Observation of Dynamical Instability for a Bose-Einstein Condensate in a Moving 1D Optical Lattice

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We have experimentally studied the unstable dynamics of a harmonically trapped Bose-Einstein condensate loaded into a 1D moving optical lattice. The lifetime of the condensate in such a potential exhibits a dramatic dependence on the quasimomentum state. This is unambiguously attributed to the onset of dynamical instability, after a comparison with the predictions of the Gross-Pitaevskii theory. Deeply in the unstable region we observe the rapid appearance of complex structures in the atomic density profile, as a consequence of the condensate phase uniformity breakdown.

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Ultracold atoms in optical lattices have proven to be a rich field of investigation for both fundamental and applicative issues of quantum mechanics. Periodic potentials are very well known in solid state physics, where the basic description of the system is the Bloch theory for a gas of noninteracting particles [1]. Cold atoms in optical lattices have opened the possibility to investigate effects not previously observable on ordinary matter crystals, such as Bloch oscillations, Wannier-Stark ladders, and Landau-Zener tunneling [2]. Bose-Einstein condensates (BECs) are particularly well suited for the investigation of these phenomena, due to the small momentum spread and the large coherence length. However, in a high density sample, such as a trapped BEC, nonlinearities induced by interactions among the atoms can complicate the picture and lead to a number of new effects, such as the observation of new quantum phases [3], the generation of solitonic propagation [4], and the observation of different kinds of instabilities. In recent years many papers have investigated the latter topic, both theoretically [5–10] and experimentally [11–13]. Many of these works have suggested that the BEC superflow can be broken not only by energetic (Landau) instability, but also by dynamical instability, occurring for condensates with repulsive interactions in periodic potentials.

In this Letter we report on the observation of dynamical instability for a trapped BEC in a moving 1D optical lattice. We have measured the characteristic rates for this instability to occur, mapping the quasimomentum space for both the lowest and excited energy bands. This quantitative analysis gives important information on the time scales limiting the investigation of coherence effects in a BEC moving in a periodic potential.

Dynamical instability is a peculiar feature of nonlinear systems. It occurs when the eigenspectrum of the excitations of the system exhibits complex frequencies. In this case, arbitrary small perturbations of the wave function may grow exponentially, eventually leading to the destruction of the initial state. The conditions for this kind of instability are satisfied by a BEC with repulsive interactions in a periodic potential. When the height of the periodic potential is not too large, the superfluid ground state of a BEC in a 1D optical lattice is well described by the Gross-Pitaevskii equation (GPE)

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{trap}}(\vec{r}) + sE_R\cos^2(kz) + \frac{4\pi\hbar^2 a}{m}|\Psi|^2\right]\Psi, \tag{1}$$

where Ψ is the complex BEC order parameter, $V_{\mathrm{trap}}(\vec{r})$ is the harmonic trapping potential, s is the height of the periodic potential in units of the recoil energy $E_R = \hbar^2k^2/2m$, $k = 2\pi/\lambda$ is the wave number of the laser beam creating the optical potential, and a is the scattering length. Among the stationary solutions of Eq. (1) there are the usual Bloch waves, i.e., plane waves with quasimomentum q and band index n, modulated in space by functions having the same periodicity of the lattice. A linear stability analysis of the GPE shows that, in certain regions of the quasimomentum space, these solutions are not dynamically stable and an exponential growth of perturbations may start [5,7,10]. The onset of dynamical instability has been considered as a possible explanation for the disruption of the atomic superflow in a past experiment [12], concerning dipole oscillations in the magnetic trap + static optical lattice. In this work we quantitatively investigate the unstable dynamics of a trapped BEC in a moving optical lattice in the regime of low lattice heights. The novelty of this work, with respect to the previous experiments [11,12], is that the implementation of a lattice moving at constant velocity allows us to accurately set the quasimomentum of the condensate and to make a precise investigation of the stability regimes in the full Brillouin zone and for different energy bands.

The experiment is performed with an elongated ^{87}Rb BEC produced in an Ioffe-Pritchard magnetic trap by means of forced rf evaporation. The trapping frequencies are $\omega_z = 2\pi \times 8.8$ Hz axially and $\omega_\perp = 2\pi \times 91$ Hz radially, with the axis of the trap oriented horizontally. Our typical condensates are made of $\simeq 3 \times 10^5$ atoms in the hyperfine ground state $|F = 1; m_F = -1\rangle$, with a peak density $n \simeq 1.2 \times 10^{14}$ cm^{-3}. The optical lattice is created by the interference of two counterpropagating laser beams derived from the same Ti:Sa laser operating at $\lambda =$ 820 nm, far detuned with respect to the Rb D1 line at $\lambda =$ 795 nm. The lattice beams are aligned along the symmetry axis of the condensate and are only slightly focused (400 μm diameter), so that the optical radial confinement may be completely neglected. The frequencies of the two beams are controlled by two acousto-optic modulators driven by two phase-locked radiofrequency generators in order to provide a stable detuning $\delta\nu$ between the two beams. The resulting interference pattern, averaged over the optical frequencies, is a standing wave moving at velocity $v = (\lambda/2)\delta\nu$. We calibrate the height of the optical lattice using the relation $s = 2\hbar\Omega_R/E_R$, where Ω_R is the measured Rabi frequency of the Bragg transition between the momentum states 0 and $2\hbar k$ induced by pulsing an optical lattice with $v = \hbar k/m$ [14]. Once the condensate has been produced, we adiabatically switch on the moving lattice loading the condensate in a state of well defined quasimomentum $q = mv/\hbar$ and band index n [15]. We let the BEC evolve in this potential for a variable time; then we switch off both the magnetic trap and the optical lattice and, after an expansion of 28 ms, we image the atomic cloud along the radial horizontal direction [16].

The number of atoms remaining in the BEC decreases exponentially as a function of the time spent in the periodic potential. We measure the lifetime of the condensate for different values of the quasimomentum q and for different energy bands. Since even a tiny thermal component may seriously limit the BEC lifetime, we use an rf shield in order to remove the hottest atoms produced by heating of the atomic sample. In this way we measure lifetimes of the order of ≈ 10 s in the lattice at $v = 0$, with no discernible thermal fraction even on long time scales.

The experimental results are analyzed according to the GPE shown in Eq. (1). As a matter of fact, one can avoid the complication of the full 3D theory by using the nonpolynomial Schrödinger equation (NPSE), a simplified 1D model that includes an effective radial-to-axial coupling [17]. The NPSE provides a more realistic description of the actual system with respect to the simple one dimensional GPE used previously [5], yielding an estimate of the instability thresholds and of the growth rates of the most unstable modes in nice agreement with that of Eq. (1) in the regime of lattice heights considered here (see [10] for details).

In Fig. 1 we show the measured loss rates (inverse of the lifetime) for the BEC in an optical lattice with $s = 0.2$ as a function of q. With increasing q, from the bottom of the first band to the zone boundary, the lifetime changes dramatically, spanning 3 orders of magnitude from $\approx$ 10 s to ≈ 10 ms. In particular, we observe a discontinuity around $q = 0.55q_B$ (where $q_B = 2\pi/\lambda$ is the boundary of the first Brillouin zone), in good correspondence with the threshold for the onset of dynamical instability calculated with our measured density and lattice height (vertical line). Deeply in the dynamically unstable regime (images of Fig. 1) we observe the appearance of some complex structures in the expanded BEC density profile, suggesting fragmentation of the Bloch wave. These interferencelike structures are particularly visible for higher

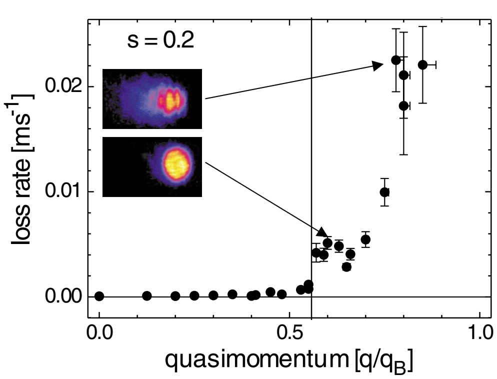

FIG. 1 (color online). Loss rates for a trapped BEC loaded in a moving optical lattice with $s = 0.2$. The vertical line corresponds to the calculated threshold for the onset of dynamical instability [10]. The images show the density distribution of the expanded cloud (the lattice is directed from right lo left). Near the zone boundary, where instability is faster, we observe the appearance of some complex structures, evidencing the loss of coherence in the BEC.

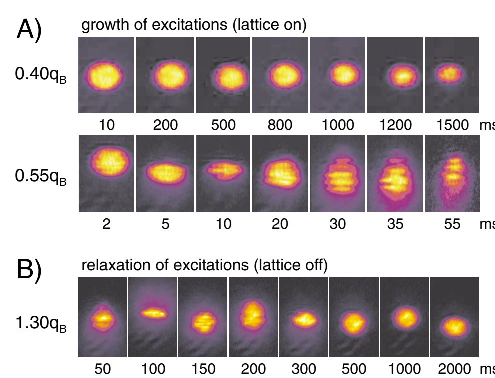

FIG. 2 (color online). (a) Evolution of the expanded atomic density profile for a lattice height $s = 1.15$ and two different quasimomentum values $q = 0.4q_B$ and $q = 0.55q_B$ below and above the threshold for dynamical instability. Note the different time scales. (b) Evolution of the expanded atomic density profile for a variable time spent in the pure magnetic trap, following 5 ms spent in an optical lattice with $s = 1.15$ and $q = 1.30q_B$, where after a few ms we observe a strong density modulation. In all these pictures the lattice is directed from top to bottom.

lattice heights and near the zone boundaries, where the evolution of instability is faster. In Fig. 2(a) we present two sequences of images showing the time evolution of the atomic density profile for a lattice height $s = 1.15$ and two different values of quasimomentum below and above the threshold for dynamical instability [18]. This behavior cannot be attributed to a simple heating of the sample. On the contrary, it may reflect the creation of phase domains induced by the growth of instabilities, which break the phase uniformity of the BEC [6]. When these structures are not just weak perturbations of the density profile, it is difficult to measure the number of atoms in the condensate directly. In order to measure it more precisely, we let the cloud evolve in the pure magnetic trap, allowing relaxation of the excitations. We have measured typical relaxation times of the order of ≈ 500 ms, after which the BEC recovers its smooth density profile, as shown in Fig. 2(b). This observation suggests the existence of mechanisms that, once the cause of instability is removed, allow the system to come back to the ground state, damping the excitations and restoring the initial coherence [19].

Our experimental technique allows us to study the dynamics in the higher energy bands and compare the peculiar behavior in each band with the results of the theoretical model. The remarkable agreement between the theory and the experiment enforces that the observed phenomena are indeed due to dynamical instability. In Fig. 3(a) we show the experimental loss rates measured in the lowest three energy bands as a function of the quasimomentum q for $s = 0.2$. These rates are compared in Fig. 3(b) with the theoretical growth rates of the most dynamically unstable modes, obtained from the linear stability analysis of the NPSE [10]. Increasing the height of the optical lattice from $s = 0.2$ to $s = 1.15$, as shown in Fig. 4, the picture in the higher bands starts to develop asymmetric features around the zone edges. This is particularly evident crossing the boundary between the second and the third Brillouin zone, where we observe a marked change in the instability rates. This nontrivial behavior cannot be attributed to single particle band structure considerations (for example, residual nonadiabaticities or interband transitions). Indeed, comparing this feature with the theoretical calculations for the growth rates of the most unstable modes, we do observe the same distinctive shape. The experimental loss rates should not be quantitatively compared with the theoretical growth rates, since these two quantities have different physical meanings: the first measures how fast the atoms are removed from the condensate; the second is the rate at which the most unstable mode grows in the linear regime, i.e., at the onset of dynamical instability. Out of the linear regime, when excitations have grown and the unstable modes are not just weak perturbations of the carrier Bloch wave, the dynamical evolution of the system cannot be explained with this perturbative approach and the full solution of the time-dependent 3D GPE is required.

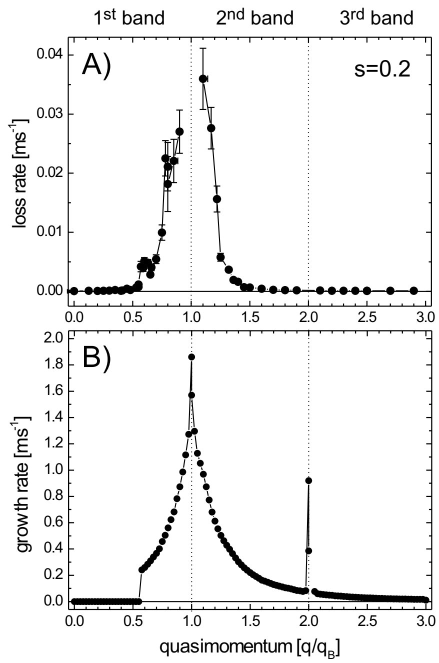

FIG. 3. (a) Experimental loss rates for a BEC loaded in a moving optical lattice with $s = 0.2$. (b) Theoretical growth rates of the most dynamically unstable modes obtained from a linear stability analysis of the NPSE [10].

However, the remarkable similarity between the experimental and theoretical curves indicates that the onset of the instability produces a significant imprinting on the subsequent dynamics of the system.

In the experiment, the BEC is trapped in the harmonic magnetic potential, in order to have a high density sample and long observation times. On the other hand, the presence of the harmonic potential may induce a variation of the quasimomentum in time, thus changing the conditions for instability. In a semiclassical approach, a force F acting on the system results in a variation of the quasimomentum q according to the law $F = dq/dt$. Integrating this equation of motion with the velocity spectrum $v(q)$ of the Bloch bands, one can demonstrate that the quasimomentum q makes small oscillations at the axial trap frequency (rescaled with the effective mass) [20]. We have taken this effect into account by including horizontal error bars in Figs. 3(a) and 4(a), representing the maximum variation of q during the experiment. In real space these oscillations correspond to a micromotion of the BEC around the center of the trap with an amplitude much less than 1 μm. Actually, this effect becomes important only when the Bloch bands significantly differ from the free particle energy, i.e., near the zone boundaries and for higher lattice heights.

We repeated the experiment in the presence of a small thermal component. We observed that, even for small values of quasimomentum ($q \gtrsim 0.05 q_B$), the atomic sample is completely destroyed in a time much shorter than the lifetime measured for the pure BEC. We attribute this

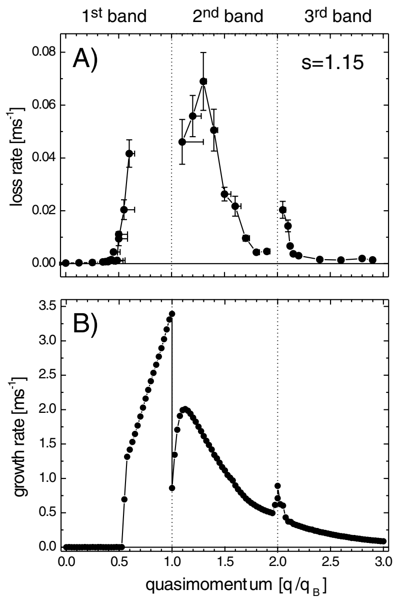

FIG. 4. (a) Experimental loss rates for a BEC loaded in a moving optical lattice with $s = 1.15$. (b) Theoretical growth rates of the most dynamically unstable modes obtained from a linear stability analysis of the NPSE [10].

behavior to the onset of energetic instability, which occurs in the presence of dissipative processes [5,10]. The residual thermal fraction surrounding the BEC can provide a mechanism for such a dissipation, thus triggering the activation of energetic instability. The time scale we measured for this mechanism to produce loss of atoms is of the order of 500 ms (for $s = 0.2$ and a thermal fraction of $\sim$30%), much longer than the lifetimes measured in the dynamically unstable regime. A detailed study of this regime will be the subject of a future publication, but the measured time scales in the two different regimes already clearly indicate that the interpretation of the early experiment [11] in terms of energetic instability was not correct. The main argument (based on the observed density distribution profile) suggesting that energetic instability was playing a role in that experiment is now also overcome by the solution of the 3D GPE [10].

In conclusion, we report the observation of dynamical instability for a harmonically trapped BEC in the presence of a 1D optical lattice. We have carried out a detailed investigation of the instability regimes for different energy bands by precisely controlling the lattice velocity. We have quantitatively studied the lifetime of the condensate in such a potential, finding good agreement between the experimental loss rates and the theoretical rates for the growth of excitations at the onset of dynamical instability. Deeply in the dynamically unstable regime we have observed the appearance of complex structures in the expanded density profile, a signature of a loss of coherence in the atomic sample. These observations clearly identify dynamical instability as the main mechanism responsible for the disruption of the matter wave superflow in the regime of small lattice heights. The quantitative analysis presented in this Letter gives important information on the characteristic time scales which limit the investigation of the coherence properties of a BEC moving in a periodic potential.

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[20] Actually, for initial q near the region in which the effective mass diverges, the system may enter a Bloch oscillationlike regime, in which q, instead of oscillating in time, grows indefinitely. The experimental points in the graphs do not refer to this regime (this is why in Fig. 4(a) we do not have points from $q = 0.6q_B$ to $q = q_B$).