# Experimental Deterministic Coherence Resonance

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We demonstrate coherence resonance in a dynamical system without external noise. The experimental evidence is reported in the low frequency fluctuations of a chaotic diode laser with optical feedback. The phenomenon is also verified numerically using the Lang-Kobayashi equations for a single solitary mode laser, without noise terms. Fast deterministic dynamics plays the role of an effective exciting noise, narrowing the resonance in the autonomous slow power drop cycles of the laser. This new result is the natural extension of deterministic stochastic resonance and noise induced coherence resonance predicted and observed in recent years.

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Stochastic resonance [1] and coherence resonance [2] are phenomena where external noise, at the appropriate amplitude, helps the occurrence of resonant responses in activated dynamical systems. For the stochastic resonances the systems are induced to improve their resonant response to an external periodic drive signal. In an apparent paradoxical effect, the presence of a good amount of noise enhances the quality of the system signal, narrowing the resonance and giving a better signal to noise ratio at the external imposed frequency. Extensively discussed in the review by Gammaitoni *et al.* [1], the effect was also demonstrated with deterministic noise [3]. Coherence resonances, originally proposed by Pikovsky and Kurths [2], occur in dynamical systems that do not need an external periodic driving. They have, into their dynamics, an autonomous limit cycle at the borderline of stability which is excitable by noise. Once excited, an excursion of the system trajectory through this orbit takes place, independent of the noise amplitude, for small noise. Systems with homoclinic chaos also have been shown to be in the category of excitable by external noise and to present stochastic and coherence resonancelike behaviors [4]. Other common excitable nonlinear systems are the ones with delayed feedback, having trajectories with fast and slow time evolution [5]. This is, for instance, the case of diode lasers with optical feedback from an external cavity [6–8]. The fast behavior is due to unstable relaxation oscillations and the slow trajectory envelope is an itinerary over unstable ruins of attractors. For these reasons, coherence resonance was demonstrated for the first time in the experiments by Giacomelli *et al.* [9] varying the amplitude of noise injected in the fixed value of the pump current of a diode lasers with optical feedback. Low frequency fluctuations (LFF), in the scale of nanosecond to tens of microseconds occur as drops in the laser averaged power. Other features characterizing this laser as an excitable system have been studied recently [10–12].

In this Letter we report coherence resonance excited by fast oscillations intrinsic to the dynamical system, instead of originating from an external noise source. This is possible in systems where a parameter produces different variation for the amplitude of the fast dynamics as compared to the variation produced in the relative position of two fixed points, one attractor node and a nearby saddle, which organize an excitable slow cycle [13]. This appears to be the case for the diode laser with optical feedback. Because of the considerable complexity of the solutions of the laser equations, we cannot present the direct proof of this argument. However, we give the indirect evidence in what follows. Without external noise terms, the numerical solutions for the laser equations give a coherence resonance behavior as the internal parameter associated to pump current is tuned. An optimal value will exist that gives best "signal to noise," as characterized by the minimum of $R = \sigma/\overline{T}$, the normalized variance of the average time between excitations cycles, defined by Pikovsky and Kurths [2]. The excited cycles are the deterministic LFF.

The equations for the laser were first proposed by Lang and Kobayashi (LK) [14] and have been largely studied since [8]. They can be written as

$$\frac{dP}{dt} = \left[G(N) - \frac{1}{\tau_p}\right]P(t) + 2\kappa\sqrt{P(t)P(t-\tau)}\cos\{\phi(t) - \phi(t-\tau) + \omega_0\tau\}, \quad (1)$$

$$\frac{d\phi}{dt} = \frac{\alpha}{2}\left[G(N) - \frac{1}{\tau_p}\right] - \kappa\sqrt{P(t-\tau)/P(t)}\sin\{\phi(t) - \phi(t-\tau) + \omega_0\tau\}, \quad (2)$$

$$\frac{dN}{dt} = J - \frac{N(t)}{\tau_s} - G(N)P(t), \quad (3)$$

where the dynamical variables are $P(t)$, the photon flux density or the square of the optical field amplitude, $\phi(t)$, the phase of the slow envelope of the field, and $N(t)$, the carrier density. The delayed feedback enters through the field amplitude and phase at time $(t-\tau)$, with $\tau$ being the round-trip time of the external cavity. The carrier lifetime is $\tau_s$, the photon lifetime is $\tau_p$, and $\omega_0$ is the solitary laser

  

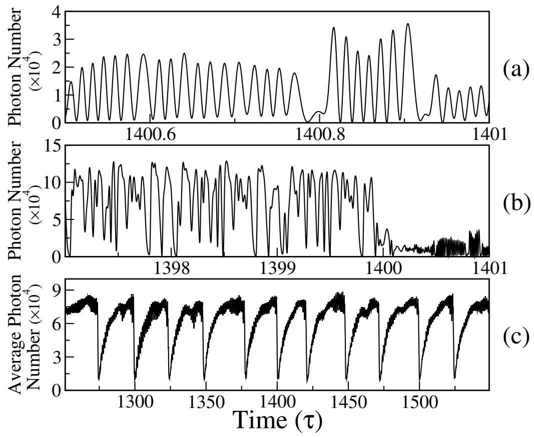


FIG. 1. Calculated segments of laser power using Lang-Kobayashi equations: (a) and (b) show the fast pulses within a fraction of the round-trip time of the external cavity; (c) is the long series averaged to simulate a 300 MHz filtering and showing many LFF events. The parameters used were chosen to give qualitative agreement with experiments: $J/J_{\mathrm{th}} = 1.013$, $\kappa = 22\ \mathrm{ns}^{-1}$, and $\tau = 6$ ns.

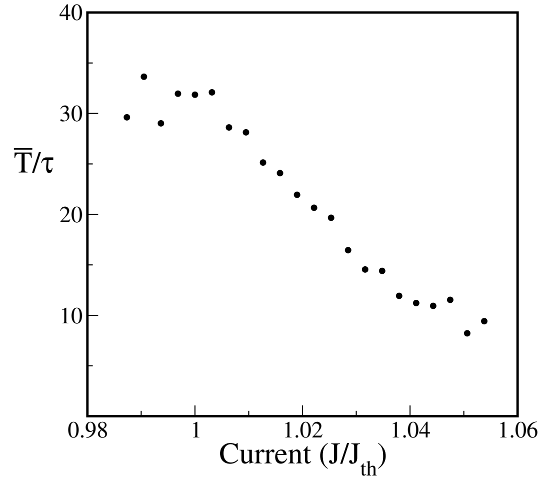


FIG. 2. Average time between LFF drops as a function of the injection current, calculated with the Lang-Kobayashi model.

frequency. The nonlinear optical gain is $G(N) = G_0(N - N_0)/(1 + \epsilon P)$, where $G_0$ is the modal gain, $N_0$ is the carrier density at transparency, and $\epsilon$ is the gain saturation coefficient. $\alpha$ is the linewidth enhancement factor, $J$ is the current density, and $\kappa$ is the feedback rate. The solitary laser threshold current results from (3) as $J_{\mathrm{th}} = (1/\tau_s)[N_0 + 1/(G_0\tau_p)]$.

Typical segments of the laser power time dependence, calculated with LK equations (3), are shown in Fig. 1. The parameters were fixed at $G_0 = 3.2 \times 10^{-6}\ \mathrm{ns}^{-1}$, $N_0 = 1.5 \times 10^8$, $\omega_0\tau = 0$, $\alpha = 3.5$, $1/\tau_p = 282\ \mathrm{ns}^{-1}$, $1/\tau_s = 1.66\ \mathrm{ns}^{-1}$, and $\epsilon = 5 \times 10^{-7}$.

With a fourth order Runge-Kutta routine the equations were integrated using time steps of $\delta t = 10^{-12}$, i.e., nearly 1/3 of the photon lifetime. This size of step was enough to assure smooth ultrafast pulses as seen in Fig. 1(a), for the bottom of a LFF and Fig. 1(b) for a region within the maximum power range. These pulses correspond to the fast chaotic mode-locked pulses predicted by von Tartwijk *et al.* [7] and were observed experimentally by Fischer *et al.* [15] and by Vaschenko *et al.* [16]. Averaging the numerical series to simulate a filter with a 300 MHz bandpass, the fast pulsations cannot be seen, but very distinct power drops, the LFF, appear as in Fig. 1(c). The variance of the fast intensity pulses was determined as a function of the pump current. It grows lineary for $0.98 < J/J_{\mathrm{th}} < 1.06$. Such a variance is not to be confused with that of the time interval between LFFs. This will be the amplitude of the effective deterministic noise responsible for coherence resonance to be shown. We emphasize that the numerical integrations were done without noise terms. A calculation with a longer round-trip time needs longer computation time, and their results are qualitatively similar to the results to be described.

Histograms for the time between LFF drops, $\overline{T}$, were calculated and the average time between drops and the corresponding variance were obtained, as a function of the pump current. The results for $\overline{T}$ are presented in Fig. 2. As predicted many years ago [6,8], noise is not necessary for the LFF to appear. However, with $J/J_{\mathrm{th}} \lesssim 1.00$, the absence of a noise term prevents the occurrence of longer time intervals between drops [17,18]. This explains why, in Fig. 2, $\overline{T}/\tau$ saturates near 30 for small pump currents.

The calculated normalized variance, $R$, is shown in Fig. 3. This quantity is the indicator of oscillation regularity [2]. This figure contains the main numerical result of this Letter. It shows that the LFF pulses become more ordered at an intermediate value of the pump current parameter, when the normalized variance goes through a minimum.

In their original work proposing coherence resonance, Pikovsky and Kurths [2] show (their Fig. 3) the noise to signal ratio, $R$, as a function of the external noise ampli-

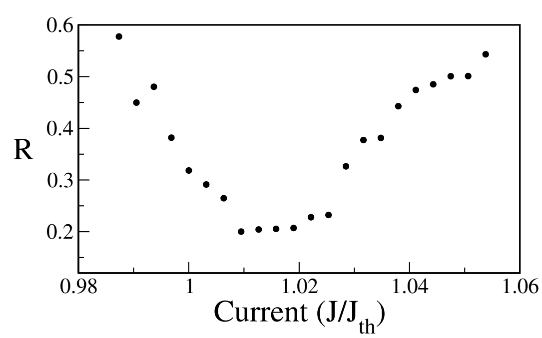


FIG. 3. Normalized variance of the time between LFF drops in the Lang-Kobayashi model, as a function of the injection current.

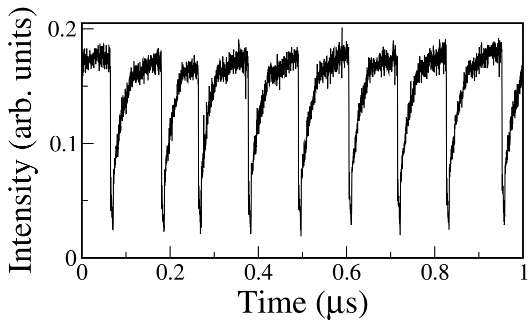


FIG. 4. Experimental segment of laser intensity with LFF power drops. The pump current was 23 mA and the round-trip time was $\tau = 6$ ns.

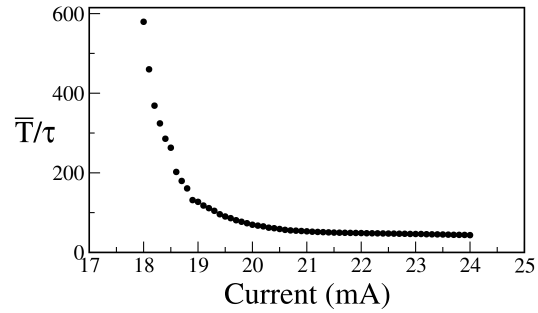


FIG. 5. Experimental average time between LFF drops as function of the injection current.

tude, $D$, for fixed system parameters. The minimum in $R$ for an intermediate value of $D$ is the signature of the effect. In the corresponding experimental demonstration, Giacomelli *et al.* [9] also give the same $R$ as a function of the noise amplitude (their Fig. 3). Here, without external noise, we give $R$ as a function of the system parameter which is the pump current, $J$. As the current affects the deterministic fast "effective noise" differently than it affects the average time between drops, such a variation of the current acts as a variation of the amplitude of the effective noise. At first, the increase on the pump current brings down the value of the variance much more than it decreases the average time. The system becomes more regular, with $R$ dropping in value. After some value of the control parameter is reached, a minimum is past, the effective noise is too big and acts to disrupt the quality of the resonance. This is the interpretation for deterministic coherence resonance observed experimentally and described next.

The experimental verification of the effect was obtained with an SDL 5401 GaAlAs diode laser, thermally stabilized to 0.01 K, emitting at 850 nm and with solitary threshold current of 17.5 mA. A feedback mirror of high reflectivity ($>90\%$) is placed 0.9 and 2.25 m from the laser creating an external cavity with 6 and 15 ns round-trip times, respectively. A collimator and a lens were placed within the cavity to reduce the beam divergence. The intensity output is detected by a 1.5 GHz bandwidth photodiode, and the data series, collected with a Tektronix TDS3032B 300 MHz oscilloscope and, for very long time series with a 12 bits A/D digital acquisition system running at 100 MHz, and connected directly to a computer memory. Different feedback levels, pump current, and external cavity length were used and the dynamics was qualitatively similar to the one reported by Sacher *et al.* [19]. Figure 4 shows an experimental segment of the laser output with LFF events. The similarity to the calculated result shown in Fig. 1(c) is evident.

From an experimental time series with more than $10^4$ LFF events, at each value of the pump current, the average time between events $\overline{T} = \overline{(t_{(i+1)} - t_i)}$, and the variance $\sigma$ were determined. From these, the experimental normalized variance $R = \sigma/\overline{T}$ were obtained. Figure 5 shows the experimental $\overline{T}$ as a function of the pump current, measured for a feedback level corresponding to a threshold reduction of 13.8% and with round-trip time of $\tau = 6$ ns. Such dependence on current is similar to what has been observed by Sukow *et al.* [20]. For currents below 18.6 mA, $\overline{T}$ decreases with a large slope. This must be due to spontaneous emission and external noise and is not observed in the LK calculations presented in Fig. 2. For higher currents, the experimental $\overline{T}$ decreases with current similar to LK calculations, but the ratio $J/J_{\rm th}$ is much larger than in numerical LK results. Such quantitative discrepancies, which remains to be studied, do not affect our observation of the experimental coherence resonance, obtained with a normalized quantity.

The corresponding normalized variance, $R$, is presented in Fig. 6. The spike drop at low currents is due to nondeterministic noise. Nondeterministic stochastic noise, with quantum and classical origin, is known to contribute to LFF at very low currents and helps regularization of the ultralow frequency fluctuations [17,18]. It is worth remembering that after the current is above threshold the laser has its upper level population clamped to the threshold value (on the average) and so the average

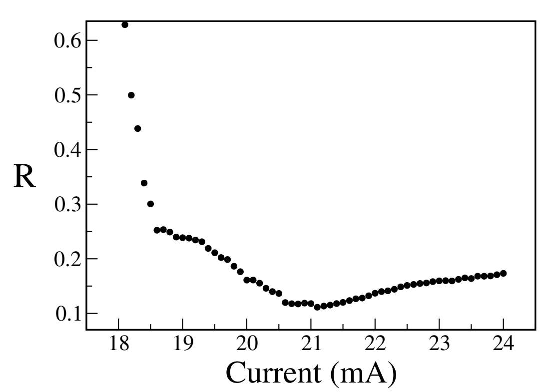


FIG. 6. Experimental normalized variance of the time between LFF drops as a function of the injection current, for a feedback delay of 6 ns.

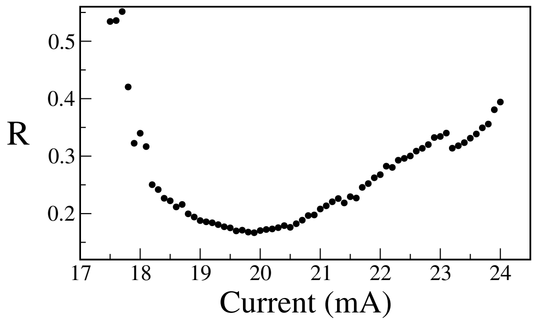


FIG. 7. Experimental normalized variance of the time between LFF drops as a function of the injection current, for a feedback delay of 15 ns.

spontaneous emission noise remains with constant amplitude [8]. Deterministic noise from the mode-locked pulsation goes on growing with current. After the value of 18.6 mA the variation of $R$ with current changes indicating that the effects of the deterministic chaotic fast mode-locked pulses take over. Beyond that value the deterministic coherence resonance manifests as the minimum in $R$ at 21 mA.

Consistent with the results shown from Lang-Kobayashi equations, within the experimental deterministic range, $R$ keeps decreasing and the LFF coherence resonance begins to broaden only after an optimal value of the current is reached. Thus the experimental resonance narrowing occurs totally within the deterministically dominated chaotic operation of the laser. Let us emphasize that a quantitative comparison of the LK results with the real laser is not possible because the laser here has shown traces of multimode (internal cavity) in its optical spectra during LFF operation.

Changing the external feedback return time and the degree of feedback amplitude modifies the position of the minimum, but for a wide range of values the shape of the $R$ dependence always shows the coherence resonance. Figure 7 shows $R$ for $\tau = 15$ ns and threshold reduction of 10.6% in the feedback amplitude. The minimum now occurs for pump current of 20 mA, again within a current range where deterministic chaos dominates the LFF dynamics. Measurements of the different current dependence for the fast pulsations and the slow averaged LFF events will be published elsewhere along with further properties of the laser, relevant to quantitative characterization of deterministic excitation phenomena in the dynamical system.

To conclude, we have demonstrated that coherence resonance occurs in deterministic systems without external noise. Experimentally, the effect appears in the deterministic chaotic low frequency fluctuations of a diode laser, optically fed back by a distant external mirror. Numerically, the effect is predicted calculating solutions for the single mode model of Lang-Kobayashi. The normalized variance of time intervals between LFF power drops was shown to decrease, passing through a minimum value, as the pump current is increased. This is an indication that the fast time scale dynamics of the laser acts as a noise source for the slow evolution of the average laser power. A similar effect must occur in other natural systems where the deterministic chaotic dynamics consists of fast variables driving nonlinear terms of coupled slow variables.

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