Perturbative Generation of a Strange-Quark Asymmetry in the Nucleon

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We point out that perturbative evolution in QCD at three loops generates a strange-antistrange asymmetry $s(x) - \bar{s}(x)$ in the nucleon's sea just from the fact that the nucleon has nonvanishing up and down quark valence densities. The recently computed three-loop splitting functions allow for an estimate of this effect. We find that a fairly sizable asymmetry may be generated. Results for analogous asymmetries in the heavy-quark sector are also presented.

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Strange quarks and antiquarks play a fundamental role in the structure of the nucleon [1]. Among the various strangeness-related properties of the nucleon, the strange "asymmetry," $s(x) - \bar{s}(x)$, in the number densities of strange quarks and antiquarks, x being the light-cone momentum fraction they carry, is of particular interest. Since the nucleon does not carry any strangeness quantum number, the integral of the asymmetry over all values of x has to vanish:

$$\langle s - \bar{s} \rangle = \int_0^1 dx[s(x) - \bar{s}(x)] = 0. \qquad (1)$$

However, there is no symmetry that would prevent the x dependences of functions $s(x)$ and $\bar{s}(x)$ from being different. One therefore can expect $s(x) \neq \bar{s}(x)$, in general.

To understand and quantify the strangeness asymmetry in the nucleon is interesting in different contexts. Some models of nucleon structure make predictions [2] for $s(x) - \bar{s}(x)$, and their confrontation with experimental measurements may perhaps give us further insight into nonperturbative dynamics of the strong interactions. For example, within the meson cloud model [2], one usually expects $s(x)$ to be larger than $\bar{s}(x)$ at large x, implying the opposite behavior at small x. Light-cone models [2], on the other hand, generically predict $s(x) - \bar{s}(x) < 0$ at large x. The various models mostly predict a fairly small value of the second moment of the strange-antistrange distribution, $|\langle x(s - \bar{s})\rangle| \sim 10^{-4}$.

As was emphasized in [3,4], the question concerning the strange asymmetry in the nucleon becomes particularly relevant in view of the "anomaly" seen by the NuTeV Collaboration in their measurement of the Weinberg angle in deeply inelastic neutrino scattering. The NuTeV result [5], $\sin^2\theta_W|_{\mathrm{NuTeV}} = 0.2277 \pm 0.0013_{(\mathrm{stat.})} \pm 0.0009_{(\mathrm{sys.})}$, deviates around 3 standard deviations from the commonly accepted value $\sin^2\theta_W = 0.2228 \pm 0.0004$ [6]. This large difference could be at least partly explained [3,4] by a positive value of $\langle x(s - \bar{s})\rangle$. Typically, a value $\langle x(s - \bar{s})\rangle \approx 0.005$ would be required if one wanted to attribute the NuTeV anomaly to the strange asymmetry alone.

The NuTeV Collaboration has determined the second moment $\langle x(s - \bar{s})\rangle$ from a lowest-order QCD analysis of neutrino data [7] and finds a negative value [8]:

$$\langle x(s - \bar{s})\rangle = -0.0027 \pm 0.0013. \qquad (2)$$

Such a value increases the discrepancy in $\sin^2\theta_W$ to a 3.7σ effect. The second moment had also been investigated in "global analyses" of unpolarized parton distributions. Reference [9] reported an improvement in the global analysis if the asymmetry $s(x) - \bar{s}(x)$ is positive at high x. They found $\langle x(s - \bar{s})\rangle = 0.002 \pm 0.0028$ at $Q^2 = 20\ \mathrm{GeV}^2$ from their best fit. A recent update of this analysis [10], however, reduces the asymmetry significantly. The most recent global QCD fit [11] finds a large uncertainty for the strange asymmetry and quotes a range $-0.001 < \langle x(s - \bar{s})\rangle < 0.004$.

The discussion reported so far regards strange-antistrange asymmetries that are generated by nonperturbative mechanisms. Then, because of the customary scaling violation, the asymmetry becomes dependent on the hard-scattering scale Q at which the nucleon is probed. In this Letter, we point out that perturbative QCD alone definitely predicts a nonvanishing and Q-dependent value of the strange-antistrange asymmetry. We show that nonsinglet evolution of the parton densities at three loops generates a strange asymmetry even if it is not present at the input scale. Thus, we can provide a prediction for the strange asymmetry $s(x) - \bar{s}(x)$ based solely on perturbative QCD. The effect arises because at that order of perturbation theory the probability of a splitting $q \to q'$ becomes different from that of $q \to \bar{q}'$ and because the nucleon has u and d valence densities. The three-loop splitting functions have very recently been published [12,13], among them the splitting function needed for our perturbative estimate of $s(x) - \bar{s}(x)$. To begin with, we write down the evolution equations and determine the solution for the generated strange asymmetry. We then

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present some numerical results for the strange asymmetry and extend the analysis to the heavy-quark sector.

It is convenient to write the evolution equations in Mellin space, becoming simply (with $a = q_i, \bar{q}_i, g$)

$$\frac{df_a^N(Q^2)}{d\ln Q^2} = \sum_b P_{ab}^N[\alpha_S(Q^2)] f_b^N(Q^2). \tag{3}$$

In the following we drop the dependence on the moment index N for simplicity. In Eq. (3) P_{ab} is the function describing the splitting $b \to a$. The splitting functions are perturbative; their perturbative series starts at $\mathcal{O}(\alpha_S)$:

$$P_{ab} = \left(\frac{\alpha_S}{4\pi}\right) P_{ab}^{(0)} + \left(\frac{\alpha_S}{4\pi}\right)^2 P_{ab}^{(1)} + \left(\frac{\alpha_S}{4\pi}\right)^3 P_{ab}^{(2)} + \mathcal{O}(\alpha_S^4).$$

Keeping just the first term yields the leading order (LO) evolution. Improving the approximation by taking into account also the second, or the second and third, terms corresponds to next-to-leading order (NLO) and next-to-next-to-leading order (NNLO) evolution, respectively.

Since $s(x) - \bar{s}(x)$ is a flavor nonsinglet (NS) quantity, we need to consider only the NS sector of evolution. We write the evolution kernels P_{ab} by adopting the notation of Ref. [12]. Because of charge conjugation invariance and flavor symmetry of QCD, one has (see, e.g., Ref. [14])

$$\begin{aligned} P_{q_i q_k} &= P_{\bar{q}_i \bar{q}_k} = \delta_{ik} P_{qq}^V + P_{qq}^S, \\ P_{q_i \bar{q}_k} &= P_{\bar{q}_i q_k} = \delta_{ik} P_{q\bar{q}}^V + P_{q\bar{q}}^S. \end{aligned} \tag{4}$$

The splitting functions P_{qq}^S and $P_{q\bar{q}}^S$ thus describe splittings in which the flavor of the quark changes. As will become clear, the effect we investigate originates from the fact that $P_{qq}^S \neq P_{q\bar{q}}^S$ starting from NNLO [14,15].

In the flavor NS sector the evolution equations (3) are diagonalized by properly introducing NS combinations of parton densities. Up to NLO it is sufficient to consider two NS combinations. Owing to the difference between P_{qq}^S and $P_{q\bar{q}}^S$, starting from NNLO it is necessary [15] to introduce the following *three* NS combinations:

$$\begin{aligned} f^{(V)} &\equiv \sum_{i=1}^{N_f} (f_{q_i} - f_{\bar{q}_i}), \\ f_{q_i}^{(\pm)} &\equiv f_{q_i} \pm f_{\bar{q}_i} - \frac{1}{N_f} \sum_{j=1}^{N_f} (f_{q_j} \pm f_{\bar{q}_j}), \end{aligned} \tag{5}$$

where N_f is the number of flavors. Each of these evolves as

$$\frac{d\ln f^{(A)}(Q^2)}{d\ln Q^2} = P^{(A)}[\alpha_S(Q^2)], \qquad (A = V, \pm), \tag{6}$$

where the evolution kernels are

$$\begin{aligned} P^{(V)} &= P_{qq}^V - P_{q\bar{q}}^V + N_f(P_{qq}^S - P_{q\bar{q}}^S), \\ P^{(\pm)} &= P_{qq}^V \pm P_{q\bar{q}}^V. \end{aligned}$$

The equations have the solutions

$$f^{(A)}(Q^2) = U^{(A)}(Q, Q_0) f^{(A)}(Q_0^2), \tag{7}$$

where $f^{(A)}(Q_0^2)$ is the parton density at the starting scale Q_0 and the evolution operator $U^{(A)}$ is given by

$$U^{(A)}(Q, Q_0) = \exp\left\{ \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} P^{(A)}[\alpha_S(q^2)] \right\}. \tag{8}$$

Using Eq. (7) with $A = -$ and $A = V$, we have

$$\begin{aligned} (f_{q_i} - f_{\bar{q}_i})(Q^2) = {} & U^{(-)}(Q, Q_0)(f_{q_i} - f_{\bar{q}_i})(Q_0^2) \\ & + \frac{1}{N_f}[U^{(V)}(Q, Q_0) \\ & - U^{(-)}(Q, Q_0)] f^{(V)}(Q_0^2). \end{aligned} \tag{9}$$

Equation (9) is the basic result in our discussion of flavor asymmetries. The key point is that Eq. (9) implies that, in the region of Q^2 where QCD perturbation theory is applicable, the flavor asymmetries $(f_{q_i} - f_{\bar{q}_i})(x, Q^2)$ must necessarily be different from zero. This is a definite, though qualitative, prediction of perturbative QCD.

In the following we simplify the notation, using $f_{q_i} \equiv q_i$ and $f_{\bar{q}_i} \equiv \bar{q}_i$, and we consider in detail the strange-quark asymmetry, $s - \bar{s}$. Equation (9) gives

$$\begin{aligned} (s - \bar{s})(Q^2) = U^{(-)}(Q, Q_0)\Bigg[& (s - \bar{s})(Q_0^2) + \frac{1}{N_f} \\ & \times \left(\frac{U^{(V)}(Q, Q_0)}{U^{(-)}(Q, Q_0)} - 1 \right) f^{(V)}(Q_0^2) \Bigg]. \end{aligned} \tag{10}$$

At LO and NLO, $U^{(V)} = U^{(-)}$, and thus any strange-quark asymmetry can be produced only by a corresponding asymmetry at the input scale Q_0 of the evolution. From NNLO, the degeneracy of $P^{(V)}$ and $P^{(-)}$ is removed:

$$P^{(V)} - P^{(-)} = N_f(P_{qq}^S - P_{q\bar{q}}^S) \equiv \left(\frac{\alpha_S}{4\pi}\right)^3 P_{ns}^{(2)S} + \mathcal{O}(\alpha_S^4), \tag{11}$$

where $P_{ns}^{(2)S}$ has recently been presented in Ref. [12]. It comes with the color structure $d^{abc} d_{abc}$, which is also new at this order. In a physical gauge, the Feynman diagrams contributing to $P_{ns}^{(2)S}$ are of the "light-by-light" scattering type, three gluons connecting the two different quark lines. Figure 1 shows some examples of (interferences of) (a) virtual and (b) real diagrams that generate the asymmetry in the evolution of quarks and antiquarks. The virtual part [e.g., Fig. 1(a)] has separately been studied [16] in the context of its contribution to the one-loop triple collinear splitting function. When the quark q_j is replaced by the antiquark $\bar{q}_j$, the Abelian-like part of the diagrams in Fig. 1 changes sign, because of the charge asymmetry produced by the exchange of three gluons in the t channel. This effect occurs in both QCD and QED, and it is a genuine quantum phenomenon.

On account of Eq. (10), even if $(s - \bar{s})(Q_0^2) = 0$, a nonvanishing strange-quark asymmetry is produced just by the perturbative QCD evolution. Here it is crucial that the total valence density of the nucleon, $f^{(V)}$, is nonvanishing due to the up and down valence quarks. Using Eqs. (8) and (11) we have, in moment space,

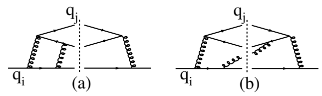

FIG. 1. Example of (a) virtual and (b) real contributions to $P_{ns}^{(2)S}$.

$$\frac{U^{(V)}(Q,Q_0)}{U^{(-)}(Q,Q_0)}-1=-\frac{1}{8\pi b_0}P_{ns}^{(2)S}\left[\left(\frac{\alpha_S(Q^2)}{4\pi}\right)^2-\left(\frac{\alpha_S(Q_0^2)}{4\pi}\right)^2\right]+\mathcal{O}(\mathrm{N^3LO}), \quad (12)$$

where $b_0=\frac{1}{12\pi}(11C_A-2N_f)$. Note that despite being a NNLO effect, the perturbative generation of $s-\bar{s}$ is a leading effect since it first occurs at this order. Under the assumption $(s-\bar{s})(Q_0^2)=0$, and neglecting heavy-quark (valence) contributions and threshold effects, the solution for the evolution equation for the strange-quark asymmetry reads to NNLO:

$$(s-\bar{s})_N(Q^2)\simeq-\frac{P_{ns,N}^{(2)S}}{8\pi b_0 N_f}\left[\left(\frac{\alpha_S(Q^2)}{4\pi}\right)^2-\left(\frac{\alpha_S(Q_0^2)}{4\pi}\right)^2\right]\times(u^{(V)}+d^{(V)})_N(Q^2), \quad (13)$$

where we have restored the Mellin moment index N. Here we have used that $U_N^{(-)}(Q,Q_0)$ simply evolves the valence input, $u^{(V)}=u-\bar{u}$ and $d^{(V)}=d-\bar{d}$, to the scale Q at LO. Assuming isospin symmetry, the sum of valence distributions is the same in the proton and the neutron and, consequently, also the perturbative strange asymmetry.

From Eq. (13) we can straightforwardly obtain predictions for $[s-\bar{s}](x,Q^2)$ by a numerical Mellin inversion, once we have chosen an initial scale and input valence densities. For our estimates we employ the low input scale $Q_0=0.51$ GeV and the u, d valence densities of the LO "radiative" parton model analysis of Ref. [17]. Threshold effects at $Q=m_c\equiv1.4$ GeV and $Q=m_b\equiv4.5$ GeV are taken into account by the full implementation of Eq. (10). Since we are considering a leading effect, the LO approximations are appropriate. The value for the initial scale is of course crucial for our results; the lower the scale, the larger will the perturbatively generated strange asymmetry be at a given higher scale Q. Our choice of a rather low input scale may be regarded as yielding the largest possible perturbative strange asymmetry. Whether or not it is indeed correct to assume that the nucleon is symmetric in s and $\bar{s}$ at a low scale is an open question; however, our motivation is to explore the new effect provided by three-loop evolution. We note that our input assumption $[s-\bar{s}](x,Q_0^2)$ is consistent with the input in Ref. [17], where actually $s(x,Q_0^2)=\bar{s}(x,Q_0^2)=0$ was assumed, resulting in a purely generated (symmetric) strange distribution which agrees reasonably well with the ones obtained in other global analyses.

Figure 2(a) shows $[s-\bar{s}](x,Q^2)$ as a function of x, for three different scales, $Q^2=2$, 10, and 100 GeV^2. For comparison, Fig. 2(b) shows the ratio of $[s-\bar{s}](x,Q^2)$ to the Martin-Roberts-Stirling-Thorne (MRST) [18] strange density $s(x,Q^2)$. The generated asymmetry is not negligible and turns out to be positive at small x and negative at large x. Since the distribution has a vanishing first moment and only one node, a *negative* second moment results:

$$\langle x(s-\bar{s})\rangle\approx-5\times10^{-4} \qquad (Q^2=20\ \mathrm{GeV}^2). \quad (14)$$

This value depends fairly little on Q^2 once $Q^2>1$ GeV^2; it then very gently decreases at large Q^2. As expected for a NNLO effect, it is quite small, somewhat smaller than the NuTeV value in Eq. (2). We note that our value lies in the band derived from a global analysis in Ref. [11].

Let us now try to put the size of the perturbatively generated strange asymmetry into a better perspective. As we discussed above, the effect becomes possible for the first time at NNLO, where $P_{ns}^{(2)S}\neq0$. This is reminiscent of a well-known effect that first arises in NLO evolution, namely, the perturbative generation of $[\bar{u}-\bar{d}](x)\neq0$ from evolution, due to $P_{q\bar{q}}^{(V)}\neq0$ at NLO [19]. Interestingly, despite being a NNLO effect, we found our $[s-\bar{s}](x,Q^2)$ to be larger than the NLO perturbative $[\bar{u}-\bar{d}](x,Q^2)$ in most of the x region, in particular, at small x where the splitting function $P_{ns}^{(2)S}$ is singular as $\ln^4 x$. Also, for the perturbative $\bar{u}-\bar{d}$ the difference $[u^{(V)}-d^{(V)}]\times(x,Q_0^2)$ of input valence densities determines the boundary condition, whereas for $s-\bar{s}$ it is their sum, according to Eq. (13). From this point of view, the perturbatively generated $[s-\bar{s}](x,Q^2)$ can actually be considered as quite large. Of course, as is well known, a much larger $\bar{u}-\bar{d}$ asymmetry than the perturbatively predicted one has been measured [20–24], which implies that nonperturbative effects outweigh the asymmetry from perturbative evolution. It is clearly possible that also in the case of $s-\bar{s}$ nonperturbative effects dominate. It is worth

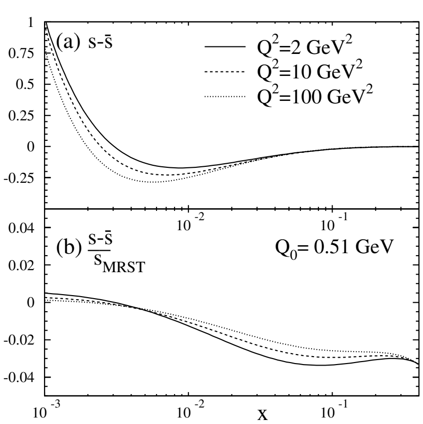

FIG. 2. (a) Strange asymmetry in the nucleon from NNLO QCD evolution for $Q^2=2$, 10, and 100 GeV^2; (b) the corresponding ratio to the LO strange density of Ref. [18].

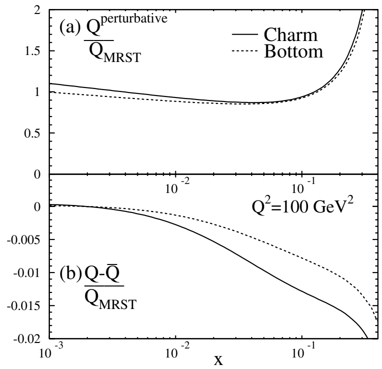

FIG. 3. (a) Ratios between the purely pertubatively generated charm and bottom densities at $Q^2 = 100\ \mathrm{GeV}^2$ to the corresponding density of Ref. [18]; (b) charm and bottom asymmetries in the nucleon generated from NNLO QCD evolution.

pointing out that the uncertainties in the perturbative strange asymmetry itself are difficult to quantify since it is effectively a LO effect. On the other hand, as we mentioned in the introduction, models of nucleon structure generally predict a very small strange asymmetry, the second moment usually being several times smaller than ours in Eq. (14). Therefore, at the very least, we expect perturbative evolution to play a significant role in relating model predictions at the (low) model scale to $s - \bar{s}$ at scales relevant for comparison to experimental data. We also note that the large-x behavior of our perturbatively generated strange asymmetry is driven by that of the evolved valence densities and of the splitting function $P_{ns}^{(2)S}(x)$. As $x \to 1$, one expects from perturbative QCD

$$[s - \bar{s}](x, Q^2) \sim (1 - x)^2 \ln(1 - x)[u^{(V)} + d^{(V)}](x, Q^2).$$

This may well be the dominant behavior at high x, even in the presence of a nonperturbative input for $s - \bar{s}$.

We finally note that our analysis may also be extended to predict the asymmetries for heavy flavors c and b. Here, the perturbative prediction may be more reliable since one will typically start the evolution from the mass of the heavy quark, which is in the perturbative region. Assuming that the charm and bottom densities vanish at the respective masses, we find the results shown in Fig. 3. Plot (a) compares the purely perturbatively generated charm and bottom densities to the results of the latest MRST LO analysis [18]. The agreement found at scales far away from the threshold for heavy-quark production, and in the relevant small x range, is a signal of the validity of the approach. One expects a similar situation to hold for the asymmetry. Plot (b) corresponds to the ratio between the generated asymmetry and the corresponding heavy-quark density. Note that for the last result we only assume that the heavy-flavor *asymmetries* vanish at the respective masses. This is a weaker assumption than that there be no initial heavy-quark distributions at all at these scales. As can be observed, the asymmetries are smaller than in the strange sector, mostly due to the larger initial scale at which the evolution begins.

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