Caloric Curve for Mononuclear Configurations

L. G. Sobotka,[1,2] R. J. Charity,[1] J. Tõke,[3] and W. U. Schröder[3]
[1]*Department of Chemistry, Washington University, St. Louis, Missouri 63130, USA*
[2]*Department of Physics, Washington University, St. Louis, Missouri 63130, USA*
[3]*Department of Chemistry, University of Rochester, Rochester, New York 14627, USA*
(Received 26 April 2004; published 21 September 2004)

The caloric curve for mononuclear configurations is studied with a schematic model. We investigate the dependence of the entropy on the density and effective-mass profiles. In finite nuclei, a plateau in the caloric curve is a consequence of decreasing density and the destruction of correlations rather than an indication of phase coexistence. The mononuclear regime is metastable with respect to binary fission at low excitation energy and with respect to multifragmentation at high excitation. The statistical framework presented here is suitable to treat scenarios where experimental conditions are set to favor a population of highly excited mononuclei.

DOI: 10.1103/PhysRevLett.93.132702 PACS numbers: 25.70.Gh, 25.70.Pq

Heavy compound nuclei (CN) are metastable objects, i.e., local mononuclear entropy maxima that are separated from the more stable dinuclear states by transition regions of significantly lower entropy. Standard statistical-model treatments of CN decay are predicated on a time-scale separation between the CN formation time and the time scales for simple (mostly single-particle) decay modes as well as the massively collective decay processes we call fission. The former are usually treated using the prescriptions offered by Weisskopf [1] or Hauser and Feshbach [2], while the latter are usually treated by a transition-state formalism initially developed for chemical reactions by Erying [3]. The distinction between decay modes can be bridged in concept [4] and practice [5]; however, the massively collective decay channels can be retarded by transient delays [6].

It has been known for almost four decades that, below approximately 1/3 of the saturation density ρ_o, α matter has a lower free energy than uniform nuclear matter [7,8]. However, mononuclear configurations at reduced density can be reasonable subjects for statistical decay treatments as long as they are either metastable or protected from massively collective decays modes by transient delays. Specifically with increasing excitation energy, an equilibrium (i.e., local maximal entropy) mononuclear density profile can be reached, the decay of which can be treated with only minor modifications to the well known formalisms, as long as a time-scale separation exists between formation and all conceivable decay modes.

This work does not deal with the important issue of time-scale separation [9], only with the gross statistical properties of mononuclei at high excitation energy. In particular, we show that the relaxation of the density profile of mononuclei, in pursuit of maximal-entropy, causes the caloric curve $T(\varepsilon)$ (where T is the statistical nuclear temperature and ε is the excitation per nucleon) to flatten out and exhibit a quasiplateau. We believe this explains the nature of the caloric curve first studied by Wada *et al.* [10] and later by Pochodzalla *et al.* [11], and for which systematics have recently been analyzed in detail by Natowitz *et al.* [12].

The approximate saturation of the statistical temperature is primarily due to density reduction, but it is also influenced by the evolution of the effective mass of nucleons in the nuclear medium. The first effect is just the sequestration of energy in the potential energy of nuclear expansion. The energy spent on expansion reduces the thermal part of the total excitation in much the same way as the collective rotational energy does in the case of high angular momenta CN.

The ratio of the effective mass to the bare nucleon mass m^*/m differs from one due to the finite range of the nuclear force and the time nonlocality of the interaction. The former effect, which is responsible for making the optical model potential energy dependent, reduces m^*/m by a density dependent factor $m_k(\rho)$ which must return to one at low density. The time nonlocality can be thought of as the coupling of low-lying surface modes to single-particle degrees of freedom [13,14]. This collective effect brings states down from high energy, increasing the many-body density of states at low excitation energy. The effective-mass factor, $m_\omega(\rho', T)$, accounting for this relocation of levels, while greater than one at low energy and localized on the surface of the quantum drop, must return to one in the limit of high excitation or low density gradient.

We confine our analysis to a one parameter description of expansion and the literature descriptions of how the effective-mass terms evolve with density and excitation. Our approach combines the physically transparent picture of maximal-entropy mononuclear configurations found in the recent work by Tõke *et al.* [15], with the effective-mass change with excitation energy found in the works of Natowitz, Shlomo, and collaborators [16].

The dominant term in the expression for the entropy of a quantum drop of degenerate Fermi liquid can be written

as [17]

$$S = 2\sqrt{aU} = 2\sqrt{a(E_T^* - E_E)} = 2\sqrt{aA(\varepsilon - \varepsilon_E)}, \quad (1)$$

where a is the level-density parameter and the thermal, total, and expansion energies are U, E_T^*, and E_E, respectively. With total particle number A, ε and ε_E are the total and expansion energies per particle. In the local density approximation (LDA) [18], the level density depends on the nuclear profile, the local Fermi momentum k, and the effective-mass [19,20],

$$a = \frac{\pi^2}{4}\sum_\tau \int \frac{\rho_\tau(r)}{[\hbar^2 k_\tau^2(r)/2m^*]} d\mathbf{r}. \quad (2)$$

The density profiles $\rho_\tau(r)$ of the two isospin partners (with index τ) are taken to be the same functional form, scaled in proportion to the number of nucleons. The native ($\epsilon = 0$ MeV) radial profiles are of the "standard" type with a Gaussian derivative,

$$\rho_n(r) = \frac{\rho_o}{2}\left[1 - \mathrm{erf}\left(\frac{r - R_o}{\sqrt{2}b}\right)\right], \quad (3)$$

with effective sharp radius $R_o = r_o A^{1/3}$ ($r_o = 1.16$ fm) and surface width $b = 1.0$ fm. The expansion is limited to the one-dimensional self-similar family, i.e., $\rho(r, c) = c^3\rho_n(cr)$. The expansion parameter c is found by maximizing the entropy,

$$\left(\frac{\partial S}{\partial c}\right)_\varepsilon = 0. \quad (4)$$

The collective energy involved in expansion is taken as the simple upside down bell shaped form, involving only the central density, suggested by Friedman [21], $\varepsilon_E(c) = \varepsilon_b(1 - \frac{\rho(0,c)}{\rho_o})^2$. We have used $\varepsilon_b = 8$ and 6 MeV in the present calculations. The energy required for expansion using $\varepsilon_b = 8$ is almost identical to that calculated (with Coulomb) using the logic of Myers and Swiatecki [22] and a nuclear matter compressibility coefficient of $K_o = 234$. In this schematic model, $\varepsilon_b = 6$ MeV simply implies a 25% reduction in the energy cost for expansion.

Execution of Eq. (4) not only finds the metastable mononuclear expansion but also ensures that the surface pressure is zero. This procedure is therefore logically different from the physically unreal but true equilibrium condition found by placing a drop in a box and having a surrounding vapor supply a pressure.

We choose the phenomenological form for the effective mass suggested by Prakash *et al.* [20] and used by De *et al.* [23]:

$$\frac{m^*}{m} = (m_k)[m_\omega] = \left(1 - \alpha\frac{\rho(r,c)}{\rho_o}\right)\left[1 - \beta(T)\frac{\rho'(r,c)}{\rho_o}\right], \quad (5)$$

with

$$\alpha = 0.3, \qquad \beta(T) = 0.4A^{1/3}\exp[-(TA^{1/3}/21)^2]. \quad (6)$$

(The T dependence requires knowing the caloric curve $[T(\varepsilon)]$. We solve this problem by iteratively starting with the $m^*/m = 1$ caloric curve. This iteration ensures that the T is uniquely determined by ε and satisfies the stationary condition.) The effective-mass factor is suppressed in the bulk, peaks at the surface [24], and degrades to one with decreasing density and increasing thermal energy. These two many-body effects, to a large extent, offset one another in near ground-state nuclei, yielding $a \approx A/8$ for unexpanded ^{197}Au, the nucleus considered here. However, the destruction of the cooperativity encoded in these two effective-mass terms does not occur on identical energy scales. While the detailed density and the excitation energy dependence of these terms are unknown, the present work shows how the gross effects captured by these terms couple with expansion to dictate the form of the caloric curve.

The excess entropies above the unexpanded ($c = 1$) native shape are shown in Fig. 1. The maximum entropy determines the equilibrium expansion and mononuclear entropy $S_M(\varepsilon)$.

The reduction of the equilibrium central density with excitation $\rho_c(\varepsilon)$ (at the extremum in entropy) is shown in Fig. 2(a) for these cases: $m^* = 1$ with $\varepsilon_b = 8$ MeV and $m^*/m = m_k(\rho)m_\omega(\rho', T)$ for both $\varepsilon_b = 8$ and 6 MeV. Consideration of $m_k(\rho)$ alone exhibits a reduction in the central density similar to that with $m_k(\rho)m_\omega(\rho', T)$, while consideration of $m_k(\rho)m_\omega(\rho')$ leads to approximately the same $\rho_c(\varepsilon)$ as with $m^*/m = 1$. Without the effective-mass terms, the $\rho_c(\varepsilon)$ dependence is almost identical with the (extended) finite-temperature Hartree-Fock calculation reported in [16]. As one should expect, the decrease in density is more substantial with the reduced energy cost of expansion. The central densities implied by the maximal-entropy procedure used here with $m^*/m = m_k(\rho)m_\omega(\rho', T)$ and $\varepsilon_b = 6$ MeV are similar to those extracted from caloric curve data (diamonds) reported by Natowitz *et al.* [16]. On the other hand, $\rho_c(\varepsilon)$ does not

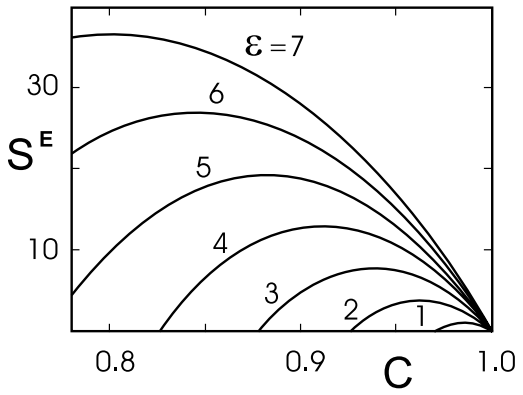

FIG. 1. Representative calculation of total excess entropy as a function of expansion.

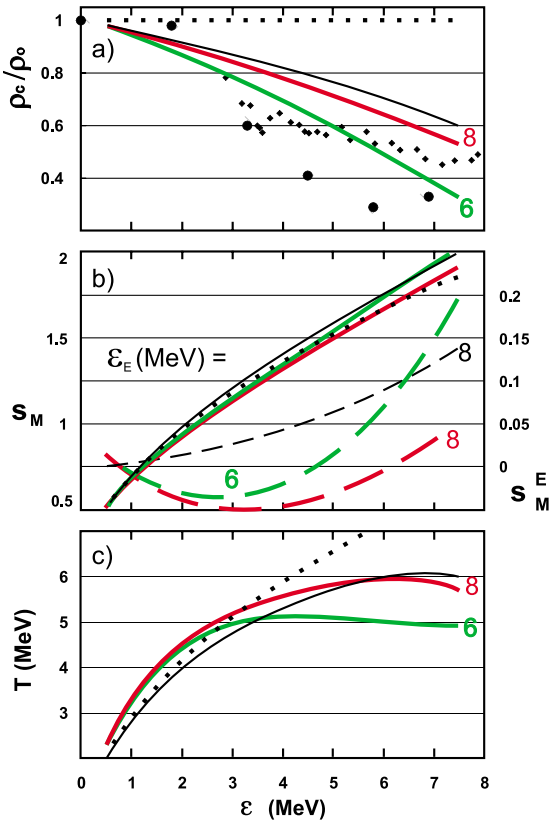

FIG. 2 (color online). (a) Central densities; (b) entropy s_M and excess entropy s_M^E (dashed line) per particle; (c) T as a function of the excitation per nucleon ε for ^{197}Au. The cases shown: unexpanded (dotted line); $m^* = 1$, $\varepsilon_b = 8$ MeV (black line); $m^* = (m_k)[m_\omega]$, $\varepsilon_b = 8$ MeV (thick dark-grey or red line) and $\varepsilon_b = 6$ MeV (thick light-grey or green line). In (a) the densities, extracted from apparent level-density parameters (diamonds) [26] and from Coulomb barriers (circles) [25], are also shown.

drop as quickly, over as narrow a range of energies, as is suggested by the analysis of Coulomb barriers (circles) in the work by Bracken, Viola, and collaborators [25].

Figure 2(b) displays the entropy per nucleon for the native density profile (dotted line), as well as the mononuclear (maximal) entropy per nucleon s_M and excess value s_M^E for $m^*/m = 1$ with $\varepsilon_b = 8$ MeV and $m^*/m = m_k(\rho)m_\omega(\rho', T)$ with both $\varepsilon_b = 8$ and 6 MeV. The excess entropy allowed by relaxing the density profile is evident as is a reduced rate of entropy growth for $\varepsilon < 3$ MeV when the effective mass is modeled by Eqs. (5) and (6). This "reduction" is due to decreasing m^*/m in the surface region with increasing ε (see discussion of Fig. 3).

The statistical temperatures [Fig. 2(c)] are found by differentiation of the maximal entropies with respect to excitation energy,

$$T = 1/\left[\frac{\partial s_M}{\partial \varepsilon}\right]. \tag{7}$$

As a result of the extremum condition used to determine the entropies used in Eq. (7), the expansion parameter c is uniquely determined by the excitation energy and the condition of zero external pressure. The latter condition implies that no thermodynamic work is done by nuclear expansion.

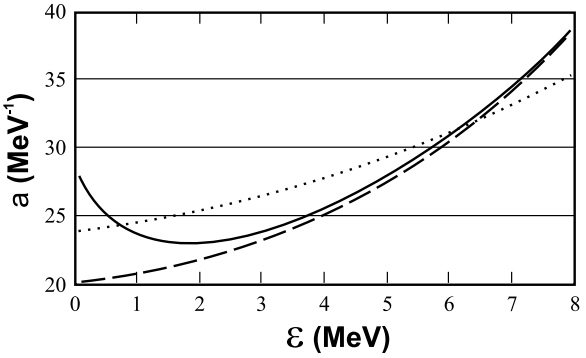

FIG. 3. Evolution of the level-density parameter a with excitation energy per nucleon ε for $m^*/m = 1$ (dotted line), (m_k) (dashed line), and $(m_k)[m_\omega]$ (solid line).

As shown in [15] and implied in [16], the relaxation of the density profile substantially flattens the temperature rise with ε (compare black dotted and solid curves). The inclusion of the effective-mass evolution increases T for $\varepsilon < 3$ MeV and decreases it for $\varepsilon > 5$ MeV, changes that give the appearance of a plateau. Decreasing the compressional energy constant reduces the value of the temperature of the pseudoplateau.

Figure 3 shows how the level-density parameter a evolves along the metastability ridge. Without effective-mass considerations ($m^*/m = 1$, dotted line) a just increases uniformly as the density drops with increasing ε. If only the m_k factor is included (dashed line), a is initially suppressed but grows faster with ε. As the density of states grows with m^*, the increase in the m_k factor provides positive feedback to the expansion process. However, adding the m_ω dependence (solid line) provides a surface enhancement at low excitation, an enhancement which dies by $\varepsilon \sim 2$ MeV. We can therefore make the following three statements. First, consideration of a momentum dependent interaction (m_k factor) is required to predict the expansion rate with ε. Second, as the change in the surface enhancement to a is essential for producing the pseudoplateau, the plateau is a finite-size effect rather than an indication of phase coexistence. Third, the height of the pseudoplateau has to do with the energy cost of expanding the finite system, and thus the surface and the Coulomb energies are both important [26].

What relevance could these metastable mononuclei have to reaction observables? At low energy, the mononuclear density of states of ^{197}Au is a reasonable subject for study because the fission barrier is sufficiently large that the mononuclear lifetime τ_M is longer than the characteristic time for equilibration within the mononuclear region of phase space τ_{eq}. At higher energies,

the time required to thermally populate the extended fission shapes can transiently suppress the fission decay width, increasing the energy region where decay within the mononuclear family dominates the decay [6].

Because of the time required for shape equilibration, the condition $\tau_{eq} < \tau_M$ will be satisfied to higher excitation energies in light ion, π, and $\overline{p}$ induced reactions than for heavy-ion reactions. The decay width of equilibrated mononuclei will be the sum of the widths for decay within the mononuclear family and outside of this family into the multifragmentation channel. However, as is the case with fission, a transient delay (this time associated with the amplification of density fluctuations [9]) is likely to initially suppress the multifragment decay and increase the energy region over which the mononuclear entropy controls the decay process. After the transient delay, the multifragment decay will contribute in proportion to the multifragment density of states. To determine the latter, one needs s_{MF}, the entropy per particle for multifragmentation, which depends on the prescription used for the free volume [27,28]. A comparison of s_M (for the case $m^*/m = 1$, at extracted temperatures) to s_{MF} as a function of volume (taken from the ideal phase space model of Das Gupta and collaborators [29]) indicates the following: (i) The volume capturing 99.75% of the mononuclear matter is much smaller than reasonable volumes for multifragmentation. (ii) s_{MF} will exceed s_M somewhere above $\varepsilon > 3$ MeV if a freeze-out volume of 3 times the unexpanded volume is used. As the freeze-out volume is much larger than the mononuclear volume, mononuclei can still be considered metastable.

Finally, if an excited nucleus expands within the mononucleus family, it can undergo an irreversible transition to the higher entropy multifragmented state. One should expect event averages sampling such a process to yield fluctuations in excess of those for a reversible process. It is therefore reasonable to suspect that the observed excessive fluctuations of the kinetic energy [30] result from an irreversible transition from a lower entropy (larger free energy in a canonical treatment) but kinetically trapped, mononuclear phase to the multifragmented phase. This is equivalent to arguing that the ensemble (the event sample) is nonergodic [31]. However, as discussed above, this transition is not required to explain a plateau in a caloric curve.

This work makes use of several simplifying assumptions. These include the LDA [18], self-similar expansion and the expression for the entropy itself (which ignores the continuum and all detailed quantum structure), all subjects that deserve further study. Nevertheless, this work shows that a near plateau in the caloric curve should be expected for a finite nuclear system due to the evolving influence of the finite range of the interaction and collective effects as the system expands.

L. G. S. would like to acknowledge fruitful discussions with Professor W. Dickhoff, Professor J. Natowitz, and Professor R. Lovett. This work was supported by the U.S. Department of Energy, Division of Nuclear Physics under Grants No. DE-FG02-87ER-40316 and No. DE-FG02-88ER-40414, for Washington University and the University of Rochester, respectively.

[1] V. Weisskopf, Phys. Rev. **52**, 295 (1937).
[2] W. Hauser and H. Feshbach, Phys. Rev. **87**, 366 (1952).
[3] H. Erying, J. Chem. Phys. **3**, 107 (1935).
[4] L. G. Moretto, Nucl. Phys. **A247**, 211 (1975).
[5] R. J. Charity, computer code GEMINI, http://www.chemistry.wustl.edu/~rc (unpublished).
[6] D. Hilscher and H. Rossner, Ann. Phys. (Paris) **17**, 471 (1992).
[7] J.W. Clark and T. P. Wang, Ann. Phys. (N.Y.) **40**, 127 (1966).
[8] B. Friedman and V. R. Pandharipande, Nucl. Phys. **A361**, 502 (1981).
[9] Ph. Chomaz, M. Colonna, and J. Randrup, Phys. Rep. **389**, 263 (2004).
[10] R. Wada *et al.*, Phys. Rev. C **39**, 497 (1989).
[11] J. Pochodzalla *et al.*, Phys. Rev. Lett. **75**, 1040 (1995).
[12] J. Natowitz *et al.*, Phys. Rev. C **65**, 034618 (2002).
[13] C. Mahaux *et al.*, Phys. Rep. **120**, 1 (1985).
[14] C. Mahaux and R. Sartor, Adv. Nucl. Phys. **20**, 1 (1991).
[15] J. Tõke, J. Lu, and W. U. Schröder, Phys. Rev. C **67**, 034609 (2003); **67**, 044307 (2003).
[16] J. Natowitz *et al.*, Phys. Rev. C **66**, 031601(R) (2002).
[17] H. A. Bethe, Phys. Rev. **50**, 332 (1936).
[18] The major problem with the LDA is its sensitivity to the tails of the density profile. The profile chosen should be less sensitive than the standard Woods-Saxon. Our choice of a heavy nucleus also reduces this problem. See S. Shlomo, Nucl. Phys. **A539**, 17 (1992).
[19] M. Barranco and J. Treiner, Nucl. Phys. **A351**, 269 (1981).
[20] M. Prakash, J. Wambach, and Z. Y. Ma, Phys. Lett. B **128**, 141 (1983).
[21] W. A. Friedman, Phys. Rev. Lett. **60**, 2125 (1988).
[22] W. D. Myers and W. J. Swiatecki, Nucl. Phys. **A587**, 92 (1995); Phys. Rev. C **57**, 3020 (1998).
[23] J. N. De, S. Shlomo, and S. Samaddar, Phys. Rev. C **57**, 1398 (1998).
[24] R.W. Hasse and P. Schuck, Phys. Lett. B **179**, 313 (1986).
[25] D. S. Bracken *et al.*, Phys. Rev. C **69**, 034612 (2004); Viola *et al.*, Phys. Rev. Lett. **93**, 132701, (2004).
[26] J. B. Natowitz *et al.*, Phys. Rev. C **66**, 031601 (2003).
[27] J. P. Bondorf *et al.*, Phys. Rep. **257**, 133 (1995).
[28] As no stationary condition is used to determine s_{MF}, the thermodynamics problem is not well posed.
[29] C. B. Das, S. Das Gupta, and A. Z. Mekjian, Phys. Rev. C **67**, 064607 (2003).
[30] M. D'Agostino *et al.*, Nucl. Phys. **A699**, 795 (2002).
[31] W. Thirring, H. Narnhofer, and H. A. Posch, Phys. Rev. Lett. **91**, 130601 (2003).