Tunable Soliton Self-Bending in Optical Lattices with Nonlocal Nonlinearity

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We address the phenomenon of soliton self-bending in Kerr-type nonlocal nonlinear media with an imprinted transverse periodic modulation of the linear refractive index. We show that the imprinted optical lattice makes possible to control the mobility of soliton by varying the depth and the frequency of the linear refractive index modulation.

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One- and two-dimensional discrete spatial solitons in media with local Kerr-type nonlinearity and transverse modulation of linear refractive index have been extensively studied during the last years (for a comprehensive review, see Ref. [1]). The interest to propagation of such solitons is dictated mainly by their rich potential for all-optical switching, power, and angle-controlled steering. In landmark recent experiments, it was demonstrated that periodic waveguide arrays (or lattices) with flexibly controlled refractive index modulation depth and waveguide separation can be created optically, in particular, in photorefractive media [2–6]. Optical lattices, whose properties can be tuned from continuous to fully discrete by variation of refractive index modulation depth, offer a number of new opportunities for optical switching architectures in comparison with those accessible with technologically imprinted arrays of evanescently coupled waveguides. Such *tunable discreteness* proved to be promising for spatial soliton management including radiative switching and parametric steering [7–9].

However, under appropriate conditions, the nonlinear response of materials might be highly nonlocal, a phenomenon that importantly affects the properties of solitons supported by such media [10]. Thus, in photorefractive crystals nonlocal diffusion nonlinearity becomes significant for narrow light beams [11–17]. Note also that nonlinear response depending not only on the light intensity but also on its derivatives is responsible for Raman self-frequency shift of temporal solitons in optical fibers [18], and appears in different models of physical systems [19,20]. Understanding the influence of nonlocality on soliton formation and propagation is therefore an important issue. In particular, spatial solitons propagating in suitable nonlocal media are known to experience self-bending through propagation [11–17], with the strength of the soliton mobility being dictated by material parameters for given input light conditions.

In this Letter we consider trapped and self-bent spatial solitons in optical lattices constituted by media with both local and nonlocal components of Kerr-type nonlinear response and harmonic transverse modulation of linear refractive index. We analyze areas of existence and basic properties of trapped solitons using an effective particle approach and computer simulations. One of the central results we put forward is the possibility to control the strength of the soliton self-bending by varying the depth and periodicity of the refractive index modulation, thus exploiting in a new way the tunable discreteness afforded by the optical lattice concept.

We address the propagation of optical radiation along the z axis in a slab waveguide with harmonic modulation of the linear refractive index in a transverse direction and first-order nonlocal contribution to nonlinear response described by the modified nonlinear Schrödinger equation for dimensionless complex field amplitude q:

$$i\frac{\partial q}{\partial \xi} = -\frac{1}{2}\frac{\partial^2 q}{\partial \eta^2} - q|q|^2 + \mu q\frac{\partial}{\partial \eta}|q|^2 - pR(\eta)q. \quad (1)$$

Here the longitudinal ξ and transverse η coordinates are scaled to the diffraction length and input beam width, respectively. The parameter μ describes the magnitude of the nonlocal component of nonlinear response; p is the guiding parameter proportional to refractive index modulation depth; the function $R(\eta) = \cos(\Omega_\eta \eta)$ stands for the transverse profile of refractive index; and Ω_η is the modulation frequency. We assume that the depth of the refractive index modulation is small compared to the unperturbed index and is of the order of the nonlinear correction to refractive index due to Kerr effect. The parameter μ is also assumed to be small, consistent with the fact that in practice the nonlocal contribution to the nonlinear response is small compared to the local one.

A qualitative picture of the soliton properties in the optical lattice with nonlocal nonlinearity can be grasped by using the effective particle approach [7–9], which is based on the evolution equation for the integral beam center $\langle \eta(\xi) \rangle = U^{-1}\int_{-\infty}^{\infty} \eta|q|^2 d\eta$, where $U = \int_{-\infty}^{\infty} |q|^2 d\eta$ is the energy flow that conserves during propagation. The equation of motion for the integral beam center $\langle\eta\rangle$ can be written in the form

$$U\frac{d^2}{d\xi^2}\langle\eta\rangle = p\int_{-\infty}^{\infty}|q|^2\frac{\partial R}{\partial\eta}d\eta + \mu\int_{-\infty}^{\infty}\left[\frac{\partial}{\partial\eta}|q|^2\right]^2 d\eta. \tag{2}$$

The approach requires substitution of expression for light field $q(\eta, \xi)$ into the right part of Eq. (2), where one assumes that the beam shape remains unchanged upon propagation in the nonlinear lattice. In the present work we are interested only in propagation of the lowest order single-humped soliton solutions of Eq. (1), and therefore we choose $q(\eta, \xi) = q_0\mathrm{sech}[\chi(\eta - \langle\eta\rangle)]\exp[i\alpha(\eta - \langle\eta\rangle) + i\phi]$, where q_0 is the amplitude, χ is the form factor or inverse beam width, α is the incident angle, and ϕ is the phase. This choice of trial function is justified since in the limit $p, \mu \to 0$ it describes soliton beams at $q_0 = \chi$. Requirement of invariable mean square beam width $d^2\langle(\eta - \langle\eta\rangle)^2\rangle/d\xi^2 = 0$ yields the expression for the amplitude q_0 corresponding to stationary propagation (see [7] for details). In particular, the lattice soliton amplitude is lower than that for soliton of uniform medium with the same width due to the lattice guiding action.

Upon substitution of sech-type trial function into Eq. (2) one gets an equation analogous to that describing motion of the pendulum under the action of constant force:

$$\frac{d^2}{d\xi^2}\langle\eta\rangle + p\Omega_\eta\frac{\pi\Omega_\eta/2\chi}{\sinh(\pi\Omega_\eta/2\chi)}\sin(\Omega_\eta\langle\eta\rangle) = \frac{8}{15}\mu q_0^2\chi^2. \tag{3}$$

This equation shows that the beam remains located (trapped) in the guiding channel of harmonic lattice when $p \geq p_{\mathrm{cr}}$, where the critical value of guiding parameter is given by $p_{\mathrm{cr}} = (16/15)|\mu|q_0^2\chi^3 \times \sinh(\pi\Omega_\eta/2\chi)/\pi\Omega_\eta^2$. Notice that p_{cr} rapidly increases with growth of the soliton form factor. The equilibrium value $\langle\eta\rangle_{\mathrm{eq}}$ corresponding to $d^2\langle\eta\rangle/d\xi^2 \equiv 0$ is given by $\langle\eta\rangle_{\mathrm{eq}} = \arcsin(p_{\mathrm{cr}}/p)$. In the case of shallow refractive index gratings $p < p_{\mathrm{cr}}$ soliton starts to travel along the lattice (i.e., solitons self-bend). Propagation of such solitons is accompanied by radiation of energy when soliton crosses lattice channels. In the limit $p \to 0$, Eq. (3) predicts that the soliton moves along a parabolic trajectory.

To obtain exact profiles of solitons supported by harmonic lattices with nonlocal component of the nonlinear response we searched for solutions of Eq. (1) in the form $q(\eta, \xi) = w(\eta)\exp(ib\xi)$, where b is the real propagation constant, and $w(\eta)$ is the real function. The resulting ordinary differential equation for function $w(\eta)$ was solved with a standard relaxation algorithm. The properties of the simplest odd solitons trapped by harmonic lattice at $p > p_{\mathrm{cr}}$ are summarized in Fig. 1. Trapped solitons exist within a finite band of propagation constants $b \in [b_{\mathrm{low}}, b_{\mathrm{upp}}]$, which shrinks with the increase of parameter μ and broadens with the increase of the guiding parameter p [Fig. 1(b)]. At the lower cutoff b_{low} lattice solitons transform into linear Bloch waves, while the existence of upper cutoff b_{upp} is related with domination of the self-bending effect on the trapping action of the lattice. The energy flow U is found to be a monotonically growing function of the propagation constant b, except for a very narrow region [not even visible in Fig. 1(a)] close to the lower existence cutoff. Figure 1(c) illustrates the typical profiles of solitons trapped by the lattice for different values of b, in the case of low-frequency lattice having larger period in comparison with soliton width $\Omega_\eta < \chi$.

Notice that at moderate energy flow the self-bending induced by the nonlocal nonlinearity is compensated by the shift of the soliton center toward the region with negative gradient of refractive index, that thus acts as a linear prism. Close to the upper cutoff the soliton profile becomes asymmetric. It is interesting to note that the shift of the soliton required for suppression of the soliton self-bending in the present physical setting is in contrast to, e.g., the curvature of the soliton wave front required to suppress the mobility of walking solitons existing in homogeneous cubic and quadratic nonlinear local media [21]. Figure 1(d) shows the profiles of solitons supported by high-frequency lattices, with $\Omega_\eta > \chi$. In this case the soliton cover several lattice channels and all of them take part in compensation of bending effect, since energy redistribution between several channels in the presence

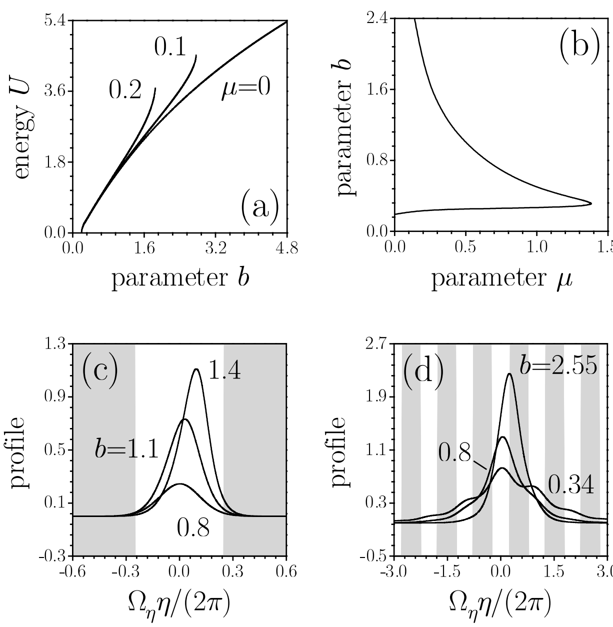

FIG. 1. (a) Energy flow versus propagation constant at $p = 1$, $\Omega_\eta = 2$. (b) Area of existence of lattice solitons at $p = 1$, $\Omega_\eta = 2$. (c) Soliton profiles in low-frequency lattice at $p = 1$, $\Omega_\eta = 0.5$, $\mu = 0.25$. (d) Soliton profiles in high-frequency lattice at $p = 1$, $\Omega_\eta = 4$, $\mu = 0.1$. In gray regions in (c) and (d) function $R(\eta) \leq 0$, while in white regions $R(\eta) > 0$.

of the nonlocal component of nonlinear response results in the appearance of the refractive index gradient along the whole guiding array. This effect is especially apparent for low-energy solitons [as in Fig. 1(d)].

To analyze stability of the obtained soliton families we searched for the perturbed solutions of Eq. (1) in a form $q(\eta, \xi) = [w(\eta) + U(\eta, \xi) + iV(\eta, \xi)]\exp(ib\xi)$. We looked for perturbations growing with the complex rate δ : $U(\eta, \xi) = \mathrm{Re}[u(\eta, \delta)\exp(\delta\xi)]$, $V(\eta, \xi) = \mathrm{Re}[v(\eta, \delta)\exp(\delta\xi)]$. Linearization of Eq. (1) around stationary solution $w(\eta)$ yields an eigenvalue problem:

$$\delta u = -\frac{1}{2}\frac{d^2 v}{d\eta^2} + bv - pRv + \left(2\mu w\frac{dw}{d\eta} - w^2\right)v,$$

$$\delta v = \frac{1}{2}\frac{d^2 u}{d\eta^2} - bu + pRu - \left(4\mu w\frac{dw}{d\eta} - 3w^3\right)u - 2\mu w^2\frac{du}{d\eta}, \tag{4}$$

which was solved numerically. Trapped solitons were found to be stable almost in the whole domain of their existence except for the narrow region near the lower cutoff b_{low} where $dU/db \leq 0$.

Besides the soliton trapping shown above, a salient physical phenomenon to be addressed is the impact of the lattice on solitons that are not trapped and thus do self-bend. This occurs when the lattice depth is too small to trap the solitons ($p < p_{\mathrm{cr}}$), so that they move across the lattice and their trajectory self-bends. To elucidate the dynamics of such phenomena we solved Eq. (1) with the split-step Fourier method using the renormalized sech-type solitons as input conditions. The central results obtained are illustrated by Fig. 2. The initial stage of self-bent soliton propagation is accompanied by slow tunneling of its energy from an input lattice channel to the neighboring one [Fig. 2(a)]. The rate of this process strongly depends on the depth of refractive index modulation and, as expected on intuitive grounds, is slowed down with the increase of the guiding parameter p [Fig. 2(b)]. The nonlocal contribution to the nonlinear response results in the progressive increase of the instantaneous propagation angle. Gradually the trajectory of soliton motion approaches a smooth parabola. It should be pointed out that self-bent soliton propagation is always accompanied by radiative losses and, when the instantaneous propagation angle approaches the Bragg one, the backward scattering grows dramatically and the soliton decays into a set of Bloch waves. The concept of *tunable self-bending* and thus its potential application to self-routing are illustrated by Fig. 3. The integral center trajectories are depicted in Fig. 3(a). The possibility to control the soliton trajectory by varying the refractive index modulation depth p, thus addressing a desired output lattice channel, is clearly apparent. This is illustrated in Fig. 3(b), which shows the output soliton position (actually the corresponding spatial shift) as a function of the guiding parameter p. Notice that there is a broad interval of values of the p parameter where the soliton shift almost linearly decreases with the growth of p. Similarly, Fig. 3(c) shows the output soliton position as the function of the lattice frequency Ω_η. For selected values of p and μ the soliton beam is trapped (immobilized) in the finite band of intermediate frequencies. On the high-frequency wing ($\Omega_\eta > 3.72$) the spatial shift grows monotonically with Ω_η, and then saturates at the level corresponding to uniform nonlinear media. At the low-frequency wing ($\Omega_\eta \to 0$), this shift also saturates at the level of uniform media, but the corresponding dependence in nonmonotonic [inset, Fig. 3(c)].

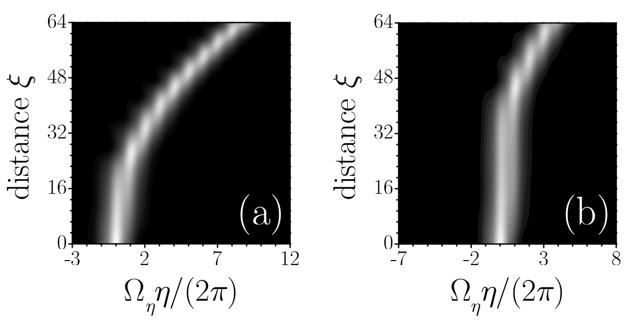

FIG. 2. Dynamics of beam propagation in harmonic lattice for $p = 0.42$ (a) and 0.5 (b) at $\Omega_\eta = 4$, $\mu = 0.045$.

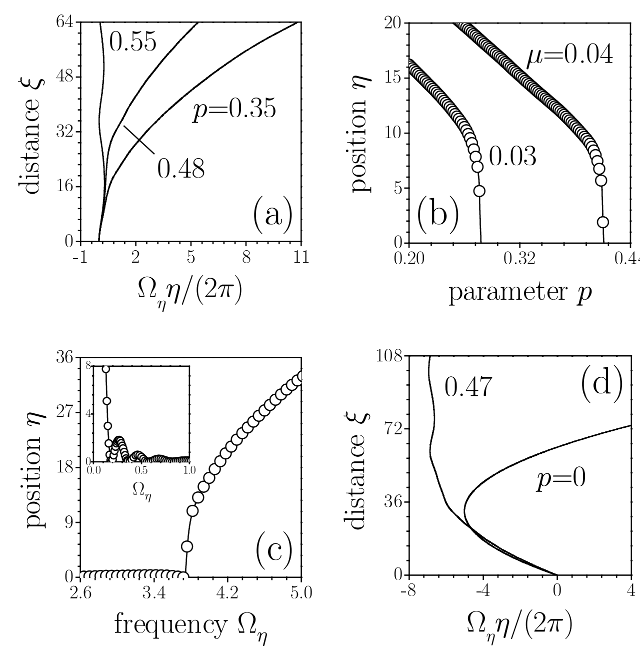

FIG. 3. (a) Trajectories of integral beam centers inside the lattice at $\Omega_\eta = 4$, $\mu = 0.045$. (b) Output beam position at $\xi = 64$ as a function of guiding parameter at $\Omega_\eta = 4$. (c) Output beam position at $\xi = 64$ as a function of modulation frequency at $p = 0.3$, $\mu = 0.04$. (d) Beam trajectories at $\Omega_\eta = 4$, $\mu = 0.03$, $\alpha = -0.5$.

All results described above were linked with solitons launched along the lattice at $\alpha = 0$. It can be readily shown that the propagation dynamics depicted in Fig. 3(a) is reversed (namely, $\xi \rightarrow -\xi$) when solitons are launched into the lattice at negative angles α. Because of radiative losses the situation is possible when the soliton becomes trapped in one of the lattice channels [Fig. 3(d)]. This is another illuminating example of controllable switching in lattices with nonlocal nonlinearity and tunable mobility.

From the point of view of experimental observation of the effects reported here, we would like to mention that nonlocal contribution to nonlinear response is typical for photorefractive crystals. Periodic lattice in such crystals can be induced optically [2–6] and, since waves of different polarizations are involved for the creation of the lattice and study of the soliton propagation, the linear lattice-creating waves will be unaffected by nonlocal diffusion nonlinearity of the crystal. The guiding parameter in photorefractive crystals can be controlled by the variation of applied voltage and the intensity of waves creating the lattice [2–6]. Actually, Eq. (1) describes propagation of light in photorefractive media only at low intensities, namely, when $I \ll I_{\mathrm{bg}}$, where I_{bg} is the intensity of background illumination. At higher intensity, saturation becomes significant. However, under such conditions self-bending dominates over lattice effects, a situation that corresponds to the opposite limit, than the one addressed here. We have verified numerically that, at power levels where nonlocal and lattice effects compete on similar footing, saturable and Kerr models yield qualitatively similar results.

To conclude, we have exposed that soliton formation in optical lattices with nonlocal nonlinearity in general, and the concept of *tunable self-bending* in particular, offers important new opportunities for the control of soliton light beams. The key feature uncovered here is the possibility to control the soliton mobility and the output lattice channel by adjusting the properties of the lattice.

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