Ultraslow Optical Solitons in a Cold Four-State Medium

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We show the formation of ultraslow optical solitons in a lifetime broadened four-state atomic medium under Raman excitation. With appropriate conditions we demonstrate, both analytically and numerically, that both bright and dark ultraslow optical solitons can occur in such a highly resonant medium with remarkable propagation characteristics. This work may open other research opportunities in condensed matter and may result in a substantial impact on technology.

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One of the fascinating manifestations of nonlinearities of nonlinear media under excitations is the existence of a class of wave propagation phenomena called soliton [1–3]. These special types of wave packets occur as the result of interplay between the dispersive and nonlinear effects in nonlinear media and can propagate undistorted over long distance. Solitons have been discovered in many branches of physics and states of matters ranging from solid medium such as optical fiber (optical wave soliton [4]) to Bose-Einstein condensed atomic vapor (matter wave soliton [5–8]).

In the optical domain, most optical solitons are produced with intense electromagnetic fields, and far-off resonance excitation schemes are generally employed in order to avoid unmanageable optical field attenuation and distortion [4]. As a consequence, optical solitons produced in this way generally travel with a speed very close to c, the speed of light in vacuum.

Recently, there has been a significant surge of research activities on wave propagation in highly resonant media [9]. One of the striking features of wave propagation in such a highly resonant medium is the significant reduction of the propagation velocity [10,11] of an optical field involved. Such an ultraslow propagation of an optical field has been shown [12–16] to lead to several new propagation effects in the field of fundamental physics, and one could envision the potential technological impact of the technique in modern optical and telecommunication engineering. In particular, well-characterized and distortion-free ultraslow optical waves may lead to important applications such as high fidelity optical buffers [17], phase shifters, transmission lines [18], switches, routers, and wavelength converters [19]. It is for this reason that ultraslow propagation of optical waves has been vigorously pursued in both fields of fundamental research and technological development.

In this Letter, we describe a class of optical solitons, *ultraslow optical solitons,* in a highly resonant nonlinear optical medium. Our study is motivated by recent reports using a conventional electromagnetically induced transparency (EIT) [9] technique to enhance Kerr nonlinearities [12–16]. In particular, this technique has been proposed for achieving a large nonlinear phase shift with very weak control optical fields [12,13]. Furthermore, the technique has been shown to be beneficial to certain nonlinear optical processes under weak driving conditions[13–16] where the ultraslow propagation[10,11] is a dominant feature. These enhancement effects under ultraslow propagation conditions naturally lead to the question of whether the technique can also facilitate the formation of optical solitons in a highly resonant nonlinear medium. Here, we present a systematic study to address this question.

We consider a lifetime broadened four-state atomic system that interacts with a weak, pulsed probe field ($|1\rangle \rightarrow |3\rangle$ transition) and two strong, continuous-wave (cw) control fields ($|2\rangle \rightarrow |3\rangle$ and $|2\rangle \rightarrow |4\rangle$ transitions), respectively (Fig. 1) [20]. A similar system has been used in Refs. [12,13,21] in the context of cross phase modulation (XPM). Our system and the study based on it, however, are drastically different from those works. First and foremost is that we are interested in demonstrating the formation of ultraslowly propagating optical solitons in a highly resonant medium. Second, to achieve this goal, a time-dependent treatment of the nonlinear propagation beyond the usual zeroth order adiabatic approximation and steady-state treatment must be a central feature of the theory. As such, the propagation of the pulsed probe wave must be correctly and systematically treated to include both linear and group velocity dispersions. The latter is one of the key aspects in the formation of a soliton type of shape preserving propagation. Such a time-dependent approach, however, reveals that under weak driving conditions significant probe pulse spread and attenuation occur [22] even for the input probe pulse length on the order of $\tau \simeq 10^{-5}$ s. These are detrimental effects for any wave propagation scheme including the XPM process proposed. Third, a XPM process requires an approximation scheme that is accurate only to the first order in the field being modulated (the probe field in this

case). To be able to predict the existence of ultraslow solitons, one must systematically include the nonlinear terms up to the third order in the probe field. We demonstrate that the significant probe field spread and attenuation due to group velocity dispersion can be balanced by a (self-modulation) Kerr nonlinear effect, leading to the formation of ultraslow optical solitons of the probe field. Finally, our system is a Raman scheme [22,23] with a large one-probe-photon detuning. This is an important departure from the conventional EIT scheme used in Refs. [12,13,21] where zero one-probe-photon detuning is required. As we show, the introduction of a large one-probe-photon detuning is critical to parameter selections in demonstrating the formation of ultraslow optical solitons. To the best of our knowledge, these new features have never been reported.

We begin by writing, in the interaction picture, the atomic equations of motion and wave equation for the time-dependent probe field Ω_p as

$$\frac{\partial A_1}{\partial t} = i\Omega_p^* A_3, \tag{1a}$$

$$\frac{\partial A_2}{\partial t} = -\gamma_2 A_2 + i\Omega_C^* A_3 + i\Omega_B^* A_4, \tag{1b}$$

$$\frac{\partial A_3}{\partial t} = -(i\Delta\omega_p + \gamma_3)A_3 + i\Omega_C A_2 + i\Omega_p A_1, \tag{1c}$$

$$\frac{\partial A_4}{\partial t} = -(i\Delta\omega_B + \gamma_4)A_4 + i\Omega_B A_2, \tag{1d}$$

$$\frac{\partial \Omega_p}{\partial z} + \frac{1}{c}\frac{\partial \Omega_p}{\partial t} = i\kappa_{13} A_3 A_1^*. \tag{1e}$$

Here, $2\Omega_j (j = p, B, C)$ are the Rabi frequencies for the relevant transitions, $\Gamma_i = 2\gamma_i$ $(i = 2, 3, 4)$ is the decay rate of level $|i\rangle$, and $\kappa_{13} = 2N\omega_p |D_{31}|^2/(\hbar c)$ with N and D_{31} being the concentration and dipole moment for the transition $|1\rangle \rightarrow |3\rangle$, respectively. In deriving Eqs. (1a)–(1e), we have taken slowly varying envelope approximation for the pulsed probe field, defined $\Delta\omega_B = \omega_{42} - \omega_B$, and one-probe-photon detuning $\Delta\omega_p = \omega_{31} - \omega_p$, respectively. In addition, we have assumed $\Delta\omega_{21} = \omega_{21} + \omega_C - \omega_p = 0$; therefore, the two-photon resonance is always maintained.

In order to obtain a clear picture on the interplay between dispersion and nonlinear effects of the atomic system under excitation, we first examine the dispersion properties of the atomic system described. This requires a perturbation treatment of the system response to the first order of the weak probe field Ω_p while keeping all orders due to control fields. Later, we demonstrate effects due to higher order Ω_p required to balance the dispersion effect so that the formation of ultraslow optical solitons can occur.

Let us assume that $A_j = \sum_k A_j^{(k)}$, where $A_j^{(k)}$ is the kth order part of A_j in terms of Ω_p. Within adiabatic following framework it can be shown that $A_j^{(0)} = \delta_{j1}$ (δ_{jm} is the Kronecker δ symbol) and $A_1^{(1)} = 0$. Thus, to the first order of Ω_p, only Eqs. (1b)–(1e) need to be considered. Taking time Fourier transform of Eqs. (1b)–(1e) and keeping terms up to the first order of Ω_p, we obtain

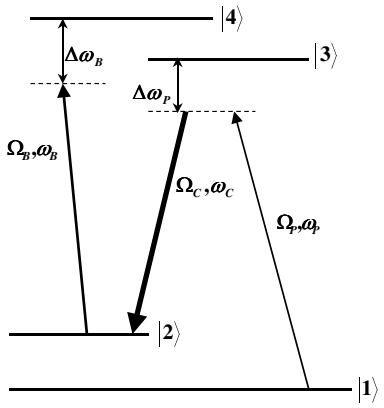

FIG. 1. Energy level diagram and excitation scheme of a lifetime broadened four-state atomic system that interacts with a weak, pulsed probe field (Rabi frequency $2\Omega_p$, optical frequency ω_p), and two cw pump lasers (Rabi frequencies $2\Omega_{B(C)}$ and optical frequencies $\omega_{B(C)}$).

$$(\omega + i\gamma_2)\beta_2^{(1)} + \Omega_C^* \beta_3^{(1)} + \Omega_B^* \beta_4^{(1)} = 0, \tag{2a}$$

$$\Omega_C \beta_2^{(1)} + (\omega - \Delta\omega_p + i\gamma_3)\beta_3^{(1)} = -\Lambda_p, \tag{2b}$$

$$\Omega_B \beta_2^{(1)} + (\omega - \Delta\omega_B + i\gamma_4)\beta_4^{(1)} = 0, \tag{2c}$$

$$\frac{\partial \Lambda_p}{\partial z} - i\frac{\omega}{c}\Lambda_p = -i\frac{\kappa_{13} D_p}{D}\Lambda_p, \tag{2d}$$

where $\beta_j^{(1)}$ $(j = 2, 3, 4)$ and Λ_p are the Fourier transforms of $A_j^{(1)}$ and Ω_p, respectively. ω is the Fourier transformation variable, $D_p = |\Omega_B|^2 - (\omega + i\gamma_2) \times (\omega - \Delta\omega_B + i\gamma_4)$, and $D = |\Omega_C|^2(\omega - \Delta\omega_B + i\gamma_4) + |\Omega_B|^2(\omega - \Delta\omega_p + i\gamma_3) - (\omega + i\gamma_2)(\omega - \Delta\omega_p + i\gamma_3) \times (\omega - \Delta\omega_B + i\gamma_4)$.

Equations (2a)–(2d) can be solved analytically, yielding

$$\Lambda_p(z, \omega) = \Lambda_p(0, \omega)\exp(iKz), \tag{3}$$

where

$$K = \frac{\omega}{c} - \frac{\kappa_{13} D_p}{D} = K_0 + K_1\omega + K_2\omega^2 + \mathcal{O}(\omega^3). \tag{4}$$

The physical interpretation of the dispersion coefficients in Eq. (4) is rather clear. $K_0 = \phi + i\alpha/2$ describes the phase shift ϕ per unit length and absorption coefficient α of the probe field, $K_1 = 1/V_g$ gives the propagation velocity, and K_2 represents the group velocity dispersion which contributes both to the probe pulse shape change and additional loss of probe field intensity [22]. We note that by taking $\Delta\omega_p = 0$, $\gamma_2 \simeq 0$, and $|\Delta\omega_B| \gg \gamma_4, \gamma_3$, $K_0 L = \Phi_{\mathrm{XPM}} + i\alpha L/2$ reduces exactly to

$$\Phi_{\mathrm{XPM}} \simeq \frac{\kappa_{13}|\Omega_B|^2}{|\Omega_C|^2 \Delta\omega_B} L, \qquad \alpha \simeq 2\frac{\kappa_{13}|\Omega_B|^2 \gamma_4}{|\Omega_C|^2 \Delta\omega_B^2},$$

as given in Ref. [12] under steady-state approximation.

Our objective is to search for the formation of a shape preserving propagation, i.e., an optical soliton of the probe field. For this purpose, we must systematically keep terms up to ω^2 in Eq. (4). For a Gaussian input probe pulse of duration τ, i.e., $\Omega_p(0,t) = \Omega_{p0}\exp(-t^2/\tau^2)$, we obtain from Eq. (3) after carrying out inverse transfer [22]

$$\Omega_p(z,t) = \frac{\Omega_{p0}}{\sqrt{b_1 - ib_2}}\exp\left[iK_0 z - \frac{(t-K_1 z)^2}{\tau^2(b_1 - ib_2)}\right], \quad (5)$$

where $b_1 \equiv b_1(z) = 1 + 4z\mathrm{Re}[K_2]/\tau^2$, $b_2 \equiv b_2(z) = 4z\mathrm{Im}[K_2]/\tau^2$. Equation (5) clearly shows that linear and group velocity dispersion effects contribute to the propagation velocity, pulse attenuation, and spread, as expected.

With the group velocity dispersion coefficient obtained, we now turn to the investigation of the nonlinear effect of the system and search for an effective remedy and realistic parameters to demonstrate the interplay between dispersion and nonlinear effect and the formation of ultraslow optical solitons.

A detailed analysis of the nonlinear polarization in Eq. (1e) reveals that the nonlinear Kerr effect due to self-modulation may offer such an effective remedy. We now show that, under appropriate conditions, self-modulation can precisely balance group velocity dispersion in the ultraslow propagation regime, leading to the formation of ultraslow optical solitons of the probe field.

Following Ref. [1], we take a trial function $\Omega_p(z,t) = \Omega_P(z,t)\exp(iK_0 z)$ and substitute it in Eqs. (1a)–(1e) to obtain the nonlinear wave equation of the slowly varying envelope $\Omega_P(z,t)$,

$$-i\left[\frac{\partial}{\partial z} + \frac{1}{V_g}\frac{\partial}{\partial t}\right]\Omega_P + K_2\frac{\partial^2}{\partial t^2}\Omega_P = \mathrm{NLT}. \quad (6)$$

where the nonlinear term $\mathrm{NLT} = -\kappa_{31}A_3^{(1)}e^{-iK_0 z}\sum_{j=2}^{4}|A_j^{(1)}|^2$ (note that we have used $A_k^{(0)} = \delta_{k1}$ in deriving this expression). It is straightforward to show that under the standard mapping [1–3], i.e., $K - K_0 \to -i\partial/\partial z$ and $\omega \to i\partial/\partial t$, Eq. (6) reduces exactly to Eq. (4) with $\mathrm{NLT} = 0$. Using ($j = 2, 3, 4$)

$$A_j^{(1)} = \frac{\Omega_B\Omega_C^*\delta_{j4} - D_{p0}\delta_{j3} - (\Delta\omega_B + i\gamma_4)\Omega_C\delta_{j2}}{D_0}\Omega_p, \quad (7)$$

where $D_0 \equiv D(\omega = 0) = -|\Omega_C|^2(\Delta\omega_B - i\gamma_4) - |\Omega_B|^2(\Delta\omega_p - i\gamma_3) - i\gamma_2(\Delta\omega_p - i\gamma_3)(\Delta\omega_B - i\gamma_4)$ and $D_{p0} \equiv D_p(\omega = 0) = |\Omega_B|^2 + \gamma_2\gamma_4 + i\gamma_2\Delta\omega_B$, we obtain from Eq. (6) (we define $\xi = z$ and $\eta = t - z/V_g$)

$$i\frac{\partial}{\partial\xi}\Omega_P - K_2\frac{\partial^2}{\partial\eta^2}\Omega_P = We^{-\alpha\xi}|\Omega_P|^2\Omega_P, \quad (8)$$

where $\alpha = 2\mathrm{Im}(K_0) = 2\kappa_{13}\mathrm{Im}(D_{p0}/D_0)$ and

$$W = -\frac{\kappa_{13}D_{p0}}{D_0}\frac{|D_{p0}|^2 + |\Omega_C|^2(|\Omega_B|^2 + \Delta\omega_B^2 + \gamma_4^2)}{|D_0|^2}. \quad (9)$$

A close inspection of Eq. (8) shows that if a reasonable and realistic set of parameters can be found so that $\exp(-\alpha L) \simeq 1$, $K_2 = K_{2r} + iK_{2i}$ with $K_{2r} \gg K_{2i}$, and $W = W_r + iW_i$ with $W_r \gg W_i$, then Eq. (8) can be cast into the standard nonlinear Schrödinger equation

$$i\frac{\partial}{\partial\xi}\Omega_P - K_{2r}\frac{\partial^2}{\partial\eta^2}\Omega_P = W_r|\Omega_P|^2\Omega_P, \quad (10)$$

which admits [1–3] solutions describing bright and dark solitons [including N-soliton ($N = 1, 2, 3, \ldots$] for bright solitons [3]) depending on the sign of the product $K_{2r}W_r$. The fundamental bright soliton (i.e., $N = 1$) is given by

$$\Omega_P = \Omega_{P0}\mathrm{sech}(\eta/\tau)\exp(-i\xi W_r|\Omega_{P0}|^2/2), \quad (11)$$

where $\mathrm{sech}(\eta/\tau)$ is the hyperbolic secant function. The amplitude Ω_{P0} and width τ are arbitrary constants, subjecting only to the constraint $|\Omega_{P0}\tau|^2 = 2K_{2r}/W_r$. We note that the assumption of $|\Omega_{P0}\tau|^2 \ll |\Omega_C\tau|^2$ has been used in deriving Eq. (9). Therefore, the width τ should be chosen to satisfy $2K_{2r}/W_r \ll |\Omega_C\tau|^2$.

We now present numerical examples to demonstrate the existence of ultraslow bright and dark solitons in the system studied. We consider a system where the decay rates are $\Gamma_2 = 2\gamma_2 \simeq 2. \times 10^3\ \mathrm{s}^{-1}$, $\Gamma_3 = 2\gamma_3 \simeq 1.2 \times 10^8\ \mathrm{s}^{-1}$, and $\Gamma_4 = 2\gamma_4 \simeq 2.5 \times 10^8\ \mathrm{s}^{-1}$.

We first consider the case of dark solitons. Taking [24] $\kappa_{13} = 1.0 \times 10^9\ \mathrm{cm}^{-1}\,\mathrm{s}^{-1}$, $2\Omega_B = 2.4 \times 10^8\ \mathrm{s}^{-1}$, $2\Omega_C = 6.0 \times 10^8\ \mathrm{s}^{-1}$, $\Delta\omega_p \simeq -1.0 \times 10^9\ \mathrm{s}^{-1}$, and $\Delta\omega_B \simeq -1.2 \times 10^9\ \mathrm{s}^{-1}$, we obtain $\Phi_{\mathrm{XPM}}/L \simeq -0.117\ \mathrm{rad\,cm}^{-1}$, $\alpha \simeq 0.023\ \mathrm{cm}^{-1}$, and $V_g/c \simeq 3.8 \times 10^{-3}$. With this set of parameters, the standard nonlinear Schrödinger equation (10) with $K_{2r}W_r < 0$ is well characterized, and hence we have demonstrated the existence of dark solitons that travel with ultraslow group velocities in the cold medium. In Fig. 2 we have plotted $|\Omega_P/\Omega_{P0}|^2e^{-\alpha\xi}$ as a function of η/τ and ξ/l for $\tau = 10^{-5}$ s and $l = 1$ cm with the parameters given above. The left panel shows the dark soliton solution obtained directly from Eq. (8), whereas the right panel is the result obtained from the standard nonlinear Schrödinger equation (10) [hence the fundamental soliton solution Eq. (11)]. It is remarkable that this set of parameters has generated the soliton wave form that agrees excellently with that of the fundamental dark soliton for a propagation distance of $\xi = z = 3$ cm.

For bright solitons, we take $\Delta\omega_B \simeq 1.2 \times 10^9\ \mathrm{s}^{-1}$ with all other parameters given above unchanged. In this case, we obtain $\Phi_{\mathrm{XPM}}/L \simeq 0.1513\ \mathrm{rad\,cm}^{-1}$, $\alpha \simeq 0.039\ \mathrm{cm}^{-1}$, and $V_g/c \simeq 2.2 \times 10^{-3}$. Using this set of parameters and following the above described procedures, it is can be shown numerically that the standard nonlinear Schrödinger equation (10) with $K_{2r}W_r > 0$ is well characterized and the formation of ultraslow bright soliton occurs.

It is worth noting that the above described parameter sets also lead to a negligible loss of the probe field for both

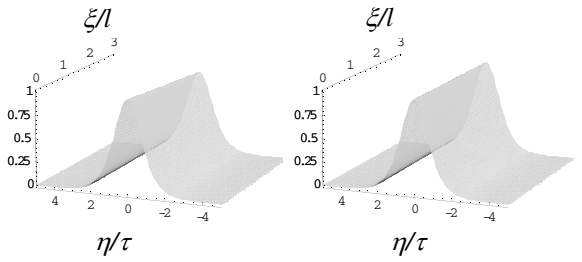

FIG. 2. The left panel is the plot of $|\Omega_P/\Omega_{P0}|^2 e^{-\alpha\xi}$, with Ω_P obtained numerically from Eq. (8), versus dimensionless time η/τ and distance ξ/l with $l = 1$ cm and $\tau = 1.0 \times 10^{-6}$ s for bright soliton formation. Other parameters are explained in the main text. The right panel shows the single bright soliton $|\Omega_P/\Omega_{P0}|^2 = \mathrm{sech}^2(\eta/\tau)$ given in Eq. (11) for the same parameters.

bright and dark solitons described. Indeed, Fig. 2 shows negligible attenuation and spreading of the probe field soliton after a propagation distance of 3 cm. This is a remarkable propagation effect in such a highly resonant system.

We further note that the fundamental soliton given in Eq. (11) has a width and amplitude satisfying $|\Omega_{P0}\tau| = \sqrt{2K_{2r}/W_r} \simeq 13$. We thus have established the existence of higher order [3] ultraslow ($V_g/c \sim 10^{-3}$) bright solitons.

We finally note that it is possible, by choosing different control field Rabi frequency and detunings, to further reduce the group velocities of the demonstrated solitons. For instance, taking $2\Omega_B = 2.4 \times 10^7\ \mathrm{s}^{-1}$, and $2\Omega_C = 6.0 \times 10^7\ \mathrm{s}^{-1}$ with other parameters unchanged, we obtain $V_g/c \simeq 2.27 \times 10^{-5}$ while both Φ_{XPM} and α remain nearly the same. A further reduction of group velocity, however, may decrease the allowable range of the width of bright solitons. In this example, we obtain $|\Omega_{P0}\tau| = \sqrt{2K_{2r}/W_r} \simeq 129.5$, which together with the constraint $|\Omega_{P0}\tau|^2 \ll |\Omega_C\tau|^2, |\Omega_B\tau|^2$, limits the range of the soliton's width to $\tau > 10^{-5}$ s.

In summary we have demonstrated that the significant probe field spreading and attenuation due to group velocity dispersion can be precisely balanced by the (self-modulation) Kerr nonlinear effect. This interplay between the dispersion and nonlinear effects results in the formation of optical solitons that traverse the cold atomic medium with ultraslow group velocities. With our parameters for bright solitons the nonlinear phase shift achievable is quite substantial, $\Phi_{\mathrm{XPM}}/L \simeq 0.1513\ \mathrm{rad\,cm}^{-1}$. This shows that optical soliton propagation technique can be an effective way to achieve large nonlinear phase shift, yet maintain shape invariant propagation of the optical field.

Ultraslow optical solitons may also occur in inhomogeneously broadened media, such as warm vapors, and solid media such as optical waveguide structures. The former requires a systematic inclusion and treatment of Doppler broadening effect, whereas the latter requires a good understanding and treatment of various relaxation mechanism in solid materials. The Raman scheme described here represents an important departure from the conventional EIT scheme and may lead to new phenomena that manifest themselves under well controlled balance between dispersion and nonlinear effects in these media in an ultraslow propagation regime.

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