial x$ the propagations before and after the barrier; $\Psi_{\rm free}$ is the wave function in the absence of the barrier and $(\hbar/im)\partial/\partial x$ results from the path decomposition expansion [25] used to define $\tau_{\rm i}$ and $\tau_{\rm f}$. Using Eq. (12) and noting that $\Psi_{\rm free}(x, t) = \int (dk/\sqrt{2\pi})\psi(k) e^{ikx - iEt/\hbar}$ with $E \equiv \hbar^2 k^2/2m$, we can calculate the right-hand side of Eq. (5) to have

$$D_{\rm p}(\tau', \tau) = \left(\frac{\hbar}{m}\right)^2 K(d, \tau') K^*(d, \tau) \times \int dk |\psi(k)|^2 k^2 e^{-iE(\tau-\tau')/\hbar}. \quad (13)$$

Note that $K(d, \tau)$ can be written as

$$K(d, \tau) = \int \frac{dk}{2\pi} e^{-iE\tau/\hbar} T(k, V_0), \quad (14)$$

which follows from the eigenfunction expansion of the propagator for $x_0 \leq 0$ and $x \geq d$, i.e., $K(x, t; x_0, 0) = \int (dk/2\pi) T(k, V_0) e^{ik(x-d-x_0) - iEt/\hbar}$ (throughout the text, the transmission amplitude is defined in such a way that the stationary solution for $x \geq d$ is given by $Te^{ik(x-d)}$).

In the monochromatic limit $|\psi(k)|^2 \to \delta(k - k_0)$, the decoherence functional is factorized as

$$D_{\rm i}(\tau', \tau) = A_{\rm i}(\tau') A_{\rm i}^*(\tau), \quad (15)$$

where, with $E_0 = \hbar^2 k_0^2/2m$,

$$A_{\rm r}(\tau) = e^{-iV_0\tau/\hbar} \int \frac{dV}{2\pi\hbar} e^{iV\tau/\hbar} T(k_0, V), \quad (16)$$

$$A_{\rm p}(\tau) = \frac{\hbar k_0}{m} e^{iE_0\tau/\hbar} \int \frac{dk}{2\pi} e^{-iE\tau/\hbar} T(k, V_0). \quad (17)$$

We now derive $\tau_{\rm LM}$, $\tau_{\rm BL}$, $\tau_{\rm BW}$, and $\tau_{\rm PM}$ from $D_{\rm i}(\tau', \tau)$ by calculating Eq. (8) for some specific F's. First, let us put $F = \tau'$. For the case of resident time (i = r), we substitute Eq. (15) with Eq. (16) into Eq. (8) and use

$$\int_0^\infty d\tau A_{\rm r}(\tau) = T_0, \quad (18)$$

$$\int_0^\infty d\tau \tau A_{\rm r}(\tau) = i\hbar \frac{\partial T_0}{\partial V_0}, \quad (19)$$

where $T_0 \equiv T(k_0, V_0)$. To prove Eq. (18), use Eq. (16) and replace $\int_0^\infty d\tau$ by $\int_{-\infty}^{\infty} d\tau$, which is allowable because $T(k_0, V)$ has simple poles only in the upper half of the complex V plane [26]. The V_0 derivative of Eq. (18) gives Eq. (19). Using Eqs. (18) and (19) and noting that $P = |T_0|^2$ in the monochromatic limit, we have $I_{\rm r}[\tau'] = i\hbar\partial \ln T_0/\partial V_0$. For the case of passage time (i = p), we substitute Eq. (15) with Eq. (17) into Eq. (8) and use

$$\int_0^\infty d\tau A_{\rm p}(\tau) = T_0, \quad (20)$$

$$\int_0^\infty d\tau \tau A_{\rm p}(\tau) = \frac{\hbar}{i}\left(\frac{\partial T_0}{\partial E_0} - \frac{T_0}{2E_0}\right) \quad (21)$$

to have $I_{\rm p}[\tau'] = -i\hbar\partial \ln T_0/\partial E_0 + i\hbar/2E_0$, where Eq. (20) can be proved by using the Mittag-Leffler expansion of $T(k, V_0)$ [Eq. (24) of Ref. [27]], the details of which will be given elsewhere. The obtained $I_{\rm i}[\tau']$ can be rewritten as follows:

$$I_{\rm r}[\tau'] = \tau_{\rm LM} - i\tau_{\rm BL}, \quad I_{\rm p}[\tau'] = \tau_{\rm BW} - i(\tau_{\rm PM} - \tau_0), \quad (22)$$

where $\tau_{\rm LM} = -\hbar\partial\theta_0/\partial V_0$, $\tau_{\rm BL} = -\hbar\partial \ln|T_0|/\partial V_0$, $\tau_{\rm BW} = \hbar\partial\theta_0/\partial E_0$, $\tau_{\rm PM} = \hbar\partial \ln|T_0|/\partial E_0$, and $\tau_0 \equiv \hbar/2E_0$ (note that $T_0 = |T_0|e^{i\theta_0}$). Next, we put $F = (\tau' \pm \tau)/2$. Noting that $I_{\rm i}[\tau] = I_{\rm i}^*[\tau']$, we immediately have

$$I_{\rm r}[(\tau' + \tau)/2] = \tau_{\rm LM}, \quad I_{\rm r}[(\tau - \tau')/2] = i\tau_{\rm BL}, \quad (23)$$

$$I_{\rm p}[(\tau' + \tau)/2] = \tau_{\rm BW}, \quad I_{\rm p}[(\tau - \tau')/2] = i(\tau_{\rm PM} - \tau_0). \quad (24)$$

This is the main result. Recall that $I_{\rm i}[(\tau' + \tau)/2]$ and $I_{\rm i}[\tau'\tau] - I_{\rm i}^2[(\tau' + \tau)/2]$ are, respectively, the quasiaverage and the quasivariance of tunneling times. In the monochromatic limit, the quasivariance can be shown to be equal to $|I_{\rm i}[(\tau - \tau')/2]|^2$, so that $|I_{\rm i}[(\tau - \tau')/2]|$ is the "quasideviation." We may thus interpret the result as follows: $\tau_{\rm LM}$ and $|\tau_{\rm BL}|$ are, respectively, the quasiaverage and the quasideviation of resident times, and $\tau_{\rm BW}$ and $|\tau_{\rm PM} - \tau_0|$ are, respectively, the quasiaverage and the quasideviation of passage times. To understand the physical meaning of these quasivalues, it would be necessary to explore the relationship between $D(\tau', \tau)$ and the time-dependent wave functions, where the relationship between the decoherence functional approach and the quantum shutter approach [27,28] might be helpful. In passing, $I_{\rm i}[(\tau' - \tau)^2]$, which must vanish if Eq. (7) holds, does not vanish in general.

The complex tunneling time $\tau_{\mathrm{LM}} - i\tau_{\mathrm{BL}}$, now given as $I_{\mathrm{r}}[\tau']$, has been obtained from the path integral approaches [29,30], the systematic projector approach [31], and the weak measurement approach [32]. Another complex tunneling time $\tau_{\mathrm{BW}} - i\tau_{\mathrm{PM}}(= I_{\mathrm{p}}[\tau' - i\tau_0])$ has been obtained from the energy sensitivity approach [8,33]. Note that $I_{\mathrm{r}}[\tau'\tau] = \tau_{\mathrm{LM}}^2 + \tau_{\mathrm{BL}}^2$ and $I_{\mathrm{p}}[(\tau' - i\tau_0) \times (\tau + i\tau_0)] = \tau_{\mathrm{BW}}^2 + \tau_{\mathrm{PM}}^2$.

For a wave packet initial condition, we use Eq. (10) or Eq. (13) instead of Eq. (15). The resultant $I_{\mathrm{i}}[F]$ is found to be given by $I_{\mathrm{i}}[F]$ for the monochromatic case averaged over k with the weight $|\psi(k)|^2|T(k, V_0)|^2$. For example, τ_{LM} and τ_{BL} in Eq. (23) are replaced by $\langle\tau_{\mathrm{LM}}\rangle$ and $\langle\tau_{\mathrm{BL}}\rangle$, respectively, where $\langle\cdots\rangle$ stands for the k average.

To summarize, the Larmor time and the Büttiker-Landauer time can be derived from the decoherence functional for the resident time, while the Bohm-Wigner time and the Pollak-Miller time (with an additional time τ_0) can be derived from the decoherence functional for the passage time. This is also true for over-the-barrier propagation, since we have not assumed $E_0 < V_0$. The decoherence functional approach will provide a basis for understanding various tunneling times in a unified manner without relying on specific models.

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*Electronic address: yamada@i1nws1.fuis.fukui-u.ac.jp

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