# Collective Polarization Exchanges in Collisions of Photon Clouds

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The one-loop "vacuum" Heisenberg-Euler coupling of four electromagnetic fields can lead to interesting collective effects in the collision of two photon clouds on a time scale order of magnitude faster than one estimates from the cross section and density. We estimate the characteristic time for macroscopic transformation of positive to negative helicity in clouds that are initially totally polarized and for depolarization of a polarized beam traversing an unpolarized cloud.

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Some nonlinear aspects of vacuum electrodynamics have been tested in experiments on Delbruck scattering [1], i.e., the scattering of a photon off of the Coulomb field of a nucleus, and in photon splitting [2], also in the nuclear Coulomb field.

Essentially, these effects hinge on the one-loop effective Lagrangian density for processes in which four or more electromagnetic fields of long wavelength compared to the electron Compton wavelength come together, as described by the Heisenberg-Euler interaction [3,4], the fourth order term of which is

$$L_I = \int d^3x \frac{2\alpha^2}{45m^4}[(\mathbf{E}^2 - \mathbf{B}^2)^2 + 7(\mathbf{E}\cdot\mathbf{B})^2], \qquad (1)$$

where $\alpha$ is the fine structure constant and $m$ is the mass of the electron. (We use units $\hbar = c = 1$ throughout.)

The validity of the effective interaction term (1), for long wavelength fields, can hardly be doubted. Nonetheless, its confirmation in an actual photon-photon scattering experiment would be a milestone of a kind. If one puts in the numbers for photon-photon scattering itself, the cross section is far too small to be measured with current technology. Indeed the "light by light" scattering discussed in the very interesting experiment reported in Ref. [5] was the reaction $\gamma + \gamma \rightarrow e^+ + e^-$, and does not test vacuum QED at the one-loop level. However, Kotkin and Serbo [6] have pointed out that a photon of one plane polarization, passing through a cloud of photons that are polarized in a different direction in a frame in which the collisions are head on, will experience a polarization precession with an angular frequency, $\Gamma_p = 4\alpha^2 n_\gamma \omega\omega_c/(15m^4)$, where $\omega$, $\omega_c$ are the respective frequencies of the impinging photon and the cloud and $n_\gamma$ is the number density of cloud photons. This rate is to be contrasted to the ordinary scattering rate of the impinging photon, as derived from the cross section [7], $\Gamma_s = 0.014 \times \alpha^4 m^{-8} n_\gamma \omega^3 \omega_c^3$. In all situations in which $\omega\omega_c \ll m^2$, $\Gamma_p$ is many orders of magnitude greater than $\Gamma_s$.

This polarization precession, from an effective anisotropic index of refraction, originates in the coherent interaction through forward scattering of a single beam photon with a large number of cloud photons. In this Letter, we develop the theory of another collective interaction, now between two clouds of photons, also depending on coherent forward scattering. This interaction can lead to helicity changes when photons of both clouds initially all have the same helicity, and to depolarization of one cloud when the other cloud is initially unpolarized. The rate will now turn out to be of order $\Gamma_{\rm pol}$ divided by a slowing factor $\log(N)$ where $N$ is the number of photons in a region of interaction of linear dimension $1/\Gamma_p$.

To rederive the Kotkin and Serbo result, and to lay the groundwork for the extension, we consider the complete set of momentum states $\{q_i\}$ that are occupied in the initial state in either cloud (whether singly or multiply occupied). We take $L_I$ of (1) and truncate it by keeping the parts of the fields that contain only creation and annihilation operators for this set of momenta. The momentum-conserving processes described by this interaction are just the forward scattering of beam photons from cloud photons since comoving cloud particles (or beam particles) do not scatter from each other in the interaction, (1). The result, for the effective "forward" Hamiltonian of the system, after substitution of the canonical expressions for the electromagnetic fields in terms of creation and annihilation operators into $-L_I$ of (1) and performing the space integral over a quantization volume, $V$, is

$$H_{\rm for} = GV^{-1}\sum_{j,m}\omega_j\omega_m[\zeta_j^{(1)}\tau_m^{(1)} + \zeta_j^{(3)}\tau_m^{(3)} - (11/3)\mathrm{I}_j^{(a)}\mathrm{I}_m^{(b)}], \qquad (2)$$

where $G = 2\alpha^2/15m^4$, and where the indices $j$ and $m$ extend over the momentum states defined above.

In (2) the products of photon annihilation and creation operators for the beam modes, $a_j^x$, $a_j^y$, and for the cloud modes, $b_j^x$, $b_j^y$ (where $x$ and $y$ indicate the polarization state and $j$ enumerates the set of momenta), have been reexpressed in terms of the operators,

  

$$\begin{aligned}
\mathrm{I}_j^{(a)} &= (a_j^{(x)})^\dagger a_j^{(x)} + (a_j^{(y)})^\dagger a_j^{(y)},\\
\tau_j^{(1)} &= (a_j^{x})^\dagger a_j^{y} + (a_j^{y})^\dagger a_j^{x},\\
\tau_j^{(3)} &= (a_j^{(x)})^\dagger a_j^{(x)} - (a_j^{(y)})^\dagger a_j^{(y)},
\end{aligned} \tag{3}$$

with the parallel set of definitions for the cloud operators, taking $a \to b$, $\vec{\tau} \to \vec{\zeta}$. The operators $\tau^{(1),(3)}/2$, supplemented by an operator $\tau^{(2)}/2$, which will not explicitly enter below, obey angular momentum commutation rules, as do the operators $\vec{\zeta}/2$. Since $H_{\mathrm{for}}$ only connects states of identical *unperturbed* energies, we did not include a contribution from an $H_0$ in (2). [We have made the transition from interaction Lagrangian to the interaction Hamiltonian in (2) without regard to the fact that the original $L_I$ contains time derivatives of the original canonical coordinates, $\vec{A}$. This is consistent if we take $\omega_j$ and $\omega_m$ as simple parameters in (2) and do not translate them back into time derivatives when we go to a Heisenberg picture.] To follow the polarization of a single beam photon of energy $\omega$ passing through the cloud, we can then write the Heisenberg equations for $\vec{\tau}(t)$ coming from (2) as

$$\frac{d}{dt}\vec{\tau}(t) = -2G\omega V^{-1}\vec{\tau}(t) \times [\vec{Z}(t) - \vec{v}Z^{(2)}], \tag{4}$$

where we have defined $\vec{Z} = \sum_m \omega_m \vec{\zeta}_m$ and introduced a vector $\vec{v}$, defined as a unit vector in the two direction in the internal space. [One should not confuse the three-dimensional space that we have introduced using a spinorial representation for the polarization vectors with the three-dimensional configuration space; we label vector components in the former with (1,2,3) and in the latter with $(x, y, z)$.] For the case of an isolated beam photon interacting collectively with the cloud photons, it is fairly clear that we can replace the cloud operators, $Z^{(1),(3)}$ by their expectation values, since the back reaction from beam interactions affects the cloud almost not at all. If the cloud polarization is at an angle $\theta$ to the $\hat{x}$ axis and the cloud energies are reasonably narrowly clustered around an energy $\omega_c$, we have $\langle Z^{(3)}\rangle/V = \omega_c n_\gamma \cos(2\theta)$, $\langle Z^{(1)}\rangle/V = \omega_c n_\gamma \sin(2\theta)$. Since Eqs. (4) are now linear in the operators for the beam particle, they hold for expectation values. Taking the initial condition $\langle\tau^{(3)}(0)\rangle = 1$, $\langle\tau^{(1),(2)}(0)\rangle = 0$ for an initial beam polarization in the $\hat{x}$ direction, and solving (4), we obtain the $x, x$ component of the polarization density matrix, $P_x = 1/2 + \langle\tau^{(3)}(t)\rangle/2$,

$$P_x = 1 - \frac{1}{2}\sin^2(2\theta)[1 - \cos(\Gamma_p t)], \tag{5}$$

where $\Gamma_p = 2G\omega\omega_c n_\gamma$.

Equation (5) recaptures the effects noted by Kotkin and Serbo [6], and we refer the reader to their articles for more discussion as to the possibilities of observations. To make one comparison to laboratory parameters, we define oscillation length as $\lambda = (\Gamma_p)^{-1}$ and express, in ordinary units,

$$\lambda = 1.5 \times 10^{-9}\left(\frac{E_{\mathrm{crit}}}{E}\right)^2\left(\frac{1\mathrm{MeV}}{\hbar\omega}\right)\mathrm{cm}, \tag{6}$$

where $E_{\mathrm{crit}} = m^2c^3/e\hbar$ and $E$ is the rms electric field of the cloud. In the $\omega_1 = 2.35$ eV laser used in the experiment reported in Ref. [5], the field strength was $E/E_{\mathrm{crit}} \approx 1.5 \times 10^{-6}$. In this case, taking $\hbar\omega = 100$ MeV leads to an oscillation length of $\approx 3$ cm. (The pulse length for this laser is a fraction of a millimeter; the free path for ordinary photon scattering from the cloud under these conditions is of the order of $10^9$ cm.)

We further note that this photon-cloud interaction produces no effect on the short time scale if the inital polarizations are perpendicular, and we note that if the cloud is unpolarized then there is no depolarization of the beam. Turning to the case of two colliding clouds, for which neither of these conclusions will hold, we assume for simplicity that photon densities in the two colliding groups are equal. Now we need to take the variables $\vec{\zeta}$ on the right-hand side (R.H.S.) of (2) as well as the variables $\vec{\tau}$ to be dynamic variables, rather than taking their expectation values in the initial state.

This calculation is simplest in a helicity basis, however. The forward interaction, $H_{\mathrm{for}}$, gives a matrix element for the transition in which a state of a positive helicity photon from one bath and a positive helicity photon from the other bath makes a transition to a state with two photons of negative helicities. We can easily express $H_{\mathrm{for}}$ of (2) now in terms of operators $\vec{\xi}$, $\vec{\eta}$, which act in the two-dimensional helicity spaces of the respective clouds, designated, respectively, as the "up" cloud and the "down" cloud. The components $\xi_i^{(3)}$ and $\eta_i^{(3)}$ measure the spins in the $\pm\hat{z}$ direction for the photons in the respective clouds, thus the negative of the helicity in the case of the down-moving photon. We choose both clouds to be essentially monoenergetic, with energies $\omega$ and $\omega_c$ for the respective up-moving and down-moving clouds; then we can express the forward Hamiltonian in terms of the collective coordinates, $\xi^{(\pm)} = \sum_i \xi_i^{(\pm)}$, $\eta^{(\pm)} = \sum_i \eta_i^{(\pm)}$, where $\xi^{(+)} = (\xi^{(1)} + i\xi^{(2)})/2$, etc.

By direct transformation of (2) we obtain

$$\begin{aligned}
H_{\mathrm{for}} = G\omega\omega_c V^{-1}[&2\xi^{(+)}\eta^{(-)} + 2\xi^{(-)}\eta^{(+)}\\
&- (11/3)\mathrm{I}^{(a)}\mathrm{I}^{(b)}].
\end{aligned} \tag{7}$$

Now we pose the question of what happens beginning with an initial state in which all $N$ up-moving photons have spin $+1$ and all $N$ down-moving photons have spin $-1$ in the $\hat{z}$ direction. We can proceed, as in the earlier case, by writing the equations of motion,

$$\frac{d}{dt}\xi^{(+)}(t) = -2iG\omega\omega_c V^{-1}\xi^{(3)}(t)\eta^{(+)}(t),$$
$$\frac{d}{dt}\xi^{(3)}(t) = -2iG\omega\omega_c V^{-1}[\xi^{(+)}(t)\eta^{(-)}(t) - \xi^{(-)}(t)\eta^{(+)}(t)], \tag{8}$$
$$\frac{d}{dt}\xi^{(-)}(t) = 2iG\omega\omega_c V^{-1}\xi^{(3)}(t)\eta^{(-)}(t),$$

plus the three equations in which $\vec{\tau}$ and $\vec{\zeta}$ in (8) are interchanged. In the calculation leading to (5), we proceeded to a soluble problem by taking a factorized ansatz that is equivalent, in our present problem, to the replacement,

$$\langle\xi^{(3)}(t)\eta^{(+)}(t)\rangle = \langle\xi^{(3)}(t)\rangle\langle\eta^{(+)}(t)\rangle. \tag{9}$$

But for the initial state that we are now considering, all of the mixing operators with $\pm$ superscript have expectation value zero, and it is clear that there would be no evolution in time at all were the factorization ansatz valid. We proceed instead to a calculation equivalent to solving the full coupled operator equations.

The total $\hat{z}$ component angular momentum in the new internal space in which helicity is the basis, measured by $(\xi^{(3)} + \eta^{(3)})/2$, is conserved. Thinking of the system as an assemblage of spins with an upper tier of $N$ spins all initially pointed up and a lower tier all initially pointed down, we enumerate the states that are connected to the initial state (and to each other) by the Hamiltonian of (7). Any number of the $N$ spins in the upper tier, all initially up, may be flipped, leading to $N + 1$ possibilities for the magnetic quantum number of the this tier. The operators $\vec{\xi}\cdot\vec{\xi}/4$ and $\vec{\eta}\cdot\vec{\eta}/4$ are separately conserved, each with eigenvalue $(N/2 + 1)N/2$. Therefore, for each value of $(\xi^{(3)}/2)$ in our set, there is a single upper tier configuration that enters, and a single lower tier configuration as well. We index the states by the number of flips plus one, $i$, where $i$ takes on the values $1, 2 \ldots N + 1$. We express the operator products that occur in the Hamiltonian in this basis,

$$\langle i|\xi_-\eta_+|i-1\rangle = (N-i+1)(i); \quad i = 1, \quad \ldots N+1,$$
$$\langle i+1|\xi_+\eta_-|i\rangle = (N-i+2)(i-1); \quad i = 1, \quad \ldots N+1, \tag{10}$$

which come directly from the standard angular momentum matrices. We solve numerically for a $N + 1$ component wave function $\Psi(t)$, using the Hamiltonian (7) with the substitution (10) and the initial condition $\Psi_i(0) = \delta_{i,1}$, and then calculate the measure of average helicity of the upper tier,

$$R(t) = N^{-1}\sum_{i=i}^{N}\langle\xi_3^{(i)}\rangle = \sum_{i=1}^{N+1}|\Psi_i(t)|^2(N - 2i + 2)N^{-1}. \tag{11}$$

We perform these calculations for a series of values of $N$ and show the results as a function of scaled time, $s = \Gamma_p t = 2GN\omega_c\omega t/V$. Figure 1 displays results for values of $N$ ranging from 8 to 512, equally spaced in $\log(N)$.

The data shown in the figure clearly suggest a characteristic time of order $\Gamma_p^{-1}\log(N)$ for a complete turnover of the spins. We can gain a heuristic understanding of these results. Instead of the set of operators $\vec{\xi}, \vec{\eta}$ we introduce the bilinear forms:

$$x = i\xi^{(+)}\eta^{(-)}; \qquad u = i\xi^{(-)}\eta^{(+)}; \qquad y = \eta^{(+)}\eta^{(-)},$$
$$z = \xi^{(-)}\xi^{(+)}; \qquad w = \xi^{(3)}. \tag{12}$$

Writing the Heisenberg equations of motion for these operators by taking commutators with $H_{\rm for}$ in the form (7) and making the further substitution $\eta^{(3)} = -\xi^{(3)}$, we obtain the closed set:

$$N\dot{x} = w(z + y) + w^2; \qquad \dot{u} = -\dot{x}; \qquad N\dot{y} = wx - uw,$$
$$N\dot{z} = xw - wu; \qquad N\dot{w} = -x + u, \tag{13}$$

where the derivatives are with respect to the scaled time $s$. Treating these equations as $c$-number equations [10] with the initial conditions $x = y = z = u = 0$ and $w = N$ allows us to write a single equation for $\bar{w} \equiv w/N$,

$$\frac{d^2}{ds^2}\bar{w} = -2\bar{w}(1 - \bar{w}^2) - \frac{2\bar{w}^2}{N}. \tag{14}$$

The initial condition is now $\bar{w}(0) = 1$, $\bar{w}'(0) = 0$. In Fig. 2, we plot solutions of (14) and compare with the numerical solutions to the complete equations. The fit is good for values of $R > 0.6$. We also see that in the case of solutions to (14) the equal spacing continues to values of $N \approx 10^4$, leaving little doubt of the logarithmic dependence. It is possible to understand this limit analytically from (14), capitalizing on the fact that when $N \to \infty$, the solution is the familiar kink solution in a $\lambda\phi^4$ theory in one dimension, $\bar{w} = \tanh[((t - t_0)/2)]$, then showing that for large $N$, in the time region in question, the $\bar{w}^2/N$ term

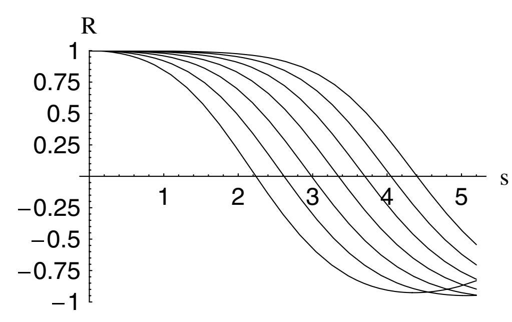


FIG. 1. The function $R(s/\Gamma_p)$ of (11), the mean helicity of the up-moving cloud, for values of $N = 8, 16, 32, 64, 128, 256, 512$ as determined from solutions of (8), plotted against the dimensionless scaled time, $s$. The curves for higher values of $N$ lie progressively farther to the right. Equal spacing of the curves indicate a transition time increasing as $\log(N)$.

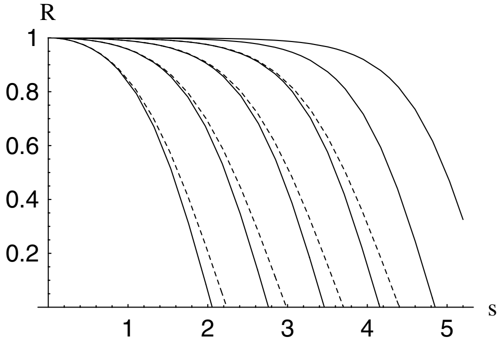


FIG. 2. The function $R(s/\Gamma_p)$ as determined from the solution for the heuristic Eq. (12) for values $N = 8, 32, 128, 512, 2048, 8192$ (solid lines). The dashed lines show the solutions of the complete equations of motion (11), as plotted in Fig. 1 for the first four values of $N$.

in (14) can be dropped in favor of changing the initial value of $w$ to $1 - 2/N$, this in turn determining $t_0 = \log(2N)$.

To summarize briefly: In many-body systems in which every particle of set $A$ interacts with every particle of set $B$, evolution times for macroscopic properties may be much faster than one would have predicted based on cross sections, even in the absence of initial phase relations among the components that one might have anticipated were necessary for such behavior. In the photon-photon system, the effect is an extension of the known index-of-refraction effects of photon polarization treated in Ref. [6]. In the detailed example treated, there is total oscillation back and forth between all positive helicities and all negative helicities in both clouds.

The case in which one cloud with 100% polarization in helicity collides with an unpolarized cloud is somewhat more complex. Here we predict partial depolarization of the polarized cloud. From (7) we see that photons in the target cloud with the opposite helicity to those of the beam cloud are effectively sterile. Therefore, we can discuss the polarization changes of the beam cloud in a manner similar to that of the calculation given above. There remains an order $\Gamma_p/\log(N)$ rate of depolarization after averaging over the configurations of polarization of the individual photons in the target cloud. This is in contrast to case of a single photon interacting with the cloud discussed at the beginning of this Letter, where depolarization takes place on the time scale $1/\Gamma_s$.

Our calculation was for an idealized system of plane wave modes in a box, with (implicit) periodic boundary conditions. Does it apply to realizable systems in which the two clouds are in contact for a time of order (box size$/c$)? It is clearly required that the characteristic time for transformation be shorter than this contact time, a criterion that is easily checked in any given situation. It is harder to answer the question, "Can the laboratory photons in the two beams really sustain a coherent interaction over the whole of the macroscopic region (of order of a cm, in the numerical example mentioned above, but now multiplied by a logarithm of the order of 100) for our process to unfold?" We do not know how to address the exact quantification of this question, although we believe that the answer is "yes" for the case of the beams from lasers and from synchrotrons. Another question concerns the role of all of the modes that we have left out in using the truncation that produced the forward Hamiltonian (2). We anticipate that over the time scale $1/\Gamma_p$, these modes create junk that does not add up to anything macroscopically due to phase oscillations, as indeed they must in our preliminary beam cloud calculation. In any case, we believe that the "speed up" through the many-body interactions that we have described here is interesting enough to warrant serious attention to some of the harder questions that arise.

Finally, we note the close similarity between the issues discussed in this Letter and in Refs. [8,9], which discussed the possibility of speeded-up flavor transformations of colliding neutrino clouds. Although the equations are quite similar, a critical difference is a term proportional to $\xi^{(3)}\eta^{(3)}$ on the R.H.S. of the analogue of (7) in the neutrino case. This term destroys the speed-up process in the simple model, with just two tiers of states, in which all the couplings between the upper tier and lower tier neutrino states (in our $N$-spin terminology) are equal to each other. In more realistic (and complex) situations, it is possible that there would be speeded evolution, however.

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[10] This is exactly the arbitrary procedure that we disparaged above for the case of the equations for the operators $\vec{\xi}$ and $\vec{\eta}$. One difference, and perhaps the key to the agreement with results of the complete solutions, is that the operators (12) keep us within the $N + 1$-dimensional subspace of states defined above, while the operators $\vec{\xi}$ and $\vec{\eta}$ do not.