X-ray Generation in Strongly Nonlinear Plasma Waves

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We show that a laser wake field in the "bubble" regime [A. Pukhov and J. Meyer-ter-Vehn Appl. Phys. B **74**, 355 (2002)], works as a compact high-brightness source of x-rays. The self-trapped relativistic electrons make betatron oscillations in the transverse fields of the bubble and emit a bright broadband x-ray radiation with a maximum about 50 keV. The emission is confined to a small angle of about 0.1 rad. In addition, we make simulations of x-ray generation by an external 28.5 GeV electron bunch injected into the bubble. γ quanta with up to GeV energies are observed in the simulation in good agreement with analytical results. The energy conversion is efficient, leading to a significant stopping of the electron bunch over 5 mm interaction distance.

The development of novel high-brightness compact x-ray sources is important for many research, industrial, and medical applications. Synchrotron light sources are the most intense x-ray sources today. In a synchrotron light source, the radiation is generated as a result of relativistic electrons scattering by a bending magnet, magnetic undulators or wigglers [1], or by high-power laser pulses (Compton scattering) [2]. Recent experiments, which explore the interaction of an intense 28.5 GeV electron beam with plasma at the Stanford Linear Accelerator Center (SLAC) [3,4], have shown that an ion channel can be successfully used as a wiggler to produce the broadband x-ray radiation: the electron beam propagating in plasma blows out the background electrons and generates an ion channel.

A relativistic electron running along the ion channel undergoes betatron oscillations about the channel axis due to the restoring force acting on the electron by the uncompensated ion charge. For small amplitudes, $r_0 k_b \ll 1$, the betatron oscillation is close to a harmonic motion with the betatron frequency $\omega_b = c k_b = \omega_p / \sqrt{2\gamma}$. Here r_0 is the radial excursion of the electron, $\omega_p = \sqrt{4\pi e^2 n_e / m}$ is the background plasma frequency, n_e is the electron density that is equal to the ion charge density in the channel, γ is the relativistic Lorentz factor of the electron, e is the electron charge, m is the electron mass, and c is the speed of light. Relativistic electrons executing betatron oscillations in the ion channel emit short-wavelength electromagnetic radiation [5,6]. Some features of this radiation spectrum have been studied in recent publications [4,7]. The fundamental wavelength of this radiation is close to $\lambda \simeq \lambda_b / (2\gamma^2)$ for small-amplitude near-axis betatron oscillations, where $\lambda_b = 2\pi/k_b$. The emission at the fundamental frequency has been called ion channel laser in the work of Whittum

et al. [8]. If the amplitude of the betatron oscillations becomes large, then the electron radiates high harmonics. If the plasma wiggler strength, $K = \gamma k_b r_0 = 1.33 \times 10^{-10}\sqrt{\gamma n_e[\text{cm}^{-3}]}r_0[\mu\text{m}]$, is so high that $K \gg 1$, then the radiation spectrum becomes quasicontinuous broadband. It is similar to the synchrotron spectrum, which is determined by the universal function $S(\omega/\omega_c)$, where $S(x) = x \int_x^\infty K_{5/3}(\xi)d\xi$ and ω_c is the critical frequency [6]. For frequencies well below the critical frequency ($\omega \ll \omega_c$), the spectral intensity increases with frequency as $\omega^{2/3}$ reaches a maximum at $\sim 0.29\omega_c$, and then drops exponentially to zero above ω_c. The critical frequency for a relativistic electron in an ion channel is $\hbar\omega_c = (3/2)\gamma^3\hbar c r_0 k_b^2 \simeq 5 \times 10^{-21}\gamma^2 n_e[\text{cm}^{-3}]r_0[\mu\text{m}]\,\text{MeV}$. The synchrotron radiation emitted from an ion channel has been observed in a recent experiment [3].

The synchrotron radiation is confined to a narrow angle $\theta_R \simeq K/\gamma$ because of the strongly relativistic motion of the electron. The averaged total power radiated by an electron undergoing betatron oscillations is [7] $\langle P_{\text{total}} \rangle \simeq e^2 c \gamma^2 k_p^4 r_0^2 / 12$. It follows from this expression that the radiated power is proportional to the squared density of ions in the channel. This fact has been confirmed in the experiment [3]. The averaged number of photons with the mean energy $\hbar\omega_c$ emitted by the electron is $\langle N_X \rangle \simeq (2\pi/9)(e^2/\hbar c)N_0 K \simeq 5.6 \times 10^{-3} N_0 K$, where N_0 is the number of betatron oscillations executed by the electron.

In the SLAC experiment [3], the ion channel has been produced by the electron beam itself in the blowout regime [9], when the electron beam density n_b is higher than the plasma density. The density of a relativistic electron beam cannot be very high because of technology reasons. This leads to a serious limitation on the gain in the radiated power, which is quadratic in plasma density.

The use of a high-power laser could overcome this limitation. The strongly nonlinear broken-wave regime has been recently observed in 3D particle-in-cell (PIC) simulations [10]. In this regime, the background electrons are completely evacuated from the first period of the plasma wave excited behind the laser pulse, and an "electron bubble" is formed. The ion density in this bubble is many orders of magnitude higher than that in a simple beam-plasma interaction. For example, the ion density in the laser-produced channel can be as high as 10^{19} cm^{-3} [10]. This is 10^5 times higher than that in the recently reported beam-plasma experiment [3]. Therefore the radiated power in the laser-produced channel may be 10^{10} times higher. The bubble moves with the group velocity of the laser pulse, which is close to the speed of light. A relativistic electron bunch injected into the bubble can propagate inside the bubble over a very long distance. Hence, in spite of the small length of the bubble itself, the electrons can oscillate in the bubble for a long time.

In the present work we show that betatron oscillations of the bunch in the transverse fields of the bubble lead to efficient x-ray generation, which can be used for developing tabletop high-brightness x-ray radiation sources.

We perform a numerical simulation of x-ray generation in laser-plasma interactions for the strongly nonlinear broken-wave regime when the bubble is formed behind the laser pulse. We use the fully 3D PIC code Virtual Laser-Plasma Laboratory [11]. The incident laser pulse is circularly polarized, has the Gaussian envelope $a(t, r) = a_0 \exp(-r_\perp^2/r_L^2 - t^2/T_L^2)$, and the wavelength $\lambda = 0.82$ μm. Here $a = eA/mc^2$ is the relativistic laser amplitude, $r_L = 8.2$ μm, $T_L = 22$ fs, $a_0 = 10$. The laser pulse propagates in a plasma with the density $n_e = 10^{19}$ cm^{-3}.

Figure 1 presents snapshots of the laser pulse (the colored scale) and the electron density (the black/white scale) at different distances. The laser pulse has passed 14 Rayleigh lengths ($Z_R = \pi r_L^2/\lambda$) after the interaction time $T_{int} = 4500\lambda/c$. Thus, the lifetime of the bubble is about $3500\lambda/c \simeq 10$ ps. Electrons, trapped in the bubble, form the relativistic bunch. We observe as the bubble stretches and the bunch elongates with time.

Despite the fact that the bunch density is higher than the background ion density, the transverse force acting on the accelerated electrons, $F_\perp$, is mainly determined by the electrostatic focusing force from the ions; see Fig. 2(a). This is because the charge force of relativistic electrons and the self-generated magnetic force almost cancel each other [12]. The energy spectrum of the electron bunch is shown in Fig. 2(b). We observe formation of the quasimonoenergetic peak [10]. At $ct = 4000\lambda$ the peak is located at 360 MeV. We calculate the corresponding wiggler strength of $K \simeq 89 \gg 1$. Thus, the electrons emit x-rays in the synchrotron regime. The number of electrons in the bunch is about 6.5×10^{10} at this time. The total energy of electrons of the bunch is about 3.3 J, that is,
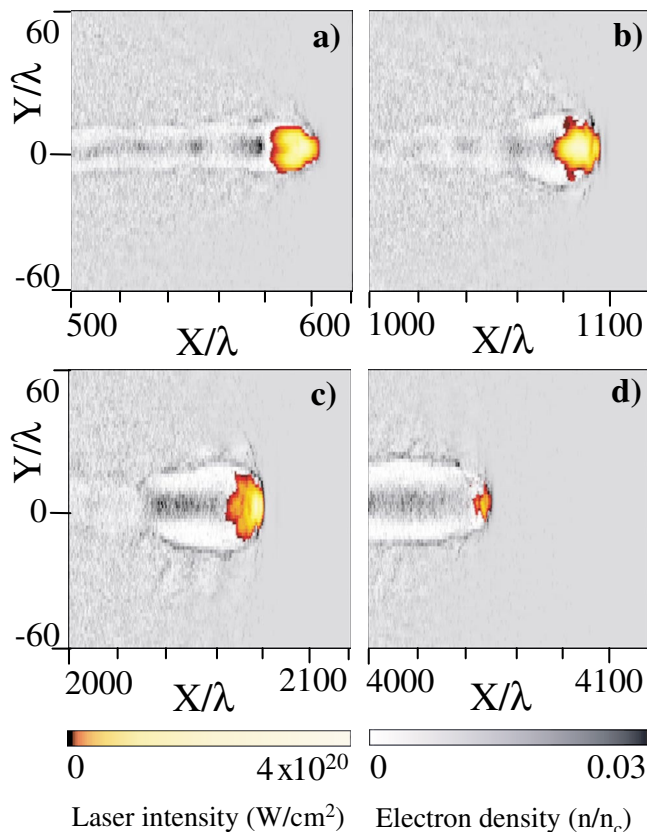

FIG. 1 (color). The evolution of the laser pulse intensity (the colored scale) and the bubble (the electron density is given in the black/white scale) in the strongly nonlinear broken-wave regime. The laser pulse propagates in a plasma layer from left to right. The plasma density and the laser intensity at (a) $ct/\lambda = 500$, (b) $ct/\lambda = 1000$, (c) $ct/\lambda = 2000$, and (d) $ct/\lambda = 4000$.
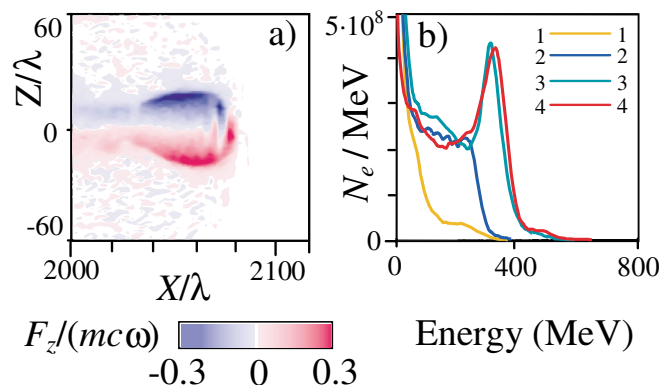

FIG. 2 (color). (a) The transversal force acting on the relativistic electrons moving in the x direction at $ct/\lambda = 2000$. (b) Temporal variation of the energy spectrum of the electron bunch: (1) $ct/\lambda = 1000$, (2) $ct/\lambda = 2000$, (3) $ct/\lambda = 3000$, (4) $ct/\lambda = 4000$.

about 20% of the laser pulse energy. The number of betatron oscillations experienced by the electrons up to this time was $N_0 = cT_{int}/\lambda_b \simeq 8.6$.

To simulate the x-ray generation we suppose that, at any given moment of time, the electron radiation spectrum is synchrotronlike [6]. The spectrum integrated over a solid angle is defined by $S(\omega/\omega_c)$. The critical frequency ω_c is given by the relation $\omega_c = (3/2)\gamma^2|F_\perp|/(mc)$; $F_\perp$ is the transversal to the electron momentum force. In our PIC code, we follow trajectories of each electron and calculate the emission during the interaction. The emitted radiation exerts a recoil on the electron [6]. The recoil force was included in the equations of electron motion in our simulations.

The synchrotron spectra at $ct = 1000\lambda$ and $ct = 4500\lambda$ are presented in Figs. 3(a) and 3(b). The surfaces shown in Figs. 3(a) and 3(b) give the number of photons within 0.1% of the bandwidth $(\Delta\hbar\omega = 10^{-3}\hbar\omega)$ per solid angle, $2\pi\sin\theta d\theta$: $\tilde{N}_X = \Delta\omega d^2N_X/(2\pi\sin\theta d\omega d\theta)$. It is seen from Fig. 3(b) that the relativistic bunch radiates highly energetic photons within a very narrow cone. The maximum of the radiation spectrum is located at about 50 keV. The analytical estimates for electron energy predict the maximum of $S(x) \simeq 0.3\hbar\omega_c \simeq 55$ keV that is in good agreement with the numerical simulation data. It is seen from Fig. 3 that the radiation from the bunch is confined within the angle $\theta \simeq 0.1$ rad and the theoretical estimate is about 0.2 rad. The photon flux (the number of photons per second in 0.1% bandwidth) and the spectral brilliance of the source at $ct = 1000\lambda$ and $ct = 4500\lambda$ are shown in Figs. 3(c) and 3(d). We can estimate the flux and the brilliance using the following formulas $\Phi \simeq (\Delta\omega_c/\omega_c)N_X(c/L_b)$ and $B \simeq \Phi/(4\pi^2\theta_R^2 S_R^2)$, where L_b is

the bunch length, $S_R \simeq \pi[r_b^2 + c^2T_{int}^2\theta_R^2/(4\pi^2)]$ is the effective source size of the radiation, and r_b is the bunch radius.

To emphasize the advantage of x-ray generation in the laser-produced ion channel in comparison with that in the self-generated channel, we perform a numerical simulation of the x-ray emission from an external 28.5 GeV electron bunch. The bunch has a diameter $2r_0 = 24.6$ μm and a length $L_b = 82$ μm with the total charge $Q_b = 5.4$ nC. The plasma and laser pulse parameters are the same as in the previous simulation. The electron beam density was much smaller than that of the background plasma, so that the laser pulse and bubble dynamics are not strongly affected by the external electron bunch. At the beginning of interaction the front of the electron bunch is close to the center of the laser pulse [see Fig. 4(a)]. The head of the bunch has overtaken the laser center by some 46λ after the interaction time $T_{int} = 4500\lambda/c$. The number of betatron oscillations during the interaction time was $N_0 = cT_{int}/\lambda_b \simeq 1.1$. It is seen from Fig. 4(b) that the laser pulse and the bubble remain structurally stable during the full interaction and the bunch is focused at this moment of time.

The synchrotron spectrum after the interaction time $T_{int} = 4500\lambda/c$ is presented in Fig. 5(a). In the present simulation we do not consider the emission from the background plasma electrons. At the given plasma density, the plasma wiggler strength parameter is about $K \simeq 817$. It is seen from Fig. 5(a) that the relativistic bunch radiates highly energetic photons within a very narrow cone. The maximum of the bunch radiation spectrum is located at about 210 MeV. The analytical estimates predict the maximum of $S(x) \simeq 0.3\hbar\omega_c \simeq 385$ MeV. The disagreement is caused by the bunch stopping because of
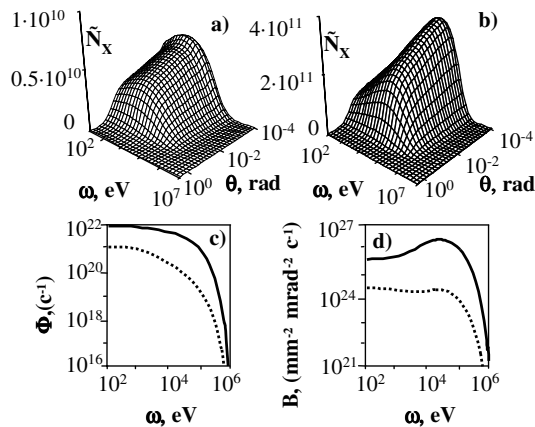
FIG. 3.   (a) The synchrotron spectrum from the plasma at $ct/\lambda = 1000$, (b) at $ct/\lambda = 4500$, (c) the photon flux (the number of photons per second in 0.1% bandwidth), and (d) the spectral brilliance. The dashed line in frames (c) and (d) corresponds to $ct/\lambda = 1000$; the solid line corresponds to $ct/\lambda = 4500$.
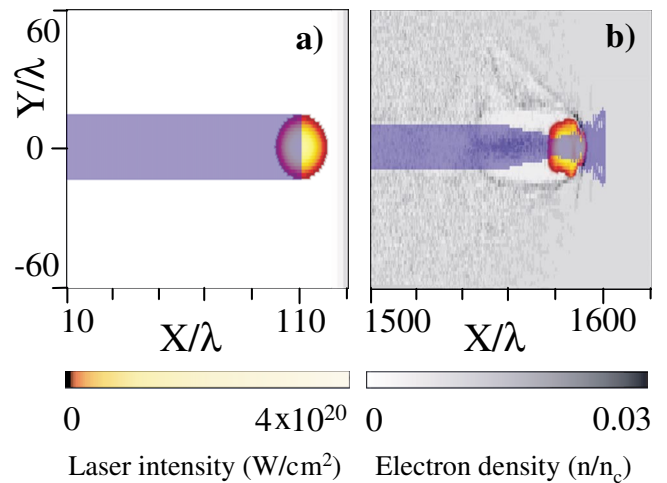
FIG. 4 (color).   Temporal evolution of the plasma density, laser intensity, and the envelope of the external 28.5 GeV electron bunch (blue): (a) at the beginning of interaction and (b) at $ct/\lambda = 1500$.
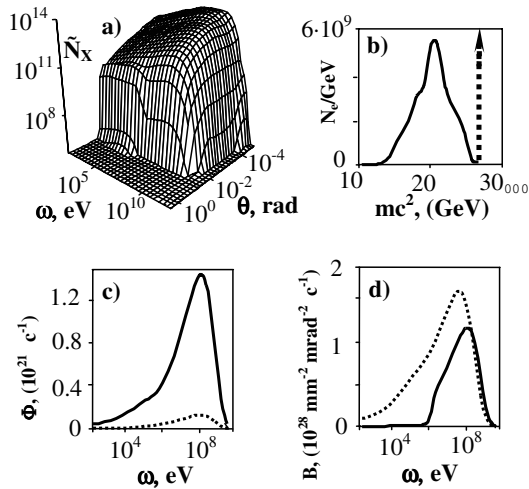
FIG. 5. (a) Synchrotron spectrum from the external 28.5 GeV electron bunch at $ct/\lambda = 4500$; (b) energy distribution of the bunch electrons: the solid line corresponds to $ct/\lambda = 4500$ while the dashed arrow marks the initial energy of the electron bunch; (c) photon flux and (d) spectral brilliance. The dashed line in frames (c) and (d) corresponds to $ct/\lambda = 500$; the solid line corresponds to $ct/\lambda = 4500$.

the radiation damping force. We also observe a significant photon flux up to the energy of 10 GeV. The radiation from the bunch is confined within the angle $\theta \simeq 10$ mrad that is close to the theoretical estimate 15 mrad. The total number of photons emitted by the bunch is about 2×10^{11}. This means that every electron of the bunch emits about six photons. The estimation for the photon number with the critical frequency ω_c is $N_X = N_e \langle N_X \rangle$, where N_e is the number of electrons in the bunch. The estimation is in good agreement with the numerical simulation results. The bunch lost about one-third of its energy after T_{int}. The energy distribution of the bunch electrons after the interaction is shown in Fig. 5(b). The photon flux and the brilliance versus the photon energy are shown in Figs. 5(c) and 5(d) . The brilliance at the beginning of interaction is slightly higher than at the end because, at the beginning, the bunch is not yet focused and, therefore, emits at small angles. It follows from Figs. 5(c) and 5(d) that the photon energy, flux, and brilliance of the x-ray emission from the laser-produced ion channel are several orders of magnitude higher than the ones observed in the self-generated ion channel [3].

In conclusion, we propose a novel compact and intense x-ray radiation source based on the strongly nonlinear broken-wave laser-plasma interaction. The brilliance of this source is some 2 orders of magnitude higher than that of the best x-ray sources available today. In addition, the radiation is polychromatic, covers that multi-keV range,

and comes in sub-100 fs pulses. This bright novel source of femtosecond x-ray pulses will have important scientific applications by enabling the direct measurement of atomic motion and structural dynamics in condensed matter on the fundamental time scale of a vibrational period. The 100 fs time scale is characteristic for atomic motion associated with ultrafast chemical reactions, nonequilibrium phase transitions, surface dynamics, and even ultrafast biological processes.

In addition, our proposed radiation source provides a sufficient number of photons per pulse to carry out these studies in a single-shot regime. This is crucially important, e.g., for the biological processes. The polychromaticity of the radiation source may allow probing of simultaneously different atomic species in complex or disordered materials.

The proposed radiation source can be a tabletop laser, and the plasma interaction length is less than a centimeter. The high ion density in the laser-plasma wiggler provides several orders of magnitudes higher energies of the x-ray photons than that observed in the recent experiment with self-generated ion channels [3] and in the designed free-electron laser (FEL)s.

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