Nonlinear Frequency Conversion in Waveguide Directional Couplers

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Nonlinear optical effects for frequency conversion require a phase-matching condition to efficiently generate a coherent field at the new wavelength. We find that the phase-matching condition can be replaced by a resonance condition when the nonlinear effect takes place in a waveguide directional coupler. We apply this theory to second-harmonic generation and find a theoretical conversion efficiency of 100%, equivalent to the perfect phase-matching condition. An example of the design of such waveguide directional coupler is presented.

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Nonlinear optical effects for frequency conversion, such as second-harmonic generation (SHG) and four-wave mixing, require a phase-matching condition to efficiently generate a coherent field at the new wavelength. Phase matching refers to the requirement that the generated electromagnetic fields along the interaction path remain in phase and thus the nonlinearly generated coherent waves are constructively summed. Several phase-matching techniques have been proposed. Perfect phase matching can occur due to the birefringence displayed by some crystals by precisely controlling the angular orientation of the crystal with respect to the propagation direction of the incident light or by precisely controlling temperature. Quasi-phase-matching (QPM) uses the spatially periodic modulation of the nonlinear optical coefficients and has achieved great success in recent years [1]. Other phase-matching methods have been achieved in waveguide structures where modal dispersion phase matching [2], anomalous-dispersion phase matching [3], and Cerenkov phase matching [4] have been demonstrated.

When two optical waveguides are close together, because of the interaction of the guided modes of each waveguide, optical power can be coupled between two waveguides. This structure is called a directional coupler, which has been intensively used in the area of integrated optics. Coupled-mode theory, which has been successfully developed to describe the directional coupler, shows that a spatially periodic modulation of light intensity occurs while a light is propagating in such structure [5].

In this Letter, we study the nonlinear frequency conversion effects in the waveguide directional coupler. These effects could include second-harmonic generation, sum frequency, difference frequency, third-harmonic generation, parametric generation, and four-wave mixing. If the phase-matching condition is not satisfied, the power of the generated wave will oscillate with the interaction length. However, the directional coupler also periodically modulates the generated radiation at the same time. As a result, the conversion efficiency of nonlinear effects can be enhanced greatly under some resonant conditions. We will apply this theory to second-harmonic generation, but similar resonant conditions also exist for other nonlinear optical effects. Theoretical conversion efficiency can be 100% for SHG, similar to the perfect phase-matching condition. We also demonstrate that this phenomenon could be feasible for applications and present an example of the design of a waveguide directional coupler. The proposed configuration has not only fundamental theoretical significance for nonlinear optics but also major technical advantages including very small beam size, large spatial overlap of fundamental and harmonic fields, long interaction lengths with low loss, high conversion efficiency of nonlinear effects, and simplicity of fabrication without any spatially periodic poling.

A simple model has been developed to describe this new theory, as shown in Fig. 1. In Fig. 1, the white circles represent the SHG wave with phase 0 and the black ones

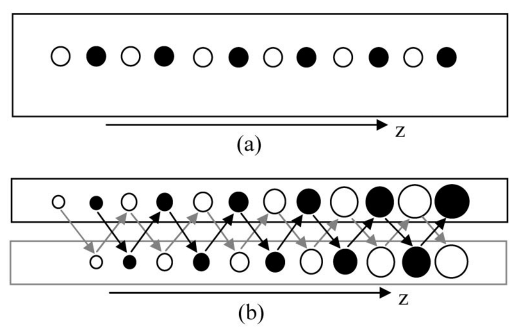

FIG. 1. A simple model describing the proposed theory. (a) Second-harmonic generation in a bulk crystal or a single waveguide. (b) Second-harmonic generation in a directional coupler. White and black circles represent the 0-phase SHG waves and the π-phase SHG waves. In (a), as the white circles propagate to the positions of the black circles, destructive interference occurs. In (b), the white circles "escape" to the lower waveguide at the positions that the black circles are generated and thus destructive interference is avoided. It is to be noted that the circles become larger along the interaction length since the new generated waves at a particular position constructively add to the waves generated ahead.

represent the SHG wave with phase π. The phase difference comes from the phase mismatching between the fundamental and the SHG wave. Obviously in a real system, the phase difference is continuous. Nevertheless, we just use two phases to simplify our discussion without losing the physical essence. Figure 1(a) describes the situation in a bulk nonlinear medium or in a single waveguide with nonzero phase mismatching. Since the SHG waves represented by the white circles are destructively interfering with those of the black circles during propagation, the SHG power cannot increase along the interaction length. However, the directional coupler provides a platform to transfer the optical power from one waveguide to another, as shown in Fig. 1(b). Here, for simplicity, it is assumed that the SHG wave is only generated in the upper waveguide. As before, there are SHG waves with 0 and π phases generated in the upper waveguide. Nevertheless, if the directional coupler is properly designed, the 0-phase SHG waves can be transferred to the lower waveguide at the positions where the π-phase SHG waves are generated, and equivalently by the π-phase SHG waves. In this way, the 0-phase wave and the π-phase wave propagate in two separated alternating paths between two waveguides. Therefore, there will be no destructive interference between the 0-phase and the π-phase SHG waves. Consequently, the power of the 0-phase wave can increase with the interaction length, and so does the π-phase SHG wave. To make this phenomenon take place, it is required that the distance between a 0-phase wave and its closest π-phase SHG wave is the same as the power-transfer length (the length needed to transfer the power in the first waveguide to the second waveguide) of the directional coupler, which results in a resonant condition discussed later on.

Assuming that the nonlinear effect and the coupling between two waveguides do not affect significantly the waveguide modes, and all the interacting modes are TE modes, the mode fields can be expressed by $\vec{E}_i^{\omega_j} = \vec{e}_y F_i^{\omega_j}(x)\exp(-i\beta_i^{\omega_j}z + i\omega_j t)$, where $\beta_i^{\omega_j}$ is the corresponding propagation constant and the subscript $i = 1, 2$ represents the ith waveguide and $j = 1, 2$ denotes the fundamental wave and the harmonic generation, respectively. The transverse mode-field profile $F_i^{\omega_j}(x)$ satisfies

$$\left(\frac{\partial^2}{\partial x^2} - \beta_i^{\omega_j}\right)F_i^{\omega_j} + \omega_j^2\mu\varepsilon_i^{\omega_j}(x)F_i^{\omega_j} = 0, \tag{1}$$

where $\varepsilon_i^{\omega_j}(x) = \varepsilon_0[n_i^{\omega_j}(x)]^2$ is the dielectric constant distribution of the ith single waveguide at frequency ω_j.

The electric field of the coupled waveguides can be expressed as the sum of the eigenmodes in each waveguide $\vec{E} = \frac{1}{2}[A_\omega(z)\vec{E}_1^\omega + A_{2\omega}(z)\vec{E}_1^{2\omega} + B_\omega(z)\vec{E}_2^\omega + B_{2\omega}(z)\vec{E}_2^{2\omega}] + \text{c.c.}$, where $A_{\omega_j}(z)$ and $B_{\omega_j}(z)$ are the slowly-varying amplitudes in waveguide 1 and waveguide 2, respectively. The wave equation obeyed by the electric field is

$$\nabla^2\vec{E} = \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} + \mu\frac{\partial^2}{\partial t^2}\left[\vec{P}_{NL}\right]. \tag{2}$$

Here, $\vec{P}_{NL}$ is the nonlinear polarization resulting from nonlinear optical susceptibilities. Finally, the coupled-amplitude equations governing the motions of the slowly-varying amplitudes are found under the slowly-varying-amplitude approximation:

$$\frac{dA_\omega}{dz} = -i\kappa_\omega B_\omega\exp(-i\Delta\beta_\omega z) - i\eta_1^\omega A_{2\omega}A_\omega^*\exp(-i\Delta k_1 z), \tag{3a}$$
$$\frac{dB_\omega}{dz} = -i\kappa_\omega A_\omega\exp(i\Delta\beta_\omega z) - i\eta_2^\omega B_{2\omega}B_\omega^*\exp(-i\Delta k_2 z), \tag{3b}$$
$$\frac{dA_{2\omega}}{dz} = -i\kappa_{2\omega} B_{2\omega}\exp(-i\Delta\beta_{2\omega} z) - i\eta_1^{2\omega} A_\omega^2\exp(i\Delta k_1 z), \tag{3c}$$
$$\frac{dB_{2\omega}}{dz} = -i\kappa_{2\omega} A_{2\omega}\exp(i\Delta\beta_{2\omega} z) - i\eta_2^{2\omega} B_\omega^2\exp(i\Delta k_2 z). \tag{3d}$$

Here, κ_ω and $\kappa_{2\omega}$ are the mode-coupling coefficients of the directional coupler for fundamental and SHG waves, respectively. $\eta_i^{\omega_j}$ includes the product of the nonlinear optical coefficient and the overlap integral of the fundamental and SHG waves. One can find the detailed expressions for these coefficients in Refs. [5–7]. The detailed derivation of the above equations from Eqs. (1) and (2) will be published elsewhere. In Eqs. (3), $\Delta\beta_{\omega_j}$ and Δk_i are measures of phase mismatching defined by the following relationship:

$$\begin{cases}\Delta\beta_{\omega_j} = \beta_2^{\omega_j} - \beta_1^{\omega_j}, \\ \Delta k_i = \beta_i^{2\omega} - 2\beta_i^\omega.\end{cases} \tag{4}$$

It is clear that $\Delta\beta_{\omega_j}$ denotes the phase mismatch between two guided modes in different waveguides at the same frequency, while Δk_i denotes the phase mismatch between the fundamental and SHG waves in the same waveguide.

To proceed in our calculation, we assume that the directional coupler consists of two identical waveguides, which results in the following equations:

$$\begin{cases} \Delta\beta_\omega = \Delta\beta_{2\omega} = 0, \\ \Delta k_1 = \Delta k_2 = \Delta k, \\ \eta_1^{\omega_j} = \eta_2^{\omega_j} = \eta^{\omega_j}. \end{cases} \tag{5}$$

To simplify the analysis further, we assume that the depletion of the fundamental wave due to the conversion of its power to SHG is negligible. Under this condition, we reach the following equation for $A_{2\omega}$:

$$\frac{d^2A_{2\omega}}{dz^2} = -\kappa_{2\omega}^2 A_{2\omega} + \frac{\eta^{2\omega}A_0^2}{4}(\Delta k + 2\kappa_\omega - \kappa_{2\omega}) \times \exp[i(\Delta k + 2\kappa_\omega)z] + \frac{\eta^{2\omega}A_0^2}{2}(\Delta k + \kappa_{2\omega}) \times \exp(i\Delta k z) + \frac{\eta^{2\omega}A_0^2}{4}(\Delta k - 2\kappa_\omega - \kappa_{2\omega}) \times \exp[i(\Delta k - 2\kappa_\omega)z] \tag{6}$$

together with the initial conditions that $A_{2\omega}(0) = 0$ and $\frac{dA_{2\omega}(0)}{dz} = -i\eta^{2\omega}A_0^2$. Here, we have assumed that $A_\omega(0) = A_0$ and $B_\omega(0) = 0$. A similar equation for the amplitude $B_{2\omega}$ can be derived.

The above equation is quite similar to the equation describing a classic undamped harmonic oscillator driven by an external harmonic force. The first term in the right side of the above equation provides harmonic motion, while the second term could be regarded as a harmonic driving force. It is well known that if the frequency of the driving force is exactly the natural frequency of the oscillator, the amplitude of the oscillation could be infinite in the steady state. The same phenomenon takes place here. The solution of $A_{2\omega}$ in the above equation could have a term proportional to z under some resonant conditions. This term will result in high-power SHG output if the waveguide length is long enough. Under the usual circumstance that $\Delta k > \kappa_\omega > \kappa_{2\omega} > 0$, it is found that there are two possible cases which result in this term. They are

$$\Delta k = \kappa_{2\omega} \tag{7}$$

and

$$\Delta k - 2\kappa_\omega = -\kappa_{2\omega}. \tag{8}$$

For example, under the condition of Eq. (7), the solution of Eq. (6) becomes

$$A_{2\omega}(z) = a_1\cos(\kappa_{2\omega}z) + a_2\sin(\kappa_{2\omega}z) + c_1\exp[i(\kappa_{2\omega} + 2\kappa_\omega)z] + c_2 z\exp(i\kappa_{2\omega}z) + c_3\exp[i(\kappa_{2\omega} - 2\kappa_\omega)z], \tag{9}$$

where $c_1 = -\frac{\eta^{2\omega}A_0^2}{8\kappa_\omega + 8\kappa_{2\omega}}$, $c_2 = -\frac{i\eta^{2\omega}A_0^2}{2}$, $c_3 = \frac{\eta^{2\omega}A_0^2}{8\kappa_\omega - 8\kappa_{2\omega}}$, $a_1 = -c_1 - c_3$, and $a_2 = -\frac{i\eta^{2\omega}A_0^2 + c_2 + ic_1(\kappa_{2\omega} + 2\kappa_\omega) + ic_3(\kappa_{2\omega} - 2\kappa_\omega)}{\kappa_{2\omega}}$. However, only the fourth term, which is proportional to z, will dominate the solution when $z > \frac{1}{|\kappa_{2\omega} - 2\kappa_\omega|}$ since the other terms will oscillate with z and their magnitudes will not increase with z. Therefore, we neglect these nondominant oscillating terms in the solution, and finally

$$A_{2\omega}(z) = -\frac{i\eta^{2\omega}A_0^2}{2}z\exp(i\kappa_{2\omega}z). \tag{10}$$

Similarly,

$$B_{2\omega}(z) = \frac{i\eta^{2\omega}A_0^2}{2}z\exp(i\kappa_{2\omega}z). \tag{11}$$

The above solutions show that the SHG amplitude will increase with the waveguide length, similar to the case of perfect phase matching. The magnitude of $A_{2\omega}(z)$ is half of that in the case of perfect phase matching in a single waveguide. This could be readily shown by setting $\kappa_{2\omega} = 0$ and $\Delta k_1 = 0$ in Eq. (3c), and the solution of Eq. (3c) describes the case of perfect phase matching in a single waveguide. If we sum the amplitudes of $A_{2\omega}(z)$ and $B_{2\omega}(z)$ after π-phase shifting the SHG output of one of these two waveguides, the total magnitude is exactly the same as in the case of perfect phase matching.

It is to be noted that in this case there is no phase matching condition required. In the situation of phase matching, the destructive interference between new generated waves at different locations is avoided by ensuring that the generated waves and pump waves have the same phase velocity. In this new configuration, the destructive interference is avoided because the 0-phase and π-phase waves are propagating in different paths. The only condition is Eq. (7), which may be regarded as a resonance condition between the coupling of two waveguides and the phase mismatching between the fundamental and SHG waves. Furthermore, this resonant condition corresponds exactly to the power-transfer relationship we described for Fig. 1(b). Even though the representation in Fig. 1(b) assumes that only one of two waveguides generates SHG, one can also obtain the same resonant condition as Eq. (7) following the same procedures. This is due to the fact that Eqs. (3a) and (3b) are not critical to obtain the resonant condition of Eq. (7), namely, the periodic exchange of the power of the fundamental wave in the two waveguides is not necessary. In addition, the phase difference between SHG outputs in these two waveguides is π when we compare Eq. (10) with Eq. (11). This is consistent to the previous discussions for Fig. 1(b). Eqs. (10) and (11) are obtained under the condition that the power of the fundamental wave is not depleted. A numerical simulation of Eqs. (3) demonstrates that the depletion of the pump wave occurs when the interaction length becomes sufficiently long and 100% conversion efficiency can be achieved.

Equation (8) is another possible resonant condition to obtain high SHG power; however, Eqs. (3a) and (3b) become critical to obtain this resonant condition. It is interesting to find that a novel quasi-phase-matching SHG technique has been investigated in Ref. [8]. The case studied in Ref. [8] is just a special case when $\kappa_{2\omega} = 0$ in Eq. (8). From Eq. (8), the condition of $\kappa_{2\omega} = 0$ results in that $\kappa_\omega = \frac{\Delta k}{2}$, which is exactly the quasi-phasing matching condition found in Ref. [8]. Our solu-

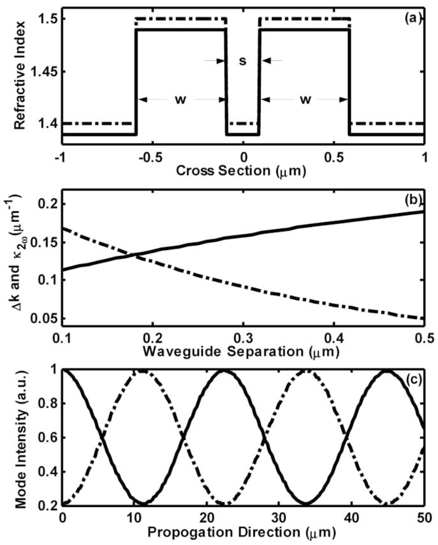

FIG. 2. (a) Refractive-index distribution of the proposed directional coupler for the fundamental wave (solid line) and the SHG wave (dashed line). (b) Phase mismatching Δk (solid line) and the coupling coefficient $\kappa_{2\omega}$ (dashed line) versus the waveguide separation. (c) Simulation of mode intensity versus the propagation distance by the beam-propagation method. This figure is for the wavelength at $1.55/2$ μm. The solid line and the dashed line represent mode intensities in waveguides 1 and 2, respectively.

tions suggest that the condition of $\kappa_{2\omega} = 0$ in Ref. [8] is not always required to obtained high-efficiency condition. Nevertheless, Eqs. (3a) and (3b) must be satisfied before we obtain this resonant condition.

We have designed a directional coupler structure with symmetric slab waveguides whose refractive-index profile is shown in Fig. 2(a). We will show that the resonant condition of Eq. (7) can be realized by this structure. The coupling constant $\kappa_{2\omega}$ is determined by

$$\kappa_{2\omega} = \frac{2h^2 p e^{-ps}}{\beta_{2\omega}(w + 2/p)(h^2 + p^2)}$$

[5], where $p^2 = \beta_{2\omega}^2 - n_b^2 k_0^2$, $h^2 = n_c^2 k_0^2 - \beta_{2\omega}^2$, and w and s are waveguide core thickness and the separation between two waveguides, as shown in Fig. 2(a). n_b and n_c are the refractive indexes of the background material and the core material, and k_0 is the wave number in the vacuum. From the above equation, it can be seen that $\kappa_{2\omega}$ increases exponentially while s decreases. Figure 2(b) demonstrates the numerical results for the coupling constant $\kappa_{2\omega}$ and the phase mismatching Δk defined by $\Delta k = \beta_{2\omega} - 2\beta_\omega$ where β_ω is the propagation constant for the fundamental TE even supermode at the wavelength 1.55 μm and $\beta_{2\omega}$ is the propagation constant for the TE_0 mode at the wavelength $1.55/2$ μm in each waveguide. As the separation s increases, the coupling constant decreases exponentially and the phase mismatching Δk increases. At $s = 0.18$ μm, the resonant condition $\kappa_{2\omega} = \Delta k$ is satisfied. To ensure that the coupled-wave equations of Eqs. (3c) and (3d) are still valid at this separation, a beam-propagation method [7] is employed to simulate the propagation of an incident wave, as shown in Fig. 2(c). The periodic exchange of power between two waveguides can be seen and the period is exactly $\pi/\kappa_{2\omega}$. This indicates that the coupled-mode equations of Eq. (3c) and (3d) are valid to describe the motion of the light at wavelength $1.55/2$ μm with this separation. It should be noted that the purpose of Fig. 2 is to show that the resonant condition Eq. (7) is feasible for an experimental test, and we do not include the nonlinear interaction between the wave at the wavelength 1.55 μm and the wave at the wavelength $1.55/2$ μm.

In conclusion, resonant phenomena similar to the driven harmonic oscillator system have been found for the nonlinear optical effects in the waveguide directional coupler. The resonant condition could be used to efficiently generate a new radiation field at another wavelength even in the case of the phase mismatching between the pump field(s) and the generated field. The resonant condition is in general $\kappa = \Delta k$, where κ is the coupling coefficient of the directional coupler for the new generated field and Δk is the phase mismatch. Meanwhile, other resonant conditions similar to Eq. (8) may be also available depending on the effect of intensity modulation of the pump wave(s) by the directional coupler. The feasibility of the directional coupler structure has been verified by a design of waveguide structures for second-harmonic generation.

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