Production of the $X(3872)$ in B-Meson Decay by the Coalescence of Charm Mesons

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If the recently discovered charmonium state $X(3872)$ is a loosely bound S-wave molecule of the charm mesons $\bar{D}^0 D^{*0}$ or $\bar{D}^{*0} D^0$, it can be produced in B-meson decay by the coalescence of charm mesons. If this coalescence mechanism dominates, the ratio of the differential rate for $B^+ \to \bar{D}^0 D^{*0} K^+$ near the $\bar{D}^0 D^{*0}$ threshold and the rate for $B^+ \to XK^+$ is a function of the $\bar{D}^0 D^{*0}$ invariant mass and hadron masses only. The identification of the $X(3872)$ as a $\bar{D}^0 D^{*0}/\bar{D}^{*0} D^0$ molecule can be confirmed by observing an enhancement in the $\bar{D}^0 D^{*0}$ invariant mass distribution near the threshold. An estimate of the branching fraction for $B^+ \to XK^+$ is consistent with observations if X has quantum numbers $J^{PC} = 1^{++}$ and if $J/\psi\pi^+\pi^-$ is one of its major decay modes.

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The recent unexpected discovery of a narrow charmonium resonance near 3.87 GeV challenges our understanding of heavy quarks and QCD. This mysterious state $X(3872)$ was discovered by the Belle Collaboration in electron-positron collisions through the B-meson decay $B^\pm \to K^\pm X$ followed by the decay $X \to J/\psi\pi^+\pi^-$ [1]. The discovery was confirmed by the CDF Collaboration using proton-antiproton collisions [2]. The X is much narrower than all other charmonium states above the threshold for decay into a pair of charm mesons. Its mass is also extremely close to the threshold for decay into the charmed mesons $\bar{D}^0 D^{*0}$ or $\bar{D}^{*0} D^0$.

The proposed interpretations of the $X(3872)$ include a D-wave charmonium state with quantum numbers $J^{PC} = 2^{--}$ or 2^{-+}, an excited P-wave charmonium state with $J^{PC} = 1^{++}$ or 1^{+-}, a "hybrid charmonium" state in which a gluonic mode has been excited, and a $\bar{D}^0 D^{*0}/\bar{D}^{*0} D^0$ molecule [3–13]. The possibility that charm mesons might form molecular states was considered some time ago [14–16]. If the binding is due to pion exchange, the most favorable channels are S wave with quantum numbers $J^{PC} = 1^{++}$ or P wave with 0^{-+} [3]. The proximity of the mass of X to the $\bar{D}^0 D^{*0}$ threshold indicates that it is extremely loosely bound. If X is an S-wave $\bar{D}^0 D^{*0}/\bar{D}^{*0} D^0$ molecule, the tiny binding energy introduces a new length scale, the $\bar{D}^0 D^{*0}$ scattering length a, that is much larger than other QCD length scales. As a consequence, certain properties of the $X/\bar{D}^0 D^{*0}/\bar{D}^{*0} D^0$ system are determined by a and are insensitive to the shorter distance scales of QCD. This phenomenon is called *low-energy universality.*

A challenge for any interpretation of the $X(3872)$ is to explain its production rate. This could be problematic for the identification of X as an S-wave $\bar{D}^0 D^{*0}/D^0 \bar{D}^{*0}$ molecule, because it can readily dissociate due to its tiny binding energy. One way to produce X is to produce $\bar{D}^0$ and D^{*0} with small enough relative momentum that they can coalesce into X. An example is the decay $\Upsilon(4S) \to Xhh'$, where h and h' are light hadrons, which can proceed through the coalescence into X of charm mesons from the 2-body decays of a virtual B and a virtual $\bar{B}$. Remarkably, low-energy universality determines the decay rate for this process in terms of hadron masses and the width Γ_B of the B meson [17]. Unfortunately, the rate is suppressed by a factor of $(\Gamma_B/m_B)^2$ and is many orders of magnitude too small to be observed.

In this Letter, we apply low-energy universality to the discovery mode $B^+ \to XK^+$ and to the process $B^+ \to \bar{D}^0 D^{*0} K^+$. We point out that the interpretation of X as an S-wave $\bar{D}^0 D^{*0}/\bar{D}^{*0} D^0$ molecule can be confirmed by observing a peak in the $\bar{D}^0 D^{*0}$ invariant mass distribution near the $\bar{D}^0 D^{*0}$ threshold in the decay $B^+ \to \bar{D}^0 D^{*0} K^+$. We also estimate the branching fraction for $B^+ \to XK^+$. The estimate is compatible with observations if X has quantum numbers $J^{PC} = 1^{++}$ and if $J/\psi\pi^+\pi^-$ is one of its major decay modes.

The mass of the X has been measured to be $m_X = 3872.0 \pm 0.6 \pm 0.5$ MeV by Belle [1] and $3871.4 \pm 0.7 \pm 0.4$ MeV by CDF [2]. It is extremely close to the $\bar{D}^0 D^{*0}$ threshold 3871.2 ± 0.7 MeV. The binding energy is $E_b = -0.5 \pm 0.9$ MeV. If the state is bound, E_b is positive, so it is likely to be less than 0.4 MeV. This is the smallest binding energy of any S-wave two-hadron bound state. The next smallest is the deuteron, a proton-neutron state with binding energy 2.2 MeV. For two hadrons whose low-energy interactions are mediated by pion exchange, the natural scale for the binding energy of a molecule is $m_\pi^2/(2\mu)$, where μ is the reduced mass of the two hadrons. For a $\bar{D}^0 D^{*0}$ molecule, this scale is about 10 MeV, so E_b is at least an order of magnitude smaller than the natural low-energy scale.

If the binding energy of X is so small, low-energy universality implies that the $X/\bar{D}^0 D^{*0}/\bar{D}^{*0} D^0$ system has properties that are determined by the $\bar{D}^0 D^{*0}$ scattering

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length a and are insensitive to the shorter distance scales of QCD. The universal binding energy of the molecule is

$$E_b \equiv m_D + m_{D^*} - m_X \simeq (2\mu a^2)^{-1}, \tag{1}$$

where $\mu = m_D m_{D^*}/(m_D + m_{D^*})$ is the reduced mass of the $\bar{D}^0$ and D^{*0}. The universal normalized momentum-space wave function at relative momentum $k \ll m_\pi$,

$$\psi(k) \simeq (8\pi/a)^{1/2}(k^2 + 1/a^2)^{-1}, \tag{2}$$

was used by Voloshin to calculate the momentum distributions for the decays $X \to \bar{D}^0 D^0 \pi^0$ and $X \to \bar{D}^0 D^0 \gamma$ [6]. The universal $\bar{D}^0 D^{*0}$ elastic scattering amplitude at relative momentum $k_{\rm cm} \ll m_\pi$ is

$$\mathcal{A}[\bar{D}^0 D^{*0} \to \bar{D}^0 D^{*0}] \simeq \frac{8\pi m_D m_{D^*}}{\mu(-1/a - ik_{\rm cm})}, \tag{3}$$

where $k_{\rm cm} \approx [2\mu(E - m_D - m_{D^*})]^{1/2}$ and E is the total energy in the center-of-momentum frame. The amplitude $\mathcal{A}[\bar{D}^{*0} D^0 \to \bar{D}^0 D^{*0}]$ for scattering to the CP conjugate state differs by the charge conjugation $C = \pm$ of the channel with the large scattering length. Another consequence of low-energy universality is that, as the binding energy E_b decreases, the probabilities for components of the wave function other than $\bar{D}^0 D^{*0}$ and $\bar{D}^{*0} D^0$ decrease as $E_b^{1/2}$ [9]. In the limit $E_b \to 0$, the state becomes $(|\bar{D}^{*0} D^0\rangle \pm |\bar{D}^0 D^{*0}\rangle)/\sqrt{2}$ if $C = \pm$. The rates for decays that do not correspond to the decay of a constituent D^{*0} or $\bar{D}^{*0}$ also decrease as $E_b^{1/2}$. This suppression may explain the surprisingly narrow width of the X.

The decay $B^+ \to XK^+$ proceeds through the weak decay $\bar{b} \to \bar{c}cs$ at very short distances. The subsequent formation of XK^+ is a QCD process that involves momenta k as low as $1/a$. The contributions from $k \sim 1/a$ are constrained by low-energy universality, but those from $k \gtrsim m_\pi$ involve the full complications of low-energy QCD. We analyze the decay $B^+ \to XK^+$ by separating short-distance effects involving $k \gtrsim m_\pi$ from long-distance effects involving $k \sim 1/a$. The decay can proceed via the short-distance 3-body decay $B^+ \to \bar{D}^0 D^{*0} K^+$ followed by the long-distance coalescence process $\bar{D}^0 D^{*0} \to X$. It can also proceed through a second pathway consisting of $B^+ \to \bar{D}^{*0} D^0 K^+$ followed by $D^0 \bar{D}^{*0} \to X$. The amplitude for the first pathway can be expressed as

$$\mathcal{A}_1[B^+ \to XK^+] = -i\sum \int \frac{d^4\ell}{(2\pi)^4} \mathcal{A}[B^+ \to \bar{D}^0 D^{*0} K^+] \times D(q+\ell, m_D) D(q_* - \ell, m_{D^*}) \times \mathcal{A}[\bar{D}^0 D^{*0} \to X], \tag{4}$$

where $q = (m_D/m_X)Q$ and $q_* = (m_{D^*}/m_X)Q$ are 4-momenta that add up to the 4-momentum Q of X and $D(p, m) = (p^2 - m^2 + i\epsilon)^{-1}$. The sum is over the spin states of the D^{*0}. This amplitude can be represented by the Feynman diagram with meson lines shown in Fig. 1. We constrain the loop integral to the small momentum region by imposing a cutoff $|\ell| < \Lambda$ in the rest frame of the virtual D^0 and $\bar{D}^{*0}$. The natural scale for the cutoff is $\Lambda \sim m_\pi$. The amplitude for $\bar{D}^0 D^{*0}$ to coalesce into X is determined by the $\bar{D}^0 D^{*0}$ scattering length a as follows:

$$\mathcal{A}[\bar{D}^0 D^{*0} \to X] = (16\pi Z m_X m_D m_{D^*}/\mu^2 a)^{1/2} \epsilon_X^* \cdot \epsilon, \tag{5}$$

where ϵ_X and ϵ are the polarization vectors of X and D^{*0} and Z is the probability for the X to be in a $\bar{D}^0 D^{*0}/\bar{D}^{*0} D^0$ state. At the $\bar{D}^0 D^{*0}$ threshold, the amplitude for $B^+ \to \bar{D}^0 D^{*0} K^+$ is constrained by Lorentz invariance to have the form

$$\mathcal{A}[B^+ \to \bar{D}^0 D^{*0} K^+] = c_1 P \cdot \epsilon^*, \tag{6}$$

where P is the 4-momentum of the B meson and c_1 is a constant. The amplitude for $B^+ \to \bar{D}^{*0} D^0 K^+$ has the same form with c_1 replaced by a constant c_2. In the $\bar{D}^0 D^{*0}$ rest frame, the integral over ℓ_0 of the two propagators in (4) is proportional to the momentum-space wave function of X. The subsequent integral over ℓ is linear in the ultraviolet cutoff Λ for the low momentum region

$$\int \frac{d^4\ell}{(2\pi)^4} D(q+\ell, m_D) D(q' - \ell, m_{D^*}) = \frac{i\mu\Lambda}{4\pi^2 m_D m_{D^*}}. \tag{7}$$

The total amplitude from the two pathways is

$$\mathcal{A}[B^+ \to XK^+] = -(Z m_X/\pi^3 m_D m_{D^*} a)^{1/2} (c_1 \pm c_2) \Lambda P \cdot \epsilon_X^*. \tag{8}$$

The sign $\pm$ corresponds to the charge conjugation $C = \pm$ of X. Heavy-quark spin symmetry implies $c_1 = c_2$ up to corrections suppressed by a factor $\Lambda_{\rm QCD}/m_D$. The interference is constructive if $C = +$ and destructive if $C = -$. The dependence of the loop amplitude (8) on Λ is canceled by a tree diagram with a $B - XK$ contact interaction whose coefficient therefore depends linearly on Λ. If the X is predominantly a $\bar{D}D^*$ molecule, there must be some value Λ_1 of the ultraviolet cutoff for which the loop amplitude dominates over the tree amplitude. Squaring

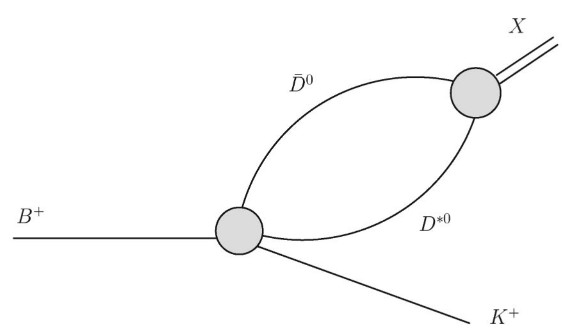

FIG. 1. Feynman diagram for $B^+ \to XK^+$ via the first pathway.

the amplitude, summing over spins, and integrating over phase space, the final result for the decay rate is

$$\Gamma[B^+ \to XK^+] = \frac{Z\lambda^{3/2}(m_B, m_X, m_K)}{64\pi^4 m_B^3 m_X^2 \mu a} |c_1 \pm c_2|^2 \Lambda_1^2, \quad (9)$$

where $\lambda(x, y, z) = x^4 + y^4 + z^4 - 2(x^2y^2 + y^2z^2 + z^2x^2)$. Because of the factor $1/a$, the decay rate scales like $E_b^{1/2}$ as $E_b \to 0$.

If another hadronic state H is close enough to the $\bar{D}^0 D^{*0}$ threshold that X has a nonnegligible probability Z_H of being in the state H, the decay can also proceed through a short-distance 2-body decay $B^+ \to HK^+$. In this case, there is an additional term $\mathcal{A}[B^+ \to HK^+] Z_H^{1/2}$ in (8). Its contribution to the decay rate also scales like $E_b^{1/2}$ as $E_b \to 0$, because Z_H scales like $E_b^{1/2}$ [9]. If $C = +$, one possibility for such a state is the excited P-wave charmonium state $\chi_{c1}(2P)$. Recent coupled-channel calculations of the charmonium spectrum suggest that $\chi_{c1}(2P)$ is likely to be well above the $\bar{D}^0 D^{*0}$ threshold [12]. We will henceforth assume that $\bar{D}^0 D^{*0}/\bar{D}^{*0} D^0$ is the only important component of the wave function and set $Z \approx 1$.

We can calculate the differential decay rate for $B^+ \to \bar{D}^0 D^{*0} K^+$ in the same way. There are again two pathways: the short-distance decay $B^+ \to \bar{D}^0 D^{*0} K^+$ followed by the long-distance scattering $\bar{D}^0 D^{*0} \to \bar{D}^0 D^{*0}$ and $B^+ \to \bar{D}^{*0} D^0 K^+$ followed by $\bar{D}^{*0} D^0 \to \bar{D}^0 D^{*0}$. The amplitude for the first pathway can be represented by the Feynman diagram with meson lines shown in Fig. 2. The calculation of the amplitude is similar to that for $B^+ \to XK^+$ except that it involves the scattering amplitude (3) instead of the coalescence amplitude (5). In the loop amplitude for $B^+ \to \bar{D}^0 D^{*0} K^+$, we keep only the term (6) that is nonzero at the $\bar{D}^0 D^{*0}$ threshold. There must be some value Λ_2 of the ultraviolet cutoff for which the loop amplitude dominates over the tree amplitude. The factor $c_1 \pm c_2$ cancels in the ratio between the amplitudes for $B^+ \to \bar{D}^0 D^{*0} K^+$ and $B^+ \to XK^+$. Our final expression for the differential decay rate is

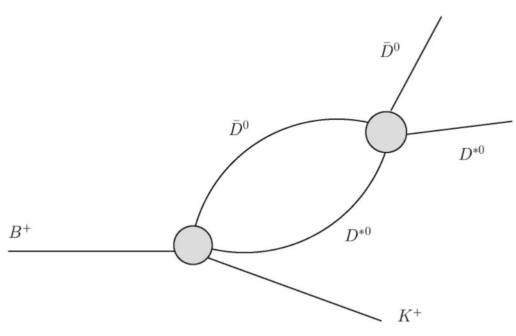

FIG. 2. Feynman diagram for $B^+ \to \bar{D} D^{*0} K^+$ via the first pathway.

$$\frac{d\Gamma}{dM_{\bar{D}D^*}}[B^+ \to \bar{D}^0 D^{*0} K^+] = \Gamma[B^+ \to XK^+] \frac{\Lambda_2^2}{\Lambda_1^2} \frac{\mu a^3 k_{\rm cm}}{\pi(1 + a^2 k_{\rm cm}^2)}, \quad (10)$$

where $M_{\bar{D}D^*}$ is the $\bar{D}^0 D^{*0}$ invariant mass and $k_{\rm cm}$ is the relative momentum in the $\bar{D}^0 D^{*0}$ rest frame:

$$k_{\rm cm} = \lambda^{1/2}(M_{\bar{D}D^*}, m_D, m_{D^*})/(2M_{\bar{D}D^*}). \quad (11)$$

In (10), we have neglected terms suppressed by $k_{\rm cm}^2/m_D^2$. The invariant mass distribution is illustrated in Fig. 3 for several values of the binding energy E_b. The distributions are normalized to one at $k_{\rm cm} = m_\pi$. As the binding energy is tuned toward 0, the peak value scales like $E_b^{-1/2}$ and the position of the peak in $M_{\bar{D}D^*} - (m_D + m_{D^*})$ scales like E_b. The observation of such an enhancement near the $\bar{D}^0 D^{*0}$ threshold would confirm the interpretation of X as a $\bar{D}^0 D^{*0}/\bar{D}^{*0} D^0$ molecule.

The BABAR Collaboration has recently measured the branching fractions for B^+ to decay into $\bar{D}^0 D^0 K^+$, $\bar{D}^0 D^{*0} K^+$, $\bar{D}^{*0} D^0 K^+$, and $\bar{D}^{*0} D^{*0} K^+$ to be $(0.19 \pm 0.03)\%$, $(0.47 \pm 0.07)\%$, $(0.18 \pm 0.07)\%$, and $(0.53 \pm 0.11)\%$, respectively [18]. We use these measurements to estimate the branching fraction for $B^+ \to XK^+$. We make the simplifying assumption that the decay amplitude factors into currents $\bar{c}\gamma^\mu(1 - \gamma_5)b$ and $\bar{s}\gamma^\mu(1 - \gamma_5)c$. Heavy-quark symmetry can then be used to express the 3-body double-charm decay amplitudes in terms of two functions $G_1(q^2)$ and $G_2(q^2)$, where q^2 is the invariant mass of the hadrons produced by the $\bar{s}\gamma^\mu(1 - \gamma_5)c$ current [19]. For example, the amplitudes for decays into $\bar{D}^0 D^{*0} K^+$ and $\bar{D}^{*0} D^0 K^+$ are

$$\begin{aligned} \mathcal{A}[B^+ \to \bar{D}^0 D^{*0} K^+] &= -iG_1 \epsilon^* \cdot (V + v) \\ &\quad - i(G_2/m_B)\epsilon_\nu^*[v_* \cdot k(V + v)^\nu - v_* \cdot (V + v)k^\nu \\ &\quad - i\epsilon^{\nu\mu\alpha\beta}(V + v)_\mu v_{*\alpha} k_\beta], \end{aligned} \quad (12)$$

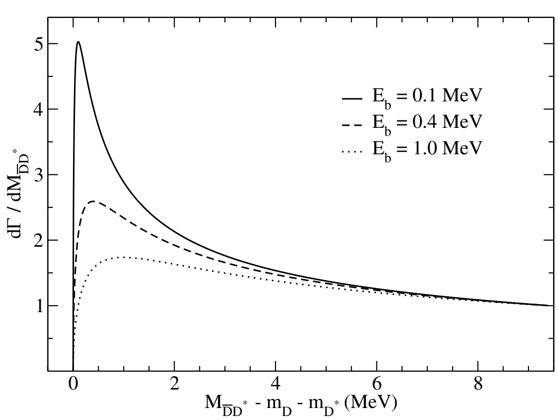

FIG. 3. The $\bar{D}^0 D^{*0}$ invariant mass distribution for $B^+ \to \bar{D}^0 D^{*0} K^+$ for three different values of the binding energy of X. The distributions are normalized to one at $k_{\rm cm} = m_\pi$.

$$\mathcal{A}[B^+ \to \bar{D}^{*0} D^0 K^+] = i(G_1 v_\mu + G_2 k_\mu / m_B)\epsilon_\nu^* [(1 + v_* \cdot V) g^{\mu\nu} - v_*^\mu V^\nu - i\epsilon^{\mu\nu\alpha\beta} v_{*\alpha} V_\beta], \quad (13)$$

where k is the 4-momentum of the K^+ and V, v_*, and v are the 4-velocities of the B^+, $\bar{D}^{*0}$ or D^{*0}, and D^0 or $\bar{D}^0$, respectively. As a further simplification, we approximate G_1 and G_2 by constants. The resulting expressions for the 3-body double-charm decay rates are

$$\Gamma[B^+ \to \bar{D}^0 D^0 K^+] = 10^{-3}\ \mathrm{MeV}[178.9|G_1|^2 + 51.8\mathrm{Re}(G_1^* G_2) + 4.37|G_2|^2], \quad (14)$$

$$\Gamma[B^+ \to \bar{D}^0 D^{*0} K^+] = 10^{-3}\ \mathrm{MeV}[49.6|G_1|^2 + 2.61\mathrm{Re}(G_1^* G_2) + 3.49|G_2|^2], \quad (15)$$

$$\Gamma[B^+ \to \bar{D}^{*0} D^0 K^+] = 10^{-3}\ \mathrm{MeV}[52.5|G_1|^2 + 1.87\mathrm{Re}(G_1^* G_2) + 2.31|G_2|^2], \quad (16)$$

$$\Gamma[B^+ \to \bar{D}^{*0} D^{*0} K^+] = 10^{-3}\ \mathrm{MeV}[221.5|G_1|^2 + 74.8\mathrm{Re}(G_1^* G_2) + 11.58|G_2|^2]. \quad (17)$$

We obtain a good fit to the BABAR branching fractions with $G_1 = 3.2 \times 10^{-6}$ and $G_2 = (-14.6 + 9.6i) \times 10^{-6}$. In the corner of phase space where the 4-velocities of $\bar{D}^0$ and D^{*0} are equal, the amplitudes (12) and (13) reduce to the form on the right side of (6) with coefficients $c_1 = c_2 = -iG_1/m_B + iG_2(m_B + m_D + m_{D^*})/m_B^2$. If X has charge conjugation $C = +$, the estimate (9) reduces to

$$\mathcal{B}[B^+ \to XK^+] \approx (2.6 \times 10^{-5}) \frac{\Lambda_1^2}{m_\pi^2}\left(\frac{E_b}{0.4\mathrm{MeV}}\right)^{1/2}. \quad (18)$$

If $C = -$, the branching fraction would be significantly smaller because of destructive interference between c_1 and c_2. We could get a more reliable result for the numerical prefactor in (18) by relaxing the factorization assumption and carrying out a Dalitz plot analysis of the 3-body decays. Since the result depends quadratically on the ultraviolet cutoff Λ_1, the best we can do is obtain an order-of-magnitude estimate of the branching fraction by setting $\Lambda_1 \approx m_\pi$.

The Belle Collaboration measured the product of the branching fractions $\mathcal{B}[B^+ \to XK^+]$ and $\mathcal{B}[X \to J/\psi\pi^+\pi^-]$ to be $(1.3 \pm 0.3) \times 10^{-5}$ [1]. Our estimate of $\mathcal{B}[B^+ \to XK^+]$ is compatible with this result if $J/\psi\pi^+\pi^-$ is one of the major decay modes of X. The experimental upper bound on the width of $X(3872)$ is $\Gamma_X < 2300$ keV. The sum of the widths for decay into $\bar{D}^0 D^0 \pi^0$ and $\bar{D}^0 D^0 \gamma$ approaches $\Gamma[D^{*0}] \approx 50$ keV in the limit $E_b \to 0$ [6]. The remaining partial widths scale as $E_b^{1/2}$. Using a coupled-channel calculation in a model in which X mixes with $J/\psi\rho$, the decay rate for $J/\psi\pi^+\pi^-$ has been estimated to be 1290 keV for $E_b = 0.7$ MeV [11]. Thus it is at least plausible that $J/\psi\pi^+\pi^-$ is one of the major decay modes. Other possible decay channels are $\eta_c\pi\pi$, radiative transitions to charmonium states, and $c\bar{c}$ annihilation decays.

We have calculated the decay rate for $B^+ \to XK^+$ and the differential decay rate for $B^+ \to \bar{D}^0 D^{*0} K^+$ near the $\bar{D}^0 D^{*0}$ threshold under the assumption that $X(3872)$ is a loosely bound S-wave $\bar{D}^0 D^{*0}/D^0 \bar{D}^{*0}$ molecule and that its production rate is dominated by the coalescence of charm mesons. Observation of a sharp peak in the $\bar{D}^0 D^{*0}$ invariant mass distribution near threshold in the decay $B^+ \to \bar{D}^0 D^{*0} K^+$ would confirm the interpretation of X as a $\bar{D}^0 D^{*0}$ molecule. Our order-of-magnitude estimate of the branching fraction for $B^+ \to XK^+$ is compatible with observations if $X(3872)$ has quantum numbers $J^{PC} = 1^{++}$ and if $J/\psi\pi^+\pi^-$ is one of its major decay modes.

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