Effects of a Vertical Magnetic Field on Particle Confinement in a Magnetized Plasma Torus

S. H. Müller,* A. Fasoli,† B. Labit, M. McGrath, M. Podestà, and F. M. Poli
Centre de Recherches en Physique des Plasmas, Ecole Polytechnique Fédérale de Lausanne, Lausanne, Switzerland
(Received 3 March 2004; published 14 October 2004)

The particle confinement in a magnetized plasma torus with superimposed vertical magnetic field is modeled and measured experimentally. The formation of an equilibrium characterized by a parallel plasma current canceling out the grad B and curvature drifts is described using a two-fluid model. Characteristic response frequencies and relaxation rates are calculated. The predictions for the particle confinement time as a function of the vertical magnetic field are verified in a systematic experimental study on the TORPEX device, including the existence of an optimal vertical field and the anticorrelation between confinement time and density.

DOI: 10.1103/PhysRevLett.93.165003 PACS numbers: 52.55.Dy, 52.55.Hc

A simple toroidal plasma with a confining magnetic field in the toroidal direction is inherently not in equilibrium because charge-dependent grad B and curvature drifts lead to an electric field that, crossed with $\mathbf{B}$, causes an outward macroscopic motion. It has been proposed since the 1960s [1] that a small superimposed vertical magnetic field can allow the particles to generate short-circuiting currents that limit the electric field. If the plasma is confined by a conducting limiter or vacuum vessel, these currents can flow in closed loops and a quasiequilibrium state may be reached, where the plasma is confined for a significantly longer time than with a purely toroidal field.

During the 1980s, this confinement scheme has received significant attention in the context of tokamak preionization (current drive) by means of microwave injection in the range of the electron cyclotron (lower hybrid) frequency [2–4]. A balance between the parallel particle loss, where the spiral-shaped magnetic field lines intersect the vacuum vessel, and the loss due to $\mathbf{E}\times\mathbf{B}$ drifts was considered by Nakao *et al.* [5]. In their model, the magnitude of the vertical magnetic field B_z is the key parameter determining the relative importance of the two loss channels. A condition for a poloidal MHD equilibrium was formulated in terms of two one-dimensional partial differential equations [6]. Surprisingly, no systematic experimental study has been undertaken to date of this fundamental plasma phenomenon, related to one of the simplest possible plasma confinement schemes by a magnetic field.

This scheme has recently regained practical importance, as several toroidal devices have become operational to investigate the important question of microinstabilities, turbulence, and anomalous transport [7–10], relevant for magnetic fusion. Naturally, a good understanding of the basic equilibrium and transport mechanisms is necessary to address these issues.

In this Letter we investigate the fundamental properties of this confinement scheme via a direct comparison between a simple theoretical model and experimental data. In the first part we present an improved version of Nakao's model, adding a time dependent treatment of the formation phase of the equilibrium and a stability analysis. In the second part we present the first systematic series of measurements of the particle confinement time as a function of the vertical magnetic field, conducted on the TORPEX device [9,10], and compare the results with the model.

A time independent magnetic field $\mathbf{B}(r)\equiv B\hat{\mathbf{n}} = B(\cos\theta\hat{\boldsymbol{\varphi}} + \sin\theta\hat{\mathbf{z}})$ is assumed, where $\{r, \varphi, z\}$ are cylindrical coordinates. Considering only leading order terms in the inverse aspect ratio, a/R, the radial coordinate r is generally approximated by the major radius R. To qualitatively demonstrate the equilibrium mechanism, the plasma is modeled as consisting of a core region, quasineutral and spatially homogeneous, and an edge region, where charge can be accumulated. Such accumulation, driven by charge-dependent fluid motion, gives rise to an electric field $\mathbf{E}(t)$ in the core plasma, whose rate of change is

$$\dot{\mathbf{E}} = (en_e/\varepsilon_0)[u_r\hat{\mathbf{r}} + (u_z + \bar{v}_{\nabla B})\hat{\mathbf{z}}]. \tag{1}$$

Here $\mathbf{u}(t)\equiv\mathbf{u}_e - \mathbf{u}_i$ is the relative fluid velocity. Toroidal drifts do not cause charge separation due to periodicity, so there is no contribution of u_φ. $\bar{v}_{\nabla B}$ is the relative velocity-averaged ∇B drift in the z direction

$$\bar{v}_{\nabla B} = -\frac{\cos^4\theta}{RB_\varphi n_e}\sum_{\beta=e,i} n_\beta\left(\frac{T_{\parallel\beta}}{e}\right)\left(1+\frac{T_{\perp\beta}}{T_{\parallel\beta}}\right) = \text{const.} \tag{2}$$

The relevant equations of motion for the fluid velocities in the core are

$$\dot{\mathbf{u}}_\alpha = \frac{q_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{u}_\alpha\times\mathbf{B}) - \bar{\boldsymbol{\nu}}_{\alpha/\beta}\cdot(\mathbf{u}_\alpha - \mathbf{u}_\beta), \qquad \alpha,\beta = e,i, \qquad \alpha\neq\beta, \tag{3}$$

to which, due to the hypothesis of uniform profiles in the region of interest, convective and pressure terms do not contribute. Equations (1)–(3) represent a simple, self-consistent model demonstrating that friction due to Coulomb collisions leads to an equilibrium and that ion perpendicular motion in the transient phase (correspond-

 0031-9007/04/93(16)/165003(4)$22.50

ing to ion polarization drifts in a single particle picture) reduces the characteristic oscillation frequency of the system significantly compared to the value that would be derived from a one-dimensional model. To retain the possibility of a temperature anisotropy, we deal with anisotropic collision frequencies $\bar{\boldsymbol{\nu}} = \bar{\nu}_\perp(\hat{\boldsymbol{\psi}}\hat{\boldsymbol{\psi}} + \hat{\mathbf{r}}\hat{\mathbf{r}}) + \bar{\nu}_\| \hat{\mathbf{n}}\hat{\mathbf{n}}$, $\hat{\boldsymbol{\psi}} \equiv \hat{\mathbf{r}} \times \hat{\mathbf{n}}$. Setting $\dot{\mathbf{u}}_\alpha = 0$, $\dot{\mathbf{E}} = 0$, we obtain the equilibrium solution $u_{\psi,e} = u_{\psi,i} = E_r/B$, $u_{r,e} = u_{r,i} = -E_z \cos\theta/B$, $u_n = -\bar{v}_{\nabla B}/\sin\theta$,

$$E_z = \frac{m_e}{e}\frac{\bar{\nu}_\|^{e/i}\bar{v}_{\nabla B}}{\sin^2\theta}, \tag{4}$$

whereas E_r is as yet undetermined. The stability properties of the equilibrium can be investigated by formulating the problem in terms of the relative velocity $\mathbf{u}$. In the coordinate system $\{\hat{\boldsymbol{\psi}}, \hat{\mathbf{r}}, \hat{\mathbf{n}}\}$ we obtain

$$\ddot{\mathbf{u}} + \begin{pmatrix} \bar{\nu}_\perp & -\Omega & 0 \\ \Omega & \bar{\nu}_\perp & 0 \\ 0 & 0 & \bar{\nu}_\| \end{pmatrix} \cdot \dot{\mathbf{u}} + \mathcal{A} \cdot \mathbf{u} = -\omega_p^2 \bar{v}_{\nabla B}\hat{\mathbf{z}},$$
$$\mathcal{A} \equiv \begin{pmatrix} \omega_p^2\cos^2\theta - \Omega_e\Omega_i & 0 & \omega_p^2 \sin\theta\cos\theta \\ 0 & \omega_p^2 - \Omega_e\Omega_i & 0 \\ \omega_p^2 \sin\theta\cos\theta & 0 & \omega_p^2 \sin^2\theta \end{pmatrix}, \tag{5}$$

where $\omega_p^2 \equiv \omega_{pe}^2 + \omega_{pi}^2$, $\omega_{p\alpha}^2 \equiv q_\alpha^2 n_\alpha/(\varepsilon_0 m_\alpha)$ are the plasma frequencies and $\bar{\boldsymbol{\nu}} \equiv \bar{\boldsymbol{\nu}}_{e/i} + \bar{\boldsymbol{\nu}}_{i/e}$, $\Omega \equiv \Omega_e + \Omega_i$, $\Omega_\alpha \equiv q_\alpha B/m_\alpha$. Clearly, this represents coupled oscillations of the three components of $\mathbf{u}$. As we expect that polarization drifts will slow down the oscillations, we search for eigenfrequencies with $\omega \ll \{\omega_p, |\Omega_e|\}$, and find for the case of practical interest, $|\sin\theta| \ll 1$:

$$\omega \simeq -i\tfrac{1}{2}\bar{\nu}_\| \pm |\Omega_e\Omega_i|^{1/2}|\sin\theta|. \tag{6}$$

The other four eigenfrequencies are of the order of the plasma frequency and therefore of little interest in this context. The typical relaxation time towards the equilibrium is thus $2/\bar{\nu}_\|$ and the typical oscillation frequency $|\Omega_e\Omega_i|^{1/2}|\sin\theta|$, independent of the plasma density and significantly lower than what would have been obtained from a one-dimensional model, namely, $\omega_p|\sin\theta|$. A numerical solution of Eq. (5) with the initial conditions $\mathbf{u}(0) = 0$, $\dot{\mathbf{u}}(0) = 0$ (Fig. 1) reveals that $E_r \to 0$ as the system approaches equilibrium, consistent with the absence of mechanisms driving a radial charge separation.

Thus two "zero order" loss mechanisms are active in a magnetized torus with a superimposed vertical field: (1) the $\mathbf{E} \times \mathbf{B}$ flow due to the equilibrium electric field and (2) the parallel particle loss where the field lines intersect the vacuum vessel. In principle, these mechanisms are coupled, and to calculate the actual outgoing fluxes, the $\mathbf{E} \times \mathbf{B}$ shifted distribution functions should be taken as upstream boundary conditions for the kinetic plasma-sheath equation [11]. However, to qualitatively demonstrate the validity of the confinement scheme we

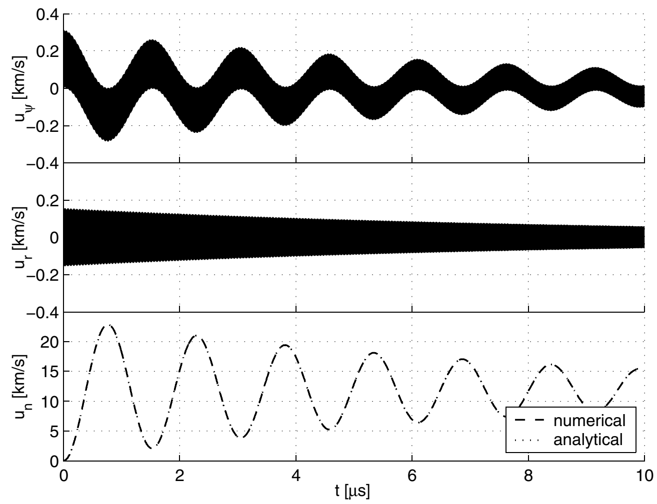

FIG. 1. Numerical solution of Eq. (5) for $B_z = 1$ mT. The symmetry of the radial oscillation entails $E_r \to 0$.

assume that the two loss channels are independent and estimate the $\mathbf{E} \times \mathbf{B}$ outgoing flux by its value in the plasma region: $\boldsymbol{\Gamma}_{\mathbf{E}\times\mathbf{B}} = n_0 v_{\mathbf{E}\times\mathbf{B}}\hat{\mathbf{r}}$, where $v_{\mathbf{E}\times\mathbf{B}} = -E_z B_\varphi/B^2 \propto |\sin\theta|^{-2}$. The parallel loss can be evaluated from the parallel flux at the magnetic presheath edge: $\boldsymbol{\Gamma}_\| = \frac{1}{2}n_0 c_s \hat{\mathbf{n}}$ [11,12], where $c_s \equiv (T_\|/m_i)^{1/2}$ is the sound speed. Here and in the following we assume $T \equiv T_e \gg T_i$. Integrating the continuity equation over the whole plasma torus and neglecting terms of order a/R we find

$$\left(\frac{dN}{dt}\right)_{\text{loss}} = -\frac{N}{\tau}, \qquad \tau = \frac{\pi}{2}\frac{a}{c_s|\sin\theta| + v_{\mathbf{E}\times\mathbf{B}}}. \tag{7}$$

Here, N is the total number of particles in the torus and τ is the confinement time, defined as the time constant of the exponential decay. The integration of the $\mathbf{E} \times \mathbf{B}$ flux has been performed only over the radial outer region $r > R$, as there is no inward flux coming from the walls. As expected from symmetry considerations, τ does not depend on the orientation of the magnetic field components, $\tau(B_z)$ is maximal for

$$|B_{z,\text{opt}}| = \left[\frac{2\bar{\nu}_\|^e |B_\varphi| m_e \sqrt{m_i T_\|}}{Re^2}\left(1 + \frac{T_\perp}{T_\|}\right)\right]^{1/3}, \tag{8}$$

$$\tau_{\max} = \frac{\pi}{3}\frac{a}{c_s}\frac{|B_\varphi|}{|B_{z,\text{opt}}|}, \tag{9}$$

where

$$\bar{\nu}_\|^e = \frac{n_e Z_{\text{eff}} e^4 \ln\Lambda}{\varepsilon_0^2 m_e^{1/2}(2\pi T_\|)^{3/2}}\frac{A - \operatorname{atan}A}{A^3}, \quad A \equiv \sqrt{\frac{T_\perp}{T_\|} - 1}, \tag{10}$$

Z_{eff} is the effective charge, and $\ln\Lambda$ is the Coulomb logarithm. $\tau_{\max}$ (9) exhibits the functional behavior

$$\tau_{\max} \propto \{a, B_\varphi^{2/3}, R^{1/3}, m_i^{1/3}, T_\|^{-1/6}, n_e^{-1/3}\}. \tag{11}$$

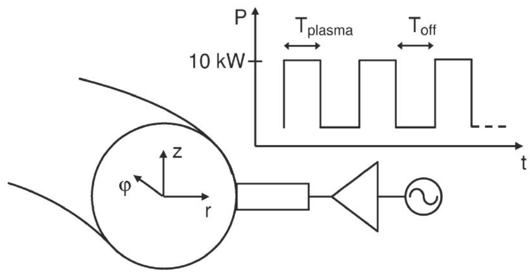

FIG. 2. Power modulation.

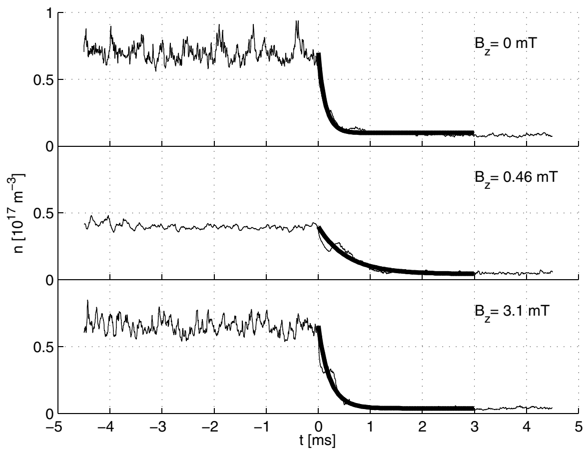

FIG. 3. Averaged plasma density decay at $r = 1.07$ m, $z = 0$, after the microwave source is turned off at $t = 0$, and exponential fit defining τ, shown for zero, optimal, and high vertical magnetic fields.

Three important conclusions can be drawn here: (1) the optimal vertical field depends only *weakly,* at most as the cubic root, on all plasma and machine parameters, (2) the confinement time can be significantly improved only by increasing the principal magnetic field or the machine size, (3) high confinement times correspond to low plasma densities.

The experimental tests have been performed on TORPEX [9,10], a toroidal device dedicated to basic plasma physics research with $R = 1$ m, $a = 0.2$ m, $|B_\varphi| \simeq 0.1$ T, $|B_z| \leq 3$ mT. Plasmas are produced and sustained for up to 100 ms by microwaves in the electron cyclotron range of frequencies, at 2.45 GHz. The power of up to 50 kW can be modulated with frequencies up to 20 kHz. Plasmas are diagnosed mainly with fixed and movable Langmuir probe arrays.

Experimentally, the particle confinement time τ can be estimated from the local exponential decays after the microwave power is turned off. We can thus verify the dependence of τ on several experimentally accessible parameters, focusing here on the vertical magnetic field that can be varied easily in the entire range where theory predicts interesting behavior. We use a hydrogen plasma made from a neutral gas pressure 3.5×10^{-5} mbar. The density is calculated from the ion saturation current measured with a movable Langmuir probe array biased at -35 V, positioned at $r = 1.07$ m, so that its tips measure eight vertical positions $z_{j=1,\ldots,8}$ simultaneously. By rotating the array we measure in a subsequent shot at $-z_j$. To improve statistics we modulate the injected power rectangularly between 0 and 10 kW at 50 Hz (Fig. 2), therefore obtaining five plasmas of 10 ms length per shot, and

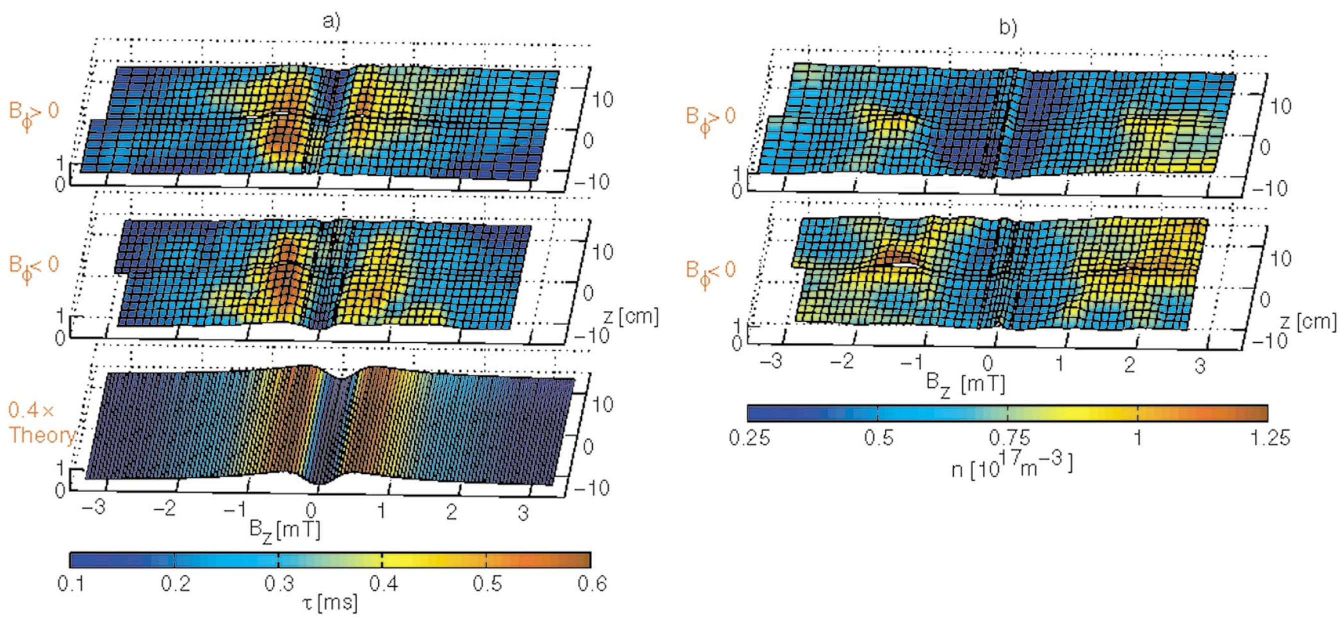

FIG. 4 (color). (a) Comparison of theoretical model predictions with experimental data on particle confinement time as a function of vertical magnetic field and vertical position. (b) Corresponding plasma density.

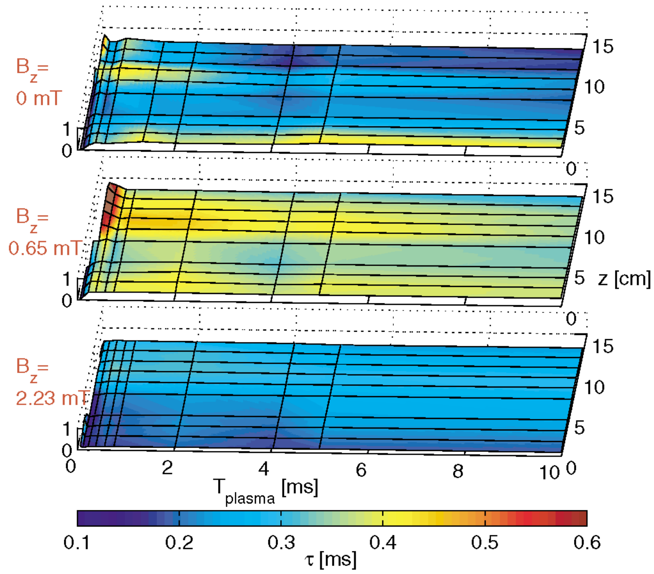

FIG. 5 (color). Confinement time vs plasma duration for three values of the vertical magnetic field, showing good stationarity. Note again that the confinement time is higher for $B_z = 0.65$ mT than for 0 and 2.23 mT.

repeat each shot twice. The average of these 15 signals is fitted with an exponential to obtain τ (Fig. 3).

The results of a scan of $B_z \in [-3, 3]$ mT and the comparison with the theoretical prediction are shown in Fig. 4 for the two orientations of the toroidal field. We clearly observe the existence of a maximum particle confinement time as a function of the vertical magnetic field. Large regions ($|z| \lesssim 7$ cm for most B_z) are identified where the departure of the confinement time from the assumed z invariance is within the experimental uncertainties. The predicted symmetries ($B_z \rightarrow -B_z, B_\varphi \rightarrow -B_\varphi$) are verified to a good degree. For a quantitative comparison we use the measured parameters $|B_\varphi| = 0.0766$ T, $n_0 \simeq 7.5 \times 10^{16}$ m^{-3}, $T_{\parallel e} \simeq 5$ eV. $Z_{\rm eff}$ is taken to be $Z_{\rm eff} \simeq 1$ as no sign of multiply ionized impurities is present in line integrated optical spectra ($\lambda \in [300, 900]$ nm). $T_{\perp e} = T_{\parallel e}$ is assumed, as the measured confinement times exceed the calculated times for temperature isotropization by about 1 order of magnitude, and $T_{\parallel i} = T_{\perp i} \simeq 1$ eV. With these parameters we obtain $\bar{\nu}_\parallel^e \simeq 2.5 \times 10^5$ s^{-1}.

The optimal vertical magnetic field according to Eq. (8) is $|B_{z,\rm opt}| \simeq 0.49$ mT, corresponding to a confinement time of $\tau_{\max} \simeq 1.5$ ms. Experimentally we find $|B_{z,\rm opt}| \sim 0.6$ mT (Fig. 4), in good agreement with theory, and $\tau_{\max} \sim 0.6$ ms, about a factor of 3 lower than the theoretical prediction. This discrepancy may be due, in addition to the simple assumption made in the model, to other mechanisms, such as microturbulence, which may contribute to the overall transport and reduce the confinement time [13]. The theoretical value given in this Letter must be understood as an upper boundary for the confinement time obtainable in the studied configuration.

By comparing Figs. 4(a) and 4(b) an anticorrelation between the confinement-time profile and the density profile is clearly observed. This is consistent with our theoretical arguments that indicate such anticorrelation: collisions hinder the short-circuiting capability of the electrons, thus leading to larger electric fields that enhance transport and losses.

We tested the stationarity of the equilibrium by reducing the duration of the plasma, $\mathcal{T}_{\rm plasma}$, from 10 ms to 60 μs, using duty-cycles $\mathcal{T}_{\rm plasma}/(\mathcal{T}_{\rm plasma} + \mathcal{T}_{\rm off})$ (Fig. 2) of 50% and 20%. We observe that the confinement time is independent on $\mathcal{T}_{\rm plasma}$ until our method to determine τ fails, as shown in Fig. 5. This is consistent with the theoretical prediction of a characteristic formation time of the equilibrium of only $2/\bar{\nu}_\parallel^e \sim 8$ μs.

In summary, a simple model based on the short-circuiting effect of the vertical magnetic field has been presented to describe the formation of an equilibrium and the background plasma confinement in a magnetized plasma torus. Several predicted features have been directly observed experimentally for the first time, including an anticorrelation between confinement time and density, and the existence of an optimal value of the vertical magnetic field for particle confinement.

We thank Professor K. Rypdal for valuable discussions. This work was partly funded by the Fonds National Suisse pour la Recherche Scientifique. M. McG. is supported by a Marie Curie Fellowship of EURATOM.

*Electronic addresses: stefan.mueller@epfl.ch; http://crppwww.epfl.ch/torpex
†Electronic address: ambrogio.fasoli@epfl.ch

[1] S. Yoshikawa, W. L. Harries, and R. M. Sinclair, Phys. Fluids **6**, 1506 (1963).
[2] S. Kubo *et al.*, Phys. Rev. Lett. **50**, 1994 (1983).
[3] F. Jobes *et al.*, Phys. Rev. Lett. **52**, 1005 (1984).
[4] K. Toi *et al.*, Phys. Rev. Lett. **52**, 2144 (1984).
[5] S. Nakao, K. Ogura, Y. Terumichi, and S. Tanaka, Phys. Lett. **96A**, 405 (1983).
[6] L. E. Zakharov and G. V. Pereverzev, Sov. J. Plasma Phys. **14**, 75 (1988).
[7] K. Rypdal, E. Gronvoll, F. Oynes, A. Fredriksen, R. J. Armstrong, J. Trulsen, and H. L. Pécseli, Plasma Phys. Controlled Fusion **34**, 1099 (1994).
[8] K. Gentle, W. Rowan, and S. Luckhardt, in *Bulletin of the APS* (AIP, New York, 2003), Vol. 48, poster LP1 74.
[9] M. Podestà, A. Fasoli, B. Labit, M. McGrath, S. Müller, and F. M. Poli, Europhys. Conf. Abstr. **27A**, P-4.164 (2003).
[10] A. Fasoli, B. Labit, M. McGrath, S. Müller, M. Podestà, and F. M. Poli, in *Bulletin of the APS* (Ref. [8]), Vol. 48, poster FP1 124.
[11] R. Chodura, Phys. Fluids **25**, 1628 (1982).
[12] P. C. Stangeby, *The Plasma Boundary of Magnetic Fusion Devices* (IOP Publishing, Bristol, 2000).
[13] F. J. Oynes, O. M. Olsen, H. L. Pecseli, A. Fredriksen, and K. Rypdal, Phys. Rev. E **57**, 2242 (1998).