Codalike Multiple Scattering of Elastic Waves in Dense Granular Media

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(Received 10 June 2003; published 7 October 2004)

We study the multiple scattering of short-wavelength ultrasound through the force networks in dry and wet glass bead packings under stress. Over long distance scales, the diffusion approximation is shown to describe adequately the transport of elastic waves dominated by shear waves. The recovered transport mean path reveals a short-range correlation of the force chains. Also we observe the drastic effect of wetting liquids on the energy dissipation in the granular medium. The relevance of these experimental findings for the seismological applications is discussed.

DOI: 10.1103/PhysRevLett.93.154303 PACS numbers: 43.35.+d, 45.70.–n, 46.40.Cd, 91.30.–f

The physics uncovered in granular materials has clear relevance to a wide variety of dissipative, nonequilibrium systems, including glasses and dense colloidal suspensions [1], and also on a much larger scale to nonlinear phenomena such as avalanche and earthquake dynamics [2]. In noncohesive granular materials, photoelastic visualization experiments and simulations show that the contact forces arising as a result of the external load are distributed in a very inhomogeneous fashion, forming a network of percolating force chains [3,4]. It is this network of contacts that determines most salient mechanical properties of a dense granular medium such as its ability to bear a load, nonlinear elastic response, and flow behavior.

Small amplitude compressional and shear acoustic waves provide a unique noninvasive probe of both the structure and the nonlinear elastic properties of the force network within real 3D granular media [5–7]. At low frequencies such that the wavelengths are very long compared to the correlation length of force chains, the granular medium is effectively a homogeneous continuum to the propagating wave [5]. At high frequencies when the wavelength decreases down to the order of the grain size, scattering effects caused by the spatial fluctuations of force chains become very significant and the effective contact medium is no longer a valid description [7,8]. In this regime, scattered sound waves are configuration specific determined by the exact structure of the force chains, which provide a sensitive tool for studying configurational variations of the contact network [6,7].

However, up to now, it remains unclear whether the multiple scattering of elastic waves in a granular medium can be described in the framework of classical diffusive propagation such as light and sound in a strongly scattering random medium, or whether it is fundamentally different than in other random media, as suggested by Liu and Nagel, due to the presence of a few dominating paths created by the force chains along which sound waves at all frequencies must pass through [6,7,9]. Addressing this question is also of considerable interest for seismological applications, which enable one to gain insight into the role of short-wavelength acoustic emission in the fault dynamics [2]. In this Letter, we test the application of the diffusion approximation to the propagation of high-frequency ultrasounds in glass bead packings under stress. We show that the multiple scattering of elastic waves allow inferring the material properties such as the correlation length of the force chain structure and the internal dissipation, on small length scales not accessible by long-wavelength coherent waves.

Experiment.—The glass beads used in our experiments are of diameter $d = 0.6$–0.8 mm, randomly deposited by pouring and stirring in a duralumin cylinder of diameter $W = 30$ mm and varying height from 7 to 15 mm. The container is closed with two fitting pistons and a normal load is applied to the granular sample across the top piston. Before the ultrasonic measurements, one cycle of loading and unloading is performed in the granular packing in order to consolidate the sample and minimize its hysteretic behavior. Statistically independent ensembles of the packing configuration are realized by stirring vigorously glass beads after each measurement and repeating carefully the same loading protocol. The volume fraction of glass beads thus obtained is found to be 0.63 ± 0.01. A *plane-wave* generating transducer of diameter 30 mm (top piston) and a small detecting transducer of diameter 2 mm are placed on the axis at the top and the bottom of the cylindrical container in direct contact with glass beads (Fig. 1). Unlike the previous experiment using

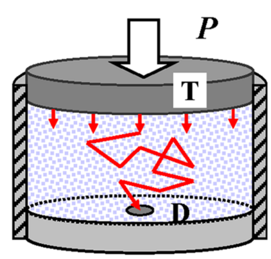

FIG. 1 (color online). Multiple scattering of elastic waves in a confined granular packing under stress P. “T” and “D” denote the large source transducer and the small detector, respectively.

a broadband short pulse [7], ten-cycle tone burst excitations of 20 μs duration centered at a frequency of 500 kHz are applied to the longitudinal source transducer. This narrow band excitation corresponds to the product of granular skeleton acoustic wave number and bead diameter, $kd = \omega d/\nu \approx 2.9$, with $\nu \approx 750$ m/s being a typical sound speed in the solid frame. At such a high frequency, one expects to deal with a strongly scattering medium.

In Fig. 2(a), we show the ultrasound propagating through a glass bead packing of thickness $L = 11.4$ mm under axial stress of $P = 750$ kPa, a typical pressure at depths of tens of meters in soils due to the weight of the overburden. As previously [7], the transmitted temporal signal consists basically of a primary coherent component and a strongly fluctuating incoherent component. The *low-frequency* coherent pulse E_P ($\sim$ 70 kHz) at the leading edge of the transmitted signal corresponds to a self-averaging effective wave propagating ballistically at compressional wave velocity $\nu_P \approx 1000$ m/s, while the *high-frequency* incoherent signal S ($\sim$ 500 kHz) arriving mostly at later times is devoted to specklelike scattered waves by the inhomogeneous distribution of force chains. The sensitivity of the coherent and incoherent waves to changes in packing configurations is shown in Fig. 2(b) by ensemble averaging the transmitted ultrasonic signals over 15 independent granular samples. Because of the random phases of the scattered signals from independent packing configurations, the scattered waves tend to cancel out each other in averaging, leaving the ballistic coherent pulse E_P which is self-averaged and consequently configuration insensitive. Moreover, another coherent signal noted as E_S survives from this averaging procedure, which propagates ballistically at a shear velocity about $\nu_S \approx 450$ m/s. As shown in the inset of Fig. 2(b), the use of large transverse transducers can lead to a considerable enhancement of this shear wave.

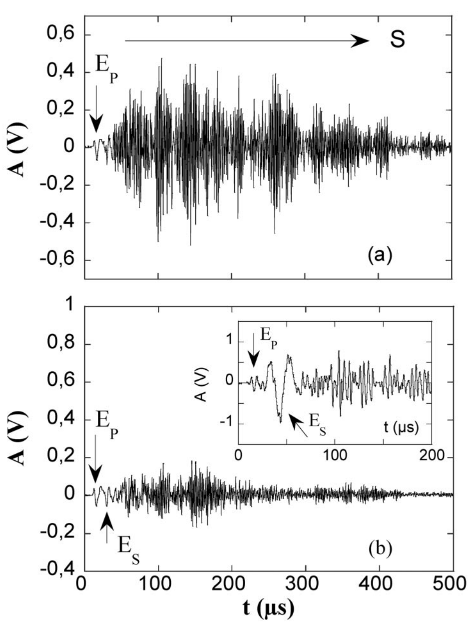

FIG. 2. Transmitted ultrasonic signals through a dry glass beads packing (a) at a given configuration, and (b) after ensemble averaging over 15 independent configurations. The inset illustrates the transmitted signal at a given configuration using the shear transducers.

To investigate the statistical characteristics of scattered waves, we follow the intensity evolution in space and time of the wave train injected into the granular medium. For each configuration we subtract the *low-frequency* coherent ballistic pulses E_P and E_S from the transmitted signal [Fig. 2(a)] by means of a high-pass (HP) filter ($f \geq$ 300 kHz), either analogical or numerical, and determine the intensity of the scattered waves by squaring the envelope of the filtered waveform. In Fig. 3, we show on a semilogarithmic scale the resulting transmitted intensity of a scattered waves ensemble averaged over 50 independent configurations for three different sample thicknesses. As the sample thickness is increased, not only the overall intensity decreases but the temporal profile broadens and the peak occurs at later times. This picture is reminiscent of the diffusively transmitted pulses of classical waves through strongly scattering random media and, in particular, the multiple acoustic scattering through a concentrated suspension of glass beads immersed in water [8]. However, we should stress the basic difference in the mode of wave transmission between this work and those in Ref. [8]. In our dry granular media the elastic waves propagate through the contact network from bead to bead, while in the glass bead suspensions the longitudinal waves propagate through the ambient fluid in the pores after scattering from the beads.

Modeling.—To ascertain the validity of the diffusion approximation, we compare the measured time profile of the average scattered intensity to the transmitted flux predicted by the *scalar* diffusion equation for the acoustic energy density U, subject to appropriate boundary conditions. In our experimental geometry, the input face of the cylindrical sample is excited uniformly by a piston-like source, which favors essentially the generation of the

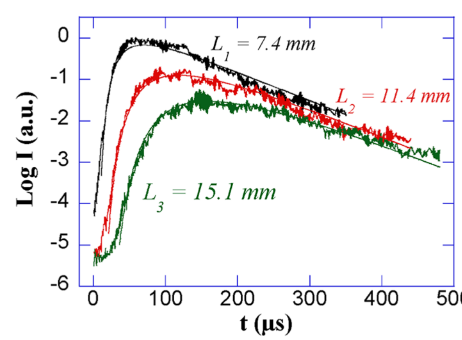

FIG. 3 (color online). The average intensity I of the scattered waves traveling across granular samples of three different thicknesses: $L = 7.4$, 11.4, 15.1 mm, respectively. Theoretical curves are given by solid lines.

fundamental mode in this cell. As shown by Page *et al.* [8], the acoustic energy density measured on the axis with its source at a distance $z = L \leq W/2$ can thus be reasonably approximated by the 1D solution to the case of a plane-wave incident on a slab geometry of thickness L. Because of the low wave velocities in the granular medium and accordingly the acoustic impedance compared to that of the duralumin cell, the reflectivity $R(\theta)$ of the cell wall averaged over the incident angles θ is very high ($\langle R \rangle \approx 0.95$). For simplicity, we assume that the cell walls are perfectly reflecting at $z = 0$ and L. The transmitted flux into the detector is accordingly given by $J(t) = (v/4)U(L, t)$ [8], leading to

$$J(t) = \frac{\nu U_0}{2L} e^{-t/\tau_a} \sum_{n=0}^{\infty} \frac{(-1)^n}{\delta_n} \cos(n\pi\ell^*/L) e^{-D(n\pi)^2 t/L^2}, \quad (1)$$

where U_0 is the deposited source energy, $D = (1/3)\nu_e l^*$ is the diffusion coefficient with ν_e the energy transport velocity and l^* the transport mean free path, τ_a is the inelastic absorption time, and $\delta_n = 2$ for $n = 0$, otherwise $\delta_n = 1$.

If we identify ν_e with a coherent wave velocity, say, the shear velocity ν_S, as discussed below, we can obtain both the diffusion coefficient D and the quality factor Q ($= 2\pi f \tau_a$) by fitting the time profile of the average transmitted intensity. The solid lines in Fig. 3 are fits to the data using the solutions of Eq. (1) with a single set of fitting parameters: $D = 0.13 \pm 0.01\ \mathrm{m^2/s}$ and $Q = 200 \pm 10$ for all three sample thicknesses. The fact that a single set of parameters describes well the experimental data for all three thickness samples strongly supports the diffusion model. The slight discrepancy between the calculations and the measurements at the late time is probably due to the coupling of the granular sample with the cell wall since the wave leakage even weak into the cell wall and its radiation back to the granular sample may influence, in particular, the decay of the average transmitted intensity at the long time scale.

In the above diffusion model, the vector nature of the underlying elastic wave motion in the granular medium is not explicitly taken into consideration. The relevant radiative transport equations (RTE) were derived in [10] using the first-principles calculations. The resulting RTE are determined in terms of the power spectral densities of the fluctuations of the material density and elastic modulus and account for both shear polarizations and mode conversion between compressional and shear modes, which occurs during each scattering event. For the range of frequency used in this study *well below* the first resonances of a glass sphere, e.g., the shearlike spheroidal mode: $f_{\mathrm{res}} \sim (\nu_S)_{\mathrm{glass}}/d \sim 4$ MHz, the granular network can be modeled as an effective random network of point masses (beads) and springs, which exhibits spatial fluctuations of both density and elastic modulus.

Thus we can qualitatively interpret the features of wave transport in granular media within the framework of RTE, though the elastic RTE deduced from the first-principles calculation is not available for amorphouslike granular media. The dominant energy component of the transmitted wave is at high frequency, which has a multiply scattered behavior approaching a diffusive regime. Over distances of propagation long compared to the transport mean free path, the mode conversion process leads to the complete depolarization of shear waves and randomization of the wave fields. In such a diffusion regime, the ratio K of energy densities of the shear (U_S) to the compressional (U_P) waves is governed by an equipartition of energy law [10], $K = U_S(t, \mathbf{r})/U_P(t, \mathbf{r}) = 2(\nu_P/\nu_S)^3$. The typical value of $\nu_P/\nu_S \geq \sqrt{3}$ yields $K \geq 10$, showing that in the diffusive regime the shear waves dominate in the scattered wave field, as is observed in the seismological data [11]. Moreover, the elastic radiative transfer equation is reduced to the single diffusion equation with a diffusion coefficient approximated to that of the shear wave $D_S = (1/3)\nu_S l_S^*$, which confirms the applicability of the above scalar diffusion model.

Material characterization.—By use of the diffusion model we can infer the structure and the properties of granular materials. The transport mean free path estimated from D and ν_e is $l_S^* \approx 0.87$ mm ($\sim d$). As l_S^* is the mean distance before the direction of wave propagation is randomized, it defines a length scale beyond which the scattering medium becomes statistically isotropic and homogeneous, and shall be large in comparison with the correlation length ξ of the medium. The fact that l_S^* is only just the grain size d leads us to state that the correlation length ξ of the force network in a 3D dense granular material is *short range,* being of the order of the bead size. This finding is in contrast with the conjecture of a long-range correlation length of the force chains [1], supported by the photoelastic visualization of the stress fields in 2D disk packings [3]. We believe that this contradiction resulting from the dimensionality of the system shall have a decisive effect on the topology of the contact network and accordingly the distribution of the stress field. Our results are consistent with a recent numerical simulation performed in 3D *elastic sphere* packings in which a continuous transition is found from exponential to Gaussian force distribution as the applied load becomes high enough ($P > 0.75$ MPa) [4]. This crossover is associated with the spatial homogenization of the force chains.

On the other hand, measurements of scattered waves allow one to assess the high-frequency viscoelastic properties of granular media. Compared to coherent wave, the diffusing wave has the advantage of separating the absorption from the scattering attenuation. In a dense granular medium, one of the major sources of dissipation is the frictional slip of the contact forces. To study this effect, we examine the influence of a small amount of wetting liquid which lubricates the contacts between the beads. We mix vacuum pump oil (viscosity $\eta \approx 20 \times 10^{-3}$ Pa s and surface tension $\gamma \approx 20 \times 10^{-3}$ N/m) with the granular sample for tens of minutes to distribute the oil uniformly among the grains. The inset of Fig. 4 illustrates a HP filtered ultrasonic signal ($f \geq 300$ kHz), transmitting through a wet glass bead packing $L = 11.4$ mm with liquid volume fraction $\phi_l \approx 0.015\%$. For such a quantity of liquids, it was revealed that the oil is basically trapped in menisci which form at asperity on the rough surfaces of grains [12]. Compared with the result obtained in a dry bead packing [Fig. 2(a)], a drastic effect on the wave amplitude is observed with high-frequency multiply scattered waves. Such a highly additional dissipation of coda-like scattered waves is presumably ascribed to the oil-induced lubrication, which reduces the static friction threshold between the grains and increases the number of potentially slipping contacts on the passage of shear waves [13]. In Fig. 4, we show for comparison the ensemble averaged acoustic intensity profiles in the dry and wet bead packings, respectively. The fit to the experimental data yields $Q_{\rm wet} = 40 \pm 2$ and $D_{\rm wet} = 0.15 \pm 0.01$ m^2/s for the wet bead packing. The low value of $Q_{\rm wet}$ is consistent with previous absorption measurements conducted at the low-frequency range [13]. As for the increase of $D_{\rm wet}$ by nearly 15% compared to the dry granular packing, it is probably related to the increasing of wave velocity of the same order of magnitude due to better bead-bead contacts caused by the lubrication effect [14].

In summary, we have shown that the multiple scattering of high-frequency elastic waves through the granular force networks can be described by the diffusion approximation over a long distance of propagation ($L \gg l^*$). From the diffusion model we find that the correlation length ξ of the force network is short range ($\xi \sim d$) in 3D granular packing under moderate stress contrary to the usual picture of long-range force correlation length ($\xi \sim 5 - 10d$) observed in a 2D granular system. A further study is to investigate the evolution of the wave transport behavior in a more tenuous granular network when the applied stress is decreased [6]. Also with the sensitive probing by the acoustic speckles, we have measured the energy dissipation of wave in the granular medium, which allows us to highlight the crucial role of a small amount of wetting liquids on its viscoelastic properties.

Finally, we point out the similarity of our experiment with the seismic wave propagation in the crusts of Earth and Moon [15]. In particular, the late-arriving coda waves in the lunar seismograms bear a striking resemblance to the multiple scattering of elastic waves in the dry granular packing. Indeed, the absence of water on the Moon causes on the upper lunar crust a highly inhomogeneous medium composed of tectonic fissures—a block-like structure held together by the pressures due to burial. We believe that some features revealed in our laboratory experiments may be used to explain some seismic observations in the high-frequency coda of local earthquakes in rocky soils and the granular medium may be useful as model systems for the characterization of seismic sources.

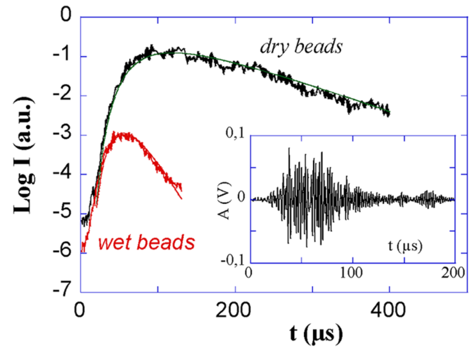

FIG. 4 (color online). Comparison of the average intensity I of the scattered waves traveling, respectively, across a dry and a wet granular packing. The solid lines are the fitting curves. The inset shows a filtered ultrasonic signal transmitting through the wet packing.

I thank P. Mills, T. Brunet, and P. Massaad for many helpful discussions.

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