Single-Electron Tunneling with Strong Mechanical Feedback

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(Received 29 April 2004; published 23 September 2004)

A harmonic nanomechanical oscillator with a high quality factor weakly coupled to a single-electron tunneling device can provide a strong feedback for electron transport. Strong feedback occurs in a narrow voltage range just above the Coulomb blockade threshold. In this regime, current is strongly modified and current noise is drastically enhanced compared to the Schottky value.

DOI: 10.1103/PhysRevLett.93.136802 PACS numbers: 73.23.Hk, 72.70.+m

Single-electron tunneling (SET) [1] has been observed in a variety of meso and nanodevices. Because of Coulomb blockade, the current at zero temperature vanishes below certain energy threshold. The threshold value can be tuned by both bias and gate voltages, and this has been widely applied in many experiments. The current above the threshold jumps if electrons tunnel to/from discrete energy levels and rises continuously if the discreteness of the energy spectrum is not resolved.

Recently, SET has been combined with nanomechanical oscillators, resulting in a new class of *nanoelectromechanical systems* (NEMS) [2]. Transport experiments with single oscillating molecules [3], suspended semiconductor beams [4] and carbon nanotubes [5] clearly demonstrate the influence of mechanical degrees of freedom on current in SET regime.

Theoretical models of NEMS elucidate the coupling of mechanical degrees of freedom to electron tunneling. This coupling comes from the dependence of either tunneling matrix elements [6–9] or electron energy [7,8,10,11] on the position x of the mechanical oscillator. We stress that in either case this coupling is generally weak. It can be quantified by a dimensionless constant $\lambda \ll 1$, which represents either the ratio of probabilities to tunnel with and without emission of an oscillator quantum, or the relative shift of the oscillator energy $\hbar\omega_0$ resulting from a single tunneling event. The fact that λ is small is well-known from solid state physics and guarantees the separation of electronic and mechanical degrees of freedom. This separation persists in equilibrium properties of NEMS [12,13]. As shown in Refs. [6,7], strong coupling leading to electromechanical instability may only occur if mechanical stiffness of the oscillator is negligible.

It is intuitively clear that in a nonequilibrium system even weak coupling can become relevant. For both mechanisms of coupling, stochastic tunneling of electrons produces a stochastic driving force acting on the oscillator. If the quality factor Q of the oscillator is sufficiently high, this weak force can still swing the oscillator to amplitudes exceeding quantum values. Since the coupling is weak, this does not yet imply that this large amplitude would in its turn significantly affect tunneling. Can one have *strong mechanical feedback* in NEMS with weak coupling? This question has not been answered yet, with literature mainly concentrating on effects perturbative in weak coupling.

In this Letter, we show that the strong feedback regime occurs in a well-defined region right above the Coulomb blockade threshold provided that the quality factor is sufficiently high. The current in this region is modified by mechanical oscillations by a value of the order of the current itself. The most pronounced signature of this regime is the giant enhancement of the current noise as compared to the Schottky value–a fundamental noise scale in nanostructures [14].

Let us preface the quantitative discussion with qualitative arguments. The SET device can be in different charge states (see Fig. 1). In close vicinity of the Coulomb threshold, only two of these states (”0” and “1”) are relevant. We consider a mechanical oscillator with frequency ω_0, mass M, and quality factor $Q \gg 1$. The equilibrium position of the oscillator depends on the charge state of the SET device, providing the coupling between charges and the oscillator. We characterize the coupling by an extra force F exerted on the oscillator in charge state “1”. This coupling is weak, so that the dimensionless parameter $\lambda \equiv F^2/\hbar M\omega_0^3 \ll 1$. This guarantees that the equilibrium properties of the system are only slightly affected by the coupling. Let us now con-

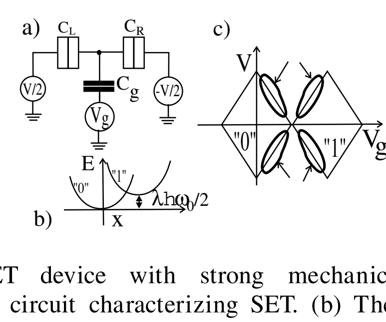

FIG. 1. SET device with strong mechanical feedback. (a) Electric circuit characterizing SET. (b) The equilibrium position of the oscillator depends on the charge state, providing weak ($\lambda \ll 1$) coupling. (c) Coulomb diamonds of the SET device. Strong feedback regime is expected close to the Coulomb blockade threshold. Our results concern the areas marked with arrows.

 0031-9007/04/93(13)/136802(4)$22.50

sider tunneling events that switch the charge states with the rate Γ. Since the force is different in different states, this results in a force in the form of a random telegraph signal acting on the oscillator. It is convenient to assume that (i) this force is classical and (ii) there are many tunneling events during the oscillation period, $\omega_0 \lesssim \Gamma$. The random kicks result in a net energy transfer to the oscillator, $dE/dt \simeq F^2/M\Gamma \simeq \lambda\hbar\omega_0^3/\Gamma$. The energy balance between dissipation and this transfer gives $dE/dt = E\omega_0/Q$, yielding a typical energy accumulated in the oscillator, $E \simeq QF^2/M\Gamma\omega_0 \simeq \hbar\omega_0 Q\lambda(\omega_0/\Gamma)$. At high Q, the amplitude of the resulting oscillations, $\zeta = \sqrt{2E/M\omega_0^2}$ can reach arbitrary high values. The applicability of this classical estimation requires $E \gg \hbar\omega_0$, so that $Q \gg \Gamma/(\omega_0\lambda)$.

The tunneling rate is a function of the energy difference W available for tunneling. This energy difference is affected by the position x of the oscillator, $W \to W - Fx$. This implies that the amplitude excited by the random force gives a strong feedback on the tunneling within the region defined by $W \lesssim W_c \simeq F\zeta \simeq (F^2/M)\sqrt{Q/\Gamma\omega_0^3} \simeq \hbar\omega_0\lambda\sqrt{Q\omega_0/\Gamma}$. Here, the classical reasoning works provided $W_c \gg \hbar\omega_0$, so that $Q \gg \Gamma/(\omega_0\lambda^2)$. This restriction on Q is stronger than the previous one. To prevent thermal smearing of the strong feedback region, the electron temperature should satisfy $k_B T \lesssim W_c$.

In this region, oscillations modify the current by a value of order of the average current itself, $I \simeq e\Gamma$. It is crucial to note that the amplitude of the oscillations fluctuates at the time scale set by damping Q/ω_0, the longest relevant time scale. The current noise in the strong feedback region is thus estimated to be $S \simeq I^2 Q/\omega_0 \simeq eI(Q\Gamma/\omega_0)$, which is much bigger than the Schottky value $S \simeq 2eI$ outside this region [15].

Let us move to the quantitative description. For an example model, we refer the reader to Ref. [12] where the energetics of a suspended carbon nanotube have been studied in detail by means of theory of elasticity and orthodox Coulomb blockade theory. We restrict ourselves to two charge states $n = 0, 1$, thus concentrating on the vicinity of the Coulomb blockade threshold (implying $W_c \ll E_C$), and one oscillator mode ω_0. In this form, the description becomes generic for *any* NEMS in the SET regime.

In the classical limit we consider, the state of the system is fully described by the joint distribution function $P_n(x, v, t)$, with x and v being the position and velocity of the oscillator (see, e.g., Refs [15,16]). This distribution function obeys the following master equation,

$$\frac{\partial P_n}{\partial t} + \left\{v\frac{\partial}{\partial x} + \frac{\partial}{\partial v}\frac{\mathcal{F}}{M}\right\}P_n - \mathrm{St}[P] = 0; \qquad (1)$$

$$\mathcal{F} = -M\omega_0^2 x - \frac{M\omega_0 v}{Q} + F_n; \qquad (2)$$

$$\mathrm{St}[P] = (2n-1)[\Gamma^{0\to1}(x)P_0 - \Gamma^{1\to0}(x)P_1]. \qquad (3)$$

The total force $\mathcal{F}$ acting on the oscillator is the sum of the elastic force, friction force, and charge-dependent coupling force, respective to the order of terms in Eq. (2). We count the position of the oscillator from its equilibrium position in the $n = 0$ state. In this case, $F_n = nF$.

The "collision integral" $\mathrm{St}[P]$ describes tunneling. The rates $\Gamma^{0\to1}(\Gamma^{1\to0})$ correspond to tunneling to (from) the Coulomb island. Each of these rates is composed of the rates of tunneling via left and right junctions, $\Gamma = \Gamma_L + \Gamma_R$. The position dependence of the rates is assumed to be due to a position dependence of energy differences available for each tunneling process [1,12]. This is valid if the typical energy difference W is sufficiently large, $W \gg \hbar\omega_0$. This position dependence is given by

$$W_{L,R}^{0\to1} = -W_{L,R}^{1\to0} = W_{L,R}^{ch,0\to1} - Fx,$$

where $W_{L,R}^{ch}$ are determined by electrostatic energy only and are given by standard "orthodox" expressions. In the limit of vanishing electron temperature, the rates are given by $\Gamma(W) = \Gamma\theta(-W)$ for tunneling via a single level and by $\Gamma(W) = (e^2R)^{-1}(-W)\theta(-W)$ for continuous spectrum of electron states in the island. Here, the $L, R, 0 \to 1, 1 \to 0$ indices are omitted for brevity, $\Gamma_{L,R}(R_{L,R})$ are tunnel rates (resistances) characterizing corresponding junctions.

Close to the Coulomb threshold electrons always tunnel in one direction (for concreteness, from the left to the right), and the slow dependence of F and $\Gamma_R[W(x)]$ on bias and gate voltages and position can be safely disregarded. The voltage and position dependence of $\Gamma_L^{0\to1}, \Gamma_L^{1\to0}$ remains essential and is given by

$$W_L^{0\to1} \approx \tilde{W} - Fx,$$

where $\tilde{W} \equiv W_L^{ch,0\to1}$ determines the energy difference to the Coulomb threshold, depends linearly on both gate and bias voltages with standard capacitance-dependent coefficients, and is positive (negative) above (below) the threshold.

We now simplify and solve Eq. (1). There are three distinct frequency scales: the inverse damping time ω_0/Q, the oscillation frequency ω_0, and the total tunneling rate $\Gamma_t = \Gamma^{0\to1} + \Gamma^{1\to0}$. We make use of the fact that $\omega_0/Q \ll \omega_0 \ll \Gamma_t$. The latter condition implies the adiabatic limit: x varies so slowly that $\Gamma(x)$ hardly changes between two tunneling events. This also means that many tunneling events occur during one period of the oscillations. Mathematically, it implies that for the "collision term" to be of the same order as the rest of the terms in Eq. (1) the distribution function should have the form

$$\begin{aligned} P_0(x, v, t) &= \frac{\Gamma^{1\to0}}{\Gamma_t}P(x, v, t) - \delta P(x, v, t), \\ P_1(x, v, t) &= \frac{\Gamma^{0\to1}}{\Gamma_t}P(x, v, t) + \delta P(x, v, t), \end{aligned} \qquad (4)$$

with $\delta P \ll P$. This allows for a regular expansion of δP in terms of ω_0/Γ_t,

$$\delta P = -\frac{F}{M}\frac{\Gamma^{0\to1}\Gamma^{1\to0}}{\Gamma_t^3}\frac{\partial P}{\partial v} + O(\omega^2/\Gamma_t^2).$$

Taking the sum of Eqs. (1), we arrive to the Fokker-Planck equation for the function P,

$$\left\{\frac{\partial}{\partial t} + v\frac{\partial}{\partial x} - \frac{\partial}{\partial v}\left(\omega_0^2 x + \frac{v\omega_0}{Q}\right)\right\}P \tag{5}$$

$$= -\frac{\tilde{F}(x)}{M}\frac{\partial P}{\partial v} + D(x)\frac{\partial P}{\partial v^2},$$

where we keep in the right-hand side the terms of zeroth and first order in ω_0/Γ_t. The first term presents the effect of the average force $\tilde{F} = F\Gamma^{0\to1}/\Gamma_t$, and it has been discussed in Ref. [6]. The effect of this term is a rather trivial one: Since $\tilde{F}$ is a function of coordinates only, it can be included into the oscillator potential energy, and just renormalizes the elastic force $M\omega_0^2 x$. In the model considered, this renormalization is small provided that $\zeta \gg F/M\omega_0^2$ and in any case does not lead to qualitatively new effects. We thus disregard this term in further consideration. It is the second term that describes the swinging of the oscillator by the stochastic time-dependent force. It may be seen as Brownian motion in velocity space characterized by the "diffusion coefficient"

$$D(x) = \frac{F^2}{M^2}\frac{\Gamma^{0\to1}\Gamma^{1\to0}}{\Gamma_t^3}.$$

In the underdamped limit $Q \gg 1$ the energy of the oscillator E is a slow variable varying at the time scale Q/ω_0 much longer than the oscillation period. We parameterize x and v by E and the oscillation phase θ,

$$x = \frac{1}{\omega_0}\sqrt{\frac{2E}{M}}\sin\theta; \qquad v = \sqrt{\frac{2E}{M}}\cos\theta,$$

and notice that the scale separation implies that $P(E,\theta) \approx P(E)$. Averaging Eq. (5) over θ, we obtain the following equation for $P(E)$,

$$\frac{\partial P}{\partial t} = \hat{\mathcal{L}}P; \hat{\mathcal{L}} \equiv \frac{\partial}{\partial E}E\left[\frac{\omega_0}{Q} + 2MD_1(E)\frac{\partial}{\partial E}\right], \tag{6}$$

with

$$D_1(E) = \frac{1}{2\pi}\int_0^{2\pi} d\theta \cos^2\theta D(x).$$

The stationary solution of Eq. (6) assumes the form

$$P(E) = A\exp\left(-\frac{\omega_0}{2QM}\int_0^E \frac{dE'}{D_1(E')}\right), \tag{7}$$

A being a normalization constant. The average electric current is evaluated with this distribution function,

$$I = e\int dx dv[\Gamma_L^{0\to1}P_0 - \Gamma_L^{1\to0}P_1] \tag{8}$$

$$= \int_0^\infty dE I(E)P(E); \qquad I(E) \equiv e\int\frac{d\theta}{2\pi}\frac{\Gamma_L^{0\to1}\Gamma_R^{1\to0}}{\Gamma_t}.$$

We now consider current noise. Mechanical noise due to electron transfer [17] and modification of electron noise by mechanical motion [9,11,16] are small by virtue of weak coupling if the correlation time of electron transfers is just a typical time between the transfers, like in shot noise. In the regime we consider, the noise is enhanced by the coupling to a slow degree of freedom—the oscillator. This provides current correlations at a much longer time scale Q/ω_0. Indeed, the current essentially depends on the energy E accumulated in the oscillator, the latter fluctuating at this time scale.

To quantify this contribution to the current noise, we notice that the time-dependent Fokker-Planck Eq. (6) is suitable for evaluating temporal correlations of any functions of E. Using the definition of low-frequency current noise, we obtain

$$S/4 = \int_0^\infty dt \langle\langle I(t)I(0)\rangle\rangle$$
$$= -\int_0^\infty dE[I(E) - I]\hat{\mathcal{L}}^{-1}[I(E) - I]P(E). \tag{9}$$

The scale of the current noise is thus $(\delta I)^2 Q/\omega_0$, δI being the E-dependent part of the current. In the strong feedback regime, $\delta I \simeq I$, and the contribution exceeds by far the typical shot noise values.

It is clear from the above qualitative discussion, that the current and noise must depend on one parameter only–the rescaled energy distance from the Coulomb blockade threshold $\tilde{W}/W_c$. This is of course confirmed by the quantitative treatment. We present the results for two cases–one discrete level and continuous spectrum–in Fig. 2 and 3, respectively. In both cases, there is no current below the threshold, $\tilde{W} < 0$. This is because the SET device remains in the state "0" with no current, so that there is no stochastic force to swing the oscillator, D_1 vanishes at low energies, and $P(E)$ is concentrated at zero

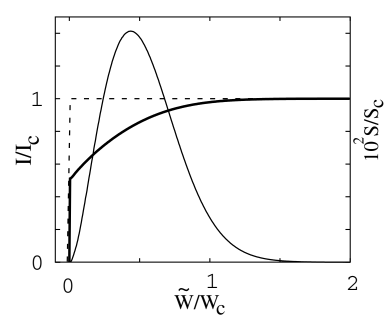

FIG. 2. Current (bold line) and noise (thin line) in the strong feedback regime for the tunneling via a single level display scaling: Rescaled current I/I_c and noise S/S_c ($S_c = 4I_c^2 Q/\omega_0$) are universal functions of the rescaled voltage. Dashed line gives the current in the absence of feedback.

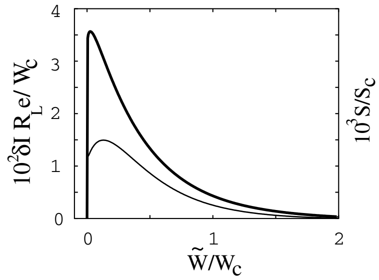

FIG. 3. Rescaled current and noise in the strong feedback regime for the tunneling via continuous spectrum. Bold curve is the difference δI between the actual current and $\tilde{W}/eR_L$, the current in the absence of feedback. Thin curve shows noise ($S_c = 4(W_c/eR_L)^2 Q/\omega_0$).

energy. Above the threshold, current and noise are not zero and strongly modified by the motion of the oscillator. Whereas the current at $\tilde{W} \gg W_c$ approaches the asymptotics in the absence of mechanical mode, the noise just vanishes at this scale approaching the much smaller shot noise values.

For the case of a single level, the voltage scale W_c reads

$$W_c^{\text{single}} = \sqrt{\frac{Q\Gamma_L\Gamma_R}{\omega_0^3\Gamma_t^3}\frac{F^2}{M}} = \hbar\omega_0\lambda\sqrt{\frac{Q\Gamma_L\Gamma_R\omega_0}{\Gamma_t^3}}. \tag{10}$$

The current jumps at the threshold to the value $I_c/2 \equiv e\Gamma_L\Gamma_R/\Gamma_t$, which is *half* of the jump it would do without coupling to the mechanical mode, and then approaches smoothly the value I_c (Fig. 2). Thus, in this case the motion of the oscillator *suppresses* the current–since during a part of the oscillation period the SET device is not operational–and broadens the upper half of the step. The noise is zero at the threshold, peaks around $0.5W_c$, and vanishes when the current is saturated.

For continuous spectrum, the smallest rate Γ_L linearly increases with energy, and $\Gamma_L \ll \Gamma_R \approx \Gamma_t$. To determine the scale W_c, we substitute $\Gamma_L = (e^2R_L)^{-1}W_c$ in Eq. (10) to obtain

$$W_c^{\text{contin}} = \frac{4QF^4}{M^2\Gamma_t^2\omega_0^3e^2R_L} = 4\hbar\omega_0\lambda^2\left(\frac{\omega_0}{\Gamma_t}\right)^2\left(\frac{\hbar}{e^2R_L}\right).$$

The current jump at the threshold equals $W_c/(eR_L3\pi^2)$, and for $\tilde{W} \gg W_c$ the current approaches from above the asymptotic value $\tilde{W}/eR_L$: The one without coupling to the oscillator. Therefore, in this situation mechanical motion *enhances* the current. The noise jumps at the threshold, develops a broad peak of approximately ≈ 1.2 times the value of the jump, and slowly vanishes.

For numerical estimates, we take $\lambda = 0.1$, which is typical for NEMS. In suspended carbon nanotubes [12], taking $\omega_0 = 10^9$ Hz and $\Gamma = 10^{10}$ Hz, we find that the values $W_c = 10\mu$ eV and $W_c = 1$ meV correspond to $Q = 10^5$ and $Q = 10^9$, respectively. In single molecular transistors, $\omega_0 = 10^{11}$ Hz. Taking $\Gamma = 10^{12}$ Hz to stay in the regime $\omega_0 \ll \Gamma$, we find that Q needed to achieve these values of W_c are, respectively, 10 and 10^5. In experiments, at low-frequency also background charge noise is present which is set-up dependent and may in some situations dominate. However, this background noise does not depend on voltage on the scale of W_c, which facilitates the observation of the current noise we discuss.

To conclude, we identify the regime of strong mechanical feedback in SET-based NEMS. This regime in the limit of weak coupling occurs in the close vicinity of the Coulomb blockade threshold and is characterized by a strongly modified current and parametrically big current noise. Both current and noise display universal scaling dependence in this regime.

We acknowledge useful discussions with H.S.J. van der Zant. This work was supported by the Netherlands Foundation for Fundamental Research on Matter (FOM) and EC FP6 funding (Contract No. FP6-2004-IST-003673). This publication reflects the views of the authors and not necessarily those of the EC. The community is not liable for any use that may be made of the information contained herein.

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