Collective Rabi Oscillations and Solitons in a Time-Dependent BCS Pairing Problem

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Motivated by recent efforts to achieve cold fermions pairing, we study the nonadiabatic regime of the Bardeen-Cooper-Schrieffer state formation. After the interaction is turned on, at times shorter than the quasiparticle energy relaxation time, the system oscillates between the superfluid and normal state. The collective nonlinear evolution of the BCS-Bogoliubov amplitudes u_p, v_p, along with the pairing function Δ, is shown to be an integrable dynamical problem which admits single soliton and soliton train solitons. We interpret the collective oscillations as Bloch precession of Anderson pseudospins, where each soliton causes a pseudospin 2π Rabi rotation.

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Dilute fermionic alkali gases cooled below degeneracy [1] are expected to host the paired BCS state [2,3]. One of the attractive features of this system is control of interaction strength achieved by using magnetically tuned Feshbach resonances [4–9] which provides access to the strong coupling BCS regime. Also, since the characteristic energy scales in atomic vapors are relatively low, while coherence times are long, one can perform time-resolved measurements on the intrinsic microscopic time scales, and explore a range of fundamentally new phenomena in the time dynamics of the paired state. These new prospects helped to revive interest in some of the basic issues of the BCS pairing problem [10–15].

In particular, one important question has to do with formation of the BCS state in cold gases [16]. The dynamics of the superconducting BCS state in metals has been a subject of long-time active interest [17]. The regimes considered can be broadly classified into two groups, with regard to how the relevant time scales compare to the quasiparticle energy relaxation time τ_ϵ and the order parameter dynamical time τ_Δ, estimated as the inverse increment of Cooper instability [18,19]. For $\tau_\epsilon \ll \tau_\Delta$, quasiparticles quickly reach local equilibrium parameterized by a time-dependent order parameter $\Delta(t)$ and thus a time-dependent Ginzburg-Landau equation for $\Delta(t)$ can be employed. However, as noted by Gor'kov and Eliashberg [20], the condition $\tau_\epsilon \ll \tau_\Delta$ holds only in relatively exotic situations, including a close proximity of a transition point, $\Delta^2/T_c \ll \hbar/\tau_\epsilon$, or a fast pair breaking (e.g., due to paramagnetic impurities).

The opposite limit, $\tau_\epsilon \gg \tau_\Delta$, which holds at temperatures not too close to critical, can be described by a Boltzmann kinetic equation for quasiparticles and a self-consistent equation for $\Delta(t)$ [21,22]. The validity of this approach requires adiabaticity on the τ_Δ time scale of both the quasiparticle distribution and the external parameters time variation. In other words, free oscillations of $|\Delta|$ characterized by $\omega \sim \tau_\Delta^{-1}$ should not be excited. At $T \ll T_c$, when $\tau_\Delta \simeq \hbar/\Delta$ [23], this is just the $\hbar\omega \ll \Delta$ criterion. (In contrast, low frequency phase oscillations are in the realm of the adiabatic picture [24,25].)

Here we analyze free oscillation of BCS pairing in a cold gas, characterized by frequencies $\hbar\omega \lesssim 2\Delta$, and treat the nonadiabatic limit not accounted for by these two approaches. The currently studied systems [26–28] are described by the nonretarded BCS pairing theory [29] which predicts $T_c = 0.3E_F e^{-1/\lambda}$, $\lambda = \frac{2}{\pi} k_F|a|$, with the scattering length a that has a resonance dependence on the external magnetic field [1]. To estimate the BCS parameter values, we consider magnetic fields not too close to the resonance where one can neglect the presence of the molecular field [5] and use the weak coupling theory. At particle density $n \approx 1.8 \times 10^{13}$ cm^{-3} [26], which corresponds to $E_F \approx 0.35$ μK, and the scattering length $a \approx -50$ nm we have $T_c \approx 0.006E_F$. An estimate of the dynamical time obtains $\tau_\Delta \simeq \hbar/\Delta_0 \approx 2$ ms, while the quasiparticle energy relaxation time is $\tau_{\epsilon\sim\Delta} \simeq \hbar E_F/\Delta_0^2 \approx 200$ ms $= 100\tau_\Delta$, consistent with weakly damped oscillations of Δ.

The cold fermionic gases present a completely new situation from yet another point of view. While relaxation rates in these systems are quite slow, the external parameters, such as the detuning from resonance, can change very quickly on the time scale of τ_Δ. This enables the BCS correlations to build up in a coherent fashion while the system is out of thermal equilibrium. In such a situation, theory must account not only for the order parameter evolution, but also for the full dynamics of individual Cooper pairs and quasiparticles. In contrast, the changes of the quasiparticle distribution in superconducting metals are described by parameter variation slow compared to τ_Δ and quasiparticle spectrum which evolves adiabatically, without exciting oscillations of Δ.

In this Letter we consider the situation when the pairing interaction is turned on abruptly on a time scale $\tau_0 \ll \tau_\Delta, \tau_\epsilon$, and explore the time evolution of the pairing instability of Fermi gas. For simplicity, we focus on the case of a gas sample size smaller than BCS correlation length $\xi = \hbar^2 k_F / m\Delta_0$ (for the parameters listed above, $\xi \simeq 24\ \mu$m is in excess of the gas sample size [26] $L \approx 18\ \mu$m). In the "zero-dimensional" limit $\xi \simeq L$ one can ignore aspects related with spatial dependence, such as inhomogeneous phase fluctuations and vortices.

At not too long times, $t \ll \tau_\epsilon$, the dynamics is governed by nondissipative equations which exhibit a nonlinear time evolution. The notion of the quasiparticle spectrum is irrelevant in this regime, and theory can rely neither on the kinetic equation, nor on the time-dependent Ginzburg-Landau equation. Our approach describes the BCS state buildup and accounts for coherent dynamics of individual Cooper pairs. We shall focus on the zero temperature case, when $\tau_\Delta \simeq \hbar/\Delta$, and show that Cooper instability results in a periodic oscillation of $|\Delta|$.

This regime can be described by the BCS Hamiltonian

$$\mathcal{H} = \sum_{\mathbf{p},\sigma} \epsilon_{\mathbf{p}} a^+_{\mathbf{p},\sigma} a_{\mathbf{p},\sigma} - \frac{\lambda(t)}{2} \sum_{\mathbf{p},\mathbf{q}} a^+_{\mathbf{p}\uparrow} a^+_{-\mathbf{p}\downarrow} a_{-\mathbf{q}\downarrow} a_{\mathbf{q}\uparrow}, \qquad (1)$$

with the coupling turned on abruptly, $\lambda(t) = \lambda\theta(t - t_*)$.

The main result of this work is that the time-dependent problem (1) is integrable. We construct a generalized time-dependent many-body BCS state, exact for the separable Hamiltonian (1), which has the form

$$|\Psi(t)\rangle = \prod_{\mathbf{p}} [u_{\mathbf{p}}(t) + v_{\mathbf{p}}(t) a^+_{\mathbf{p},\uparrow} a^+_{-\mathbf{p},\downarrow}]|0\rangle. \qquad (2)$$

The Bogoliubov mean field treatment, which gives a state of the form (2), relies on the "infinite range" form of the pairing interaction in (1) (i.e., equal coupling strength for all $(\mathbf{p}, -\mathbf{p})$, $(\mathbf{q}, -\mathbf{q})$). Since the latter does not depend on the system being in equilibrium, one can introduce a time-dependent mean field pairing function

$$\Delta(t) = \lambda \sum_{\mathbf{p}} u_{\mathbf{p}}(t) v^*_{\mathbf{p}}(t) \qquad (3)$$

The amplitudes $u_{\mathbf{p}}(t)$, $v_{\mathbf{p}}(t)$ can be obtained from the Bogoliubov–deGennes equation

$$i\partial_t \begin{pmatrix} u_{\mathbf{p}} \\ v_{\mathbf{p}} \end{pmatrix} = \begin{pmatrix} \epsilon_{\mathbf{p}} & \Delta \\ \Delta^* & -\epsilon_{\mathbf{p}} \end{pmatrix} \begin{pmatrix} u_{\mathbf{p}} \\ v_{\mathbf{p}} \end{pmatrix} \qquad (4)$$

with Δ defined self-consistently by Eq. (3).

We recall that the unpaired state is a self-consistent, albeit unstable, solution of Eqs. (4) and (3) with $\Delta = 0$:

$$u^{(0)}_{\mathbf{p}}(t) = e^{-i\epsilon_{\mathbf{p}} t}\theta(\epsilon_{\mathbf{p}}), \qquad v^{(0)}_{\mathbf{p}}(t) = e^{i\epsilon_{\mathbf{p}} t}\theta(-\epsilon_{\mathbf{p}}). \qquad (5)$$

The stability analysis [19] shows that the deviation from the unpaired state grows as $\Delta(t) \propto e^{\gamma t} e^{-i\omega t}$, with the growth exponent γ and the constant ω given by

$$1 = \lambda \sum_{\mathbf{p}} \frac{\mathrm{sgn}\epsilon_{\mathbf{p}}}{2\epsilon_{\mathbf{p}} - \zeta}, \qquad \zeta = \omega + i\gamma. \qquad (6)$$

Being similar to the BCS gap equation at $T = 0$, Eq. (6) yields the exponent close to the BCS gap value, $\gamma \simeq \Delta_0$, $\tau_\Delta \simeq \Delta_0^{-1}$. (In the weak coupling limit, $\omega = 0$ due to the particle-hole symmetry, and thus $\gamma = \Delta_0$.)

At $T = 0$, a soliton solution of Eqs. (4) and (3), can be constructed most naturally in terms of the variable

$$w_{\mathbf{p}} = \begin{cases} u_{\mathbf{p}}/v_{\mathbf{p}}, & \epsilon_{\mathbf{p}} > 0 \\ v_{\mathbf{p}}/u_{\mathbf{p}}, & \epsilon_{\mathbf{p}} < 0 \end{cases} \qquad (7)$$

Consider first the case $\epsilon_{\mathbf{p}} > 0$. From Eq. (4) we obtain

$$i\partial_t w_{\mathbf{p}} = 2\epsilon_{\mathbf{p}} w_{\mathbf{p}} + \Delta(t) - \Delta^*(t) w^2_{\mathbf{p}} \qquad (8)$$

with Δ defined by (3) and $\lim_{t\to-\infty} w_{\mathbf{p}} = \infty$ set by (5).

Motivated by the stability analysis (6), we try the ansatz $\Delta(t) = e^{-i\omega t}\alpha^{-1}(t)$ with $\alpha(t)$ real, and

$$w_{\mathbf{p}}(t) = 2\epsilon_{\mathbf{p}} f(t) - i\dot{f}(t), \qquad f(t) \equiv \frac{1}{\Delta^*} = e^{-i\omega t}\alpha(t). \qquad (9)$$

Substituting this in Eq. (8), we obtain an equation

$$i\xi_{\mathbf{p}}\dot{\alpha} + \ddot{\alpha} = \xi_{\mathbf{p}}(\xi_{\mathbf{p}}\alpha - i\dot{\alpha}) + \frac{1}{\alpha}[1 - (\xi_{\mathbf{p}}\alpha - i\dot{\alpha})^2] \qquad (10)$$

with $\xi_{\mathbf{p}} = 2\epsilon_{\mathbf{p}} - \omega$. Remarkably, the terms with $\xi_{\mathbf{p}}$ cancel, and Eq. (10) takes the same form for all the states,

$$\alpha\ddot{\alpha} = \dot{\alpha}^2 + 1, \qquad (11)$$

which justifies the ansatz (9). By a substitution $\alpha = e^{\phi}$, Eq. (11) can be brought to the form $\ddot{\phi} = e^{-2\phi}$. Integrating the latter equation, obtain $\dot{\phi}^2 + e^{-2\phi} = \gamma^2$, with γ an integration constant. This yields $\dot{\alpha}^2 = \gamma^2\alpha^2 - 1$,

$$\alpha(t) = \frac{1}{\gamma}\cosh\gamma(t - t_0), \qquad \Delta(t) = \frac{\gamma e^{-i\omega t}}{\cosh\gamma(t - t_0)}. \qquad (12)$$

Modulus $|\Delta|$ growing first, then decreasing and taking the system back to the unpaired state (Fig. 1, upper left), reflects the absence of dissipation. The peak time t_0 is set by the initial condition at the switching time.

For $\epsilon_{\mathbf{p}} < 0$, $w_{\mathbf{p}} = v_{\mathbf{p}}/u_{\mathbf{p}}$, the form of Eq. (8) remains the same up to a sign change $\epsilon_{\mathbf{p}} \to -\epsilon_{\mathbf{p}}$ and the permutation $\Delta \leftrightarrow \Delta^*$. Accordingly, the ansatz for $w_{\mathbf{p}}$ in this case is $w_{\epsilon_{\mathbf{p}}<0}(t) = 2|\epsilon_{\mathbf{p}}| f(t) - i\dot{f}(t)$, $f(t) \equiv \Delta^{-1} = e^{i\omega t}\alpha(t)$, yielding an equation identical to Eq. (11).

The last step is to analyze the requirements on this solution due to the self-consistency condition (3). For that, we rewrite Eq. (3) in terms of $w_{\mathbf{p}}(t)$ as

$$\Delta(t) = \lambda \sum_{\epsilon_{\mathbf{p}}>0} \frac{w_{\mathbf{p}}(t)}{1 + |w_{\mathbf{p}}(t)|^2} + \lambda \sum_{\epsilon_{\mathbf{p}}<0} \frac{w^*_{\mathbf{p}}(t)}{1 + |w_{\mathbf{p}}(t)|^2} \qquad (13)$$

and note that both the right and the left hand side have the same time dependence, and are equal to each other provided that the quantity $\zeta = \omega + i\gamma$ satisfies Eq. (6). This means that the gap modulus $|\Delta(t)|$ peak value is equal to the instability exponent γ defined by (6).

At $T = 0$, in the case of a constant density of states, γ is equal to the equilibrium BCS gap Δ_0, while ω vanishes due to particle-hole symmetry. Thus, remarkably, the modulus $|\Delta(t)|$ in this case peaks exactly at Δ_0. To illustrate the collective dynamics in the soliton solution, we plot the trajectories $u_{\mathbf{p}}(t)$, $v_{\mathbf{p}}(t)$ on the Bloch sphere $r_1^2 + r_2^2 + r_3^2 = 1$ using the parameterization

$$r_1 + ir_2 = 2u_{\mathbf{p}} v_{\mathbf{p}}^*; \qquad r_3 = |u_{\mathbf{p}}|^2 - |v_{\mathbf{p}}|^2 \tag{14}$$

(Fig. 1, left). Here, as well as in the soliton train solutions discussed below [Eq. (23)], each state $(u_{\mathbf{p}}, v_{\mathbf{p}})$ completes a full Rabi cycle per soliton. The trajectories, which are small loops for the pairs with large energies $\epsilon_{\mathbf{p}}$, turn into a big circle as $\epsilon_{\mathbf{p}}$ tends to the Fermi level.

To gain further insight, we reformulate the BCS approach, following Ref. [23], in terms of pseudospins associated with individual Cooper pair states. "Pauli spin" operators $\sigma_{\mathbf{p}}^{\pm} \equiv \frac{1}{2}(\sigma_{\mathbf{p}}^x \pm i\sigma_{\mathbf{p}}^y)$ can be assigned to each pair of fermions with opposite momenta as follows

$$\sigma_{\mathbf{p}}^{+} = a_{\mathbf{p}\uparrow}^{+} a_{-\mathbf{p}\downarrow}^{+}, \qquad \sigma_{\mathbf{p}}^{-} = a_{-\mathbf{p}\downarrow} a_{\mathbf{p}\uparrow}, \tag{15}$$

and $\sigma_{\mathbf{p}}^z \equiv [\sigma_{\mathbf{p}}^+, \sigma_{\mathbf{p}}^-] = a_{\mathbf{p}\uparrow}^+ a_{\mathbf{p}\uparrow} - a_{-\mathbf{p}\downarrow} a_{-\mathbf{p}\downarrow}^+$. This allows us to map the problem (1) onto an interacting spin problem

$$\mathcal{H} = \sum_{\mathbf{p}}{}' \epsilon_{\mathbf{p}} \sigma_{\mathbf{p}}^z - 2\lambda \sum_{\mathbf{p},\mathbf{q}}{}' \sigma_{\mathbf{p}}^+ \sigma_{\mathbf{q}}^- \tag{16}$$

where prime means a sum over the pairs $(\mathbf{p}, -\mathbf{p})$. Since all the spins interact with each other equally, the mean field theory here is exact, just like for the BCS problem. The mean field Hamiltonian for each spin is

$$\mathcal{H}_{\mathbf{p}} = \mathbf{b}_{\mathbf{p}} \cdot \sigma_{\mathbf{p}}, \qquad \mathbf{b}_{\mathbf{p}} = (-\Delta', -\Delta'', \epsilon_{\mathbf{p}}). \tag{17}$$

Note that, while the z component of $\mathbf{b}_{\mathbf{p}}$, given by the single particle energy, is spin-specific, the transverse component, the same for all of the spins, satisfies

$$\Delta \equiv \Delta' + i\Delta'' = \lambda \sum_{\mathbf{q}}{}' \langle \sigma_{\mathbf{q}}^+ \rangle \tag{18}$$

which is analogous to the BCS gap relation. In the ground state the spins, aligned with $\mathbf{b}_{\mathbf{p}}$, form a texture [23], with spin rotation described by the Bogoliubov angle.

The dynamical problem of interest can be cast in the form of Bloch equations for the spins,

$$\dot{\sigma}_{\mathbf{p}} = i[\mathcal{H}_{\mathbf{p}}, \sigma_{\mathbf{p}}] = 2\mathbf{b}_{\mathbf{p}} \times \sigma_{\mathbf{p}} \tag{19}$$

with $\mathbf{b}_{\mathbf{p}}$ defined self-consistently by (17) and (18). Equation (19), linearized about the texture state, describes collective excitations [23,30]. Linearized about the unpaired state, it describes Cooper instability (6).

The Bloch dynamics Eq. (19), which is linear both for operators and expectation values, is easy to simulate. In the presence of thermal noise, we observe (Fig. 2) an orderly train of the cosh solitons, indicating soliton robustness and stability (*cf.* other nonequilibrium states studied in Ref. [31]). Adding damping to the Bloch equation ensures relaxation to the ground state (Fig. 2).

We note that Volkov and Kogan [30], who investigated the weak oscillation regime, found nonexponential decay of linearized oscillations, interpreted as collisionless damping [18,30] caused by mixing of the oscillations of Δ with quasiparticle states slightly above the gap.

Let us now consider multisoliton solutions. As above, we assume the pairing function of the form $\Delta(t) = e^{-i\omega t}\Omega(t)$, with $\Omega(t)$ real. After the phase factor $e^{-i\omega t}$ is eliminated by going to Larmor frame, Eq. (19) for the average spin components $r_i = \langle \sigma_{\mathbf{p}}^i \rangle$ becomes

$$\dot{r}_1 = -\xi_{\mathbf{p}} r_2, \quad \dot{r}_2 = \xi_{\mathbf{p}} r_1 + 2\Omega r_3, \quad \dot{r}_3 = -2\Omega r_2 \tag{20}$$

$(\xi_{\mathbf{p}} = 2\epsilon_{\mathbf{p}} - \omega)$. This problem can be solved by the ansatz

$$r_1 = A_{\mathbf{p}}\Omega, \qquad r_2 = B_{\mathbf{p}}\dot{\Omega}, \qquad r_3 = C_{\mathbf{p}}\Omega^2 - D_{\mathbf{p}}. \tag{21}$$

The first and the third Eq. (20) are satisfied by (21) provided $A_{\mathbf{p}} = -\xi_{\mathbf{p}} B_{\mathbf{p}}$ and $B_{\mathbf{p}} = -C_{\mathbf{p}}$, while the second Eq. (20) is consistent with the normalization condition $r_1^2 + r_2^2 + r_3^2 = 1$, and thus yields

$$C_{\mathbf{p}}^2 \xi_{\mathbf{p}}^2 \Omega^2 + C_{\mathbf{p}}^2 \dot{\Omega}^2 + (C_{\mathbf{p}}\Omega^2 - D_{\mathbf{p}})^2 = 1. \tag{22}$$

Equation (22) will take the same form for all the spins,

$$\dot{\Omega}^2 + (\Omega^2 - \Delta_-^2)(\Omega^2 - \Delta_+^2) = 0, \qquad \Delta_- \le \Delta_+ \tag{23}$$

provided that the constants $D_{\mathbf{p}}$, $C_{\mathbf{p}}$ are chosen as $(D_{\mathbf{p}}^2 - 1)/C_{\mathbf{p}}^2 = \Delta_-^2 \Delta_+^2$, $2D_{\mathbf{p}}/C_{\mathbf{p}} = \xi_{\mathbf{p}}^2 + \Delta_-^2 + \Delta_+^2$ with the sign factor $\mathrm{sgn}\,\epsilon_{\mathbf{p}}$. Equation (23) defines an elliptic function $\Omega(t)$ oscillating periodically between Δ_- and Δ_+. At $\Delta_- \ll \Delta_+$, the solution is a train of weakly overlapping

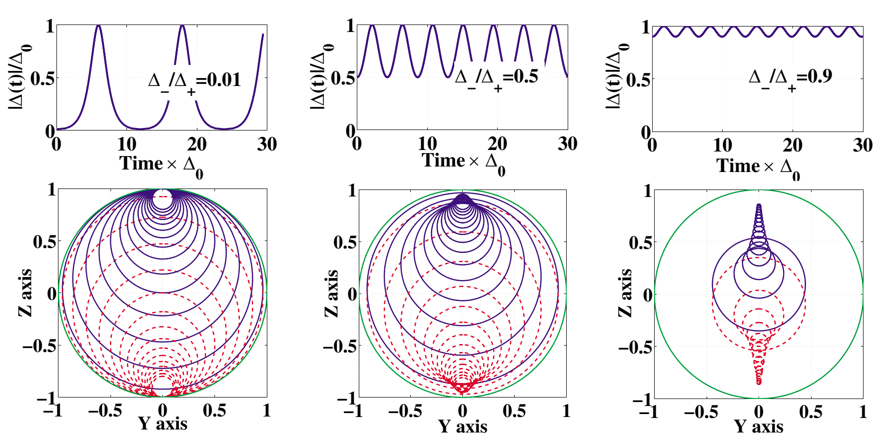

FIG. 1 (color online). Coherent BCS dynamics. Above: the soliton solutions oscillating in the limits $\Delta_- \le \Delta(t) \le \Delta_+$, (23). Below: trajectories of individual Cooper pair states on the Bloch sphere (14). Note that each state completes a full 2π Rabi cycle per soliton. The dashed-line and solid-line curves correspond to the energies $\epsilon_{\mathbf{p}}$ above and below the Fermi level.

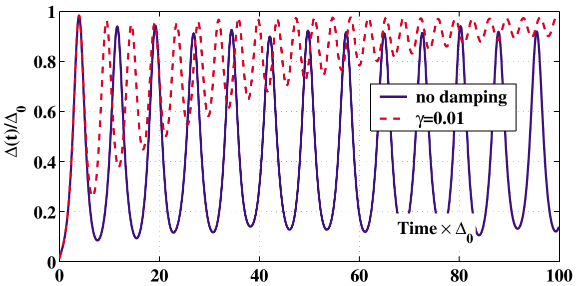

FIG. 2 (color online). Bloch dynamics $\dot{\mathbf{r}}_{\mathbf{p}} = 2\mathbf{b}_{\mathbf{p}}^{\rm eff} \times \mathbf{r}_{\mathbf{p}}$ for 10^3 spins with a constant density of states. Damping is included via $\mathbf{b}_{\mathbf{p}}^{\rm eff} = \mathbf{b}_{\mathbf{p}} - \gamma \mathbf{b}_{\mathbf{p}} \times \mathbf{r}_{\mathbf{p}}$. Initial conditions $r_{3,\mathbf{p}} = \tanh(\frac{1}{2}\beta\epsilon_{\mathbf{p}})$, $(r_1 + ir_2)_{\mathbf{p}} = e^{i\phi_{\mathbf{p}}}(1 - r_3^2)^{1/2}$, with $\beta^{-1} \equiv T = 0.1\Delta_0$ and random uncorrelated $\phi_{\mathbf{p}}$ were used to model thermal noise.

solitons (12) with $\Delta_+ = \gamma$ and $r_i(t)$ that tend to the ideal Fermi gas values $r_3 = -\mathrm{sgn}\epsilon_{\mathbf{p}}$, $r_{1,2} = 0$ between the solitons (Fig. 1, left).

The real part of the self-consistency relation (18),

$$1 = \lambda \sum_{\mathbf{p}} \frac{\xi_{\mathbf{p}} \mathrm{sgn}\epsilon_{\mathbf{p}}}{[(\xi_{\mathbf{p}}^2 + \Delta_-^2 + \Delta_+^2)^2 - 4\Delta_-^2\Delta_+^2]^{1/2}} \qquad (24)$$

fixes one of the constants $\Delta_\pm$, leaving the other one free. The ratio Δ_-/Δ_+ controls the intersoliton time separation. As it varies from 0 to 1, the soliton frequency increases, and the nonoverlapping solitons (12) gradually merge, turning into weak harmonic oscillations (Fig. 1).

The imaginary part of Eq. (18) fixes the value of the frequency shift ω (we recall that $\omega \neq 0$ in the presence of charge asymmetry). At $\Delta_- \ll \Delta_+$, Eq. (24) turns into Eq. (6) which, as we found above, defines the amplitude of a single soliton. In the opposite limit, $\Delta_- \to \Delta_+$, Eq. (24) coincides with the BCS gap equation.

There is an interesting relation between our problem and the self-induced transparency phenomenon [32]. In the latter, an optical pulse interacting with an ensemble of atoms can dissipate its energy by inducing resonant Rabi transitions in the atoms. However, when the pulse duration is tuned so that the atoms complete a full Rabi 2π cycle as the pulse goes by, the pulse sustains itself and propagates without dissipation. Our Bloch equations bear striking similarity to those of Ref. [32], where the atoms' polarization is employed to provide feedback on the pulse instead of our BCS self-consistency relation.

Before concluding, we note that the dynamics at finite temperature, in the regime described by $\tau_\epsilon \gg \tau_\Delta$, remains an open problem. In particular, we cannot rule out the possibility of chaotic behavior. The relatively simple periodic time dependence found at $T = 0$ arises due to strong coupling of the low-energy quasiparticle states and the oscillations of Δ. In contrast, at $T\tau_\Delta \gg \hbar$, only a small fraction of thermally excited states with $\epsilon_{\mathbf{p}} \sim \hbar/\tau_\Delta$ fully participate in the oscillations $\Delta(t)$, while a larger and a weaker coupled group, $\hbar/\tau_\Delta \ll \epsilon_{\mathbf{p}} \ll T$, plays a role of thermal bath, providing dissipation.

In summary, this work provides an exact solution for the BCS pairing formation problem. In the nonadiabatic regime, the dynamics is dissipationless and nonlinear. Soliton train solutions are obtained analytically and demonstrated to be generic and robust by a simulation.

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