Quark Structure and Nuclear Effective Forces

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We formulate the quark meson coupling model as a many-body effective Hamiltonian. This leads naturally to the appearance of many-body forces. We investigate the zero range limit of the model and compare its Hartree-Fock Hamiltonian to that corresponding to the Skyrme effective force. By fixing the three parameters of the model to reproduce the binding and symmetry energy of nuclear matter, we find that it allows a very satisfactory interpretation of the Skyrme force.

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The notion that quark degrees of freedom may play a role in low energy nuclear physics is largely unappreciated. The main reason is probably that the many-body formulation of nuclear physics based on point-like nucleons interacting through effective forces has proven quite successful. In this Letter, we take a radically different point of view, arguing that the nuclear effective force itself is a direct manifestation of the quark structure of the nucleon. To this end we formulate the quark meson coupling (QMC) model of the nucleus as a many-body problem. This allows us to take the limit corresponding to a zero range force which can be compared to the Skyrme force [1].

In the QMC model [2,3], the essential step is to solve for the quark structure of the nucleon under the influence of the nuclear environment. For this one considers that, in a time averaged sense, a nucleus can be described as a collection of nonoverlapping quark bags representing the nucleons. (More recently, the same ideas have been extended to a confined version of the Nambu Jona-lasinio model [4].) The interactions of the quarks with the nuclear medium are represented by the exchange of mesons between the quarks of *different* nucleons, with coupling constants treated as free parameters. As explained in Ref. [5], the σ field which is used here is not the chiral partner of the pion, and the quark-σ coupling does not break chiral invariance.

As in our previous work [3], the scalar field is denoted $\sigma(\vec{r})$, while $\omega(\vec{r})$ is the time component of the vector field, and both are taken to be time independent. In the nuclear ground state the time dependence of the fields is driven by the Fermi motion of the nucleons, so the typical frequencies are of the order of the Fermi energy, which can be neglected with respect to the high frequencies of the confined quark fields. Moreover, the space components of the ω field have their source in the velocity density of the nucleons, which is not a coherent quantity—in contrast to the nucleon density $\rho(\vec{r})$, the source of the time component. Therefore, when we solve for the nucleon structure under the influence of the medium, the dominant effect comes from the time component.

Each bag is moving in the classical fields, $\sigma(\vec{r})$, $\omega(\vec{r})$, to which the quarks are coupled. In the spirit of the Born-Oppenheimer approximation, we solve the equations of motion of the quarks in a given bag for a fixed classical position, $\vec{R}(t)$, of its center. For this we use the known Lorentz character of the σ and ω fields to transform to the instantaneous rest frame of the bag, where the static spherical cavity approximation is most appropriate. We then expand the fields around their values at the center of the bag, truncating the expansion at first order. The coupling of the quark to the constant part of the fields is solved exactly, because it amounts to a shift in the quark mass and energy. The remainder is treated as a perturbation. The rest frame energy momentum is then transformed back to the nuclear rest frame with proper account of Thomas spin precession. Keeping only terms which are quadratic in the nucleon velocity, as we do systematically throughout this work, we find the following expression for the classical energy of a nucleon with position momentum $(\vec{R}, \vec{P})$ [3]:

$$E_N(\vec{R}) = \frac{\vec{P}^2}{2M^*(\vec{R})} + M^*(\vec{R}) + g_\omega \omega(\vec{R}) + V_{so}, \tag{1}$$

with g_ω the ω-nucleon coupling constant. The spin-orbit interaction, V_{so}, is defined below. To get the dynamical mass $M^*(\vec{R})$, one has to solve the bag equations in the field $\sigma(\vec{R})$. For our purpose it is sufficient to know that it is well approximated by the expression

$$M^*(\vec{R}) = M - g_\sigma \sigma(\vec{R}) + \frac{d}{2}(g_\sigma \sigma(\vec{R}))^2, \tag{2}$$

where (M, g_σ) are the mass and the σ-nucleon coupling constant for the free nucleon and $d = 0.22R_B$, with R_B the bag radius. The last term, which represents the response of the nucleon to the applied scalar field, is an essential element of the QMC model. From our numerical studies, we know that the approximation (2) is quite accurate up to $g_\sigma \sigma = 400$ MeV, which should be sufficient for our purposes.

The energy (1) is the energy of one particular nucleon moving classically in the nuclear meson fields. Since, by hypothesis, the bags do not overlap, the total energy of the system is the sum of the energy of each of the nucleons plus the energy carried by the fields. As the latter are static, we can write

$$E_{\rm tot} = \sum_i E_N(\vec{R}_i) + E_{\rm mes}, \tag{3}$$

$$E_{\rm mes} = \frac{1}{2}\int d\vec{r}[(\nabla\sigma)^2 + m_\sigma^2\sigma^2 - (\nabla\omega)^2 - m_\omega^2\omega^2], \tag{4}$$

with m_σ, m_ω the masses of the mesons.

To simplify the expression for $E_N(\vec{R})$, we estimate the quantity $g_\sigma\sigma$ using the field equations $\delta E_{\rm tot}/\delta\sigma(\vec{r}) = 0$. Neglecting the velocity dependent terms, setting $M^* \approx M - g_\sigma\sigma$, and neglecting $(\nabla\sigma)^2$ with respect to $m_\sigma^2\sigma^2$, we find $g_\sigma\sigma(\vec{r}) \sim G_\sigma\rho^{\rm cl}(\vec{r})$, with $G_\sigma = g_\sigma^2/m_\sigma^2$ and the classical density is defined as $\rho^{\rm cl}(\vec{r}) = \sum_i\delta(\vec{r} - \vec{R}_i)$.

From our previous studies in the Hartree approximation, $G_\sigma \sim 10\ {\rm fm}^2$, which yields $g_\sigma\sigma \simeq 300$ MeV at nuclear matter density. Making a quadratic expansion of $1/M^*$ in powers of $g_\sigma\sigma$ and keeping the leading terms [6], we obtain

$$\frac{\vec{P}^2}{2M^*(\vec{R})} + M^*(\vec{R}) \approx M + \frac{\vec{P}^2}{2M} - g_\sigma\sigma(\vec{R})\left[1 - \frac{d}{2}g_\sigma\sigma(\vec{R})\right] \times\left(1 - \frac{\vec{P}^2}{2M^2}\right). \tag{5}$$

If we define the scalar density as $\rho_s^{\rm cl}(\vec{r}) = \sum_i(1 - \vec{P}_i^2/2M^2)\delta(\vec{r} - \vec{R}_i)$, we can write the total energy in the form

$$E_{\rm tot} = E_{\rm mes} + \sum_i\left[M + \frac{\vec{P}_i^2}{2M} + V_{\rm so}(i)\right] - \int d\vec{r}\rho_s^{\rm cl}\left[g_\sigma\sigma - \frac{d}{2}(g_\sigma\sigma)^2\right] + \int d\vec{r}\rho^{\rm cl}g_\omega\omega, \tag{6}$$

which will be our starting point for the many-body formulation of the QMC model. We use the equations for the mesons, $\delta E_{\rm tot}/\delta\sigma(\vec{r}) = \delta E_{\rm tot}/\delta\omega(\vec{r}) = 0$, to eliminate the meson fields from the energy, leaving a system whose dynamics depends only on the nucleon coordinates. From Eq. (6) we write the equations for the meson fields in the following forms:

$$g_\sigma\sigma = G_\sigma\rho_s^{\rm cl}(1 - dg_\sigma\sigma) + \nabla^2 g_\sigma\sigma/m_\sigma^2, \tag{7}$$

$$g_\omega\omega = G_\omega\rho^{\rm cl} + \nabla^2 g_\omega\omega/m_\omega^2, \tag{8}$$

where we have defined $G_\omega = g_\omega^2/m_\omega^2$. On the right-hand side of Eqs. (7) and (8) we have neglected the contribution of the functional derivative acting on the spin-orbit term in (6). This is because the latter was obtained as a first order perturbation and one can check that the resulting error in the final Hamiltonian is of higher order. If we insert the solutions $\omega_{\rm sol}(\vec{r})$ and $\sigma_{\rm sol}(\vec{r})$ of Eqs. (7) and (8) in the expression (6) for the energy, we get, after some algebra and omitting the irrelevant constant mass term,

$$E_{\rm tot} = \sum_i\left[\frac{\vec{P}_i^2}{2M} + V_{\rm so}(i)\right] - \frac{1}{2}\int d\vec{r}\rho_s^{\rm cl}g_\sigma\sigma_{\rm sol} + \frac{1}{2} \times \int d\vec{r}\rho^{\rm cl}g_\omega\omega_{\rm sol}. \tag{9}$$

We do not attempt to use the exact solutions of Eqs. (7) and (8) as this would lead to an intricate many-body problem that would be difficult to compare with standard nuclear physics approaches. Instead we first remark that, roughly speaking, the meson fields should follow the matter density. Therefore the typical scale for the ∇ operator acting on σ or ω is the thickness of the nuclear surface, which is about 1 fm. In so far as $1\ {\rm fm}^{-2} \ll (m_\sigma^2, m_\omega^2)$, which looks reasonable, we can consider the terms $\nabla^2 g_\sigma\sigma/m_\sigma^2$ and $\nabla^2 g_\omega\omega/m_\omega^2$ as perturbations and, *in these terms*, replace σ and ω by their first order approximation, that is, by $g_\sigma\sigma \approx G_\sigma\rho_s^{\rm cl}$ and $g_\omega\omega \approx G_\omega\rho^{\rm cl}$. The next step in solving for the σ field is to solve Eq. (7) iteratively, starting from the lowest order approximation, $g_\sigma\sigma = G_\sigma\rho_s^{\rm cl}$. When inserted into Eq. (9), this series will generate N-body forces with convergence controlled by the parameter $dg_\sigma\sigma \simeq 0.33$, according to our estimate. To simplify further we shall neglect the small difference between $\rho_s^{\rm cl}$ and $\rho^{\rm cl}$, except in the leading term. These approximations will not be difficult to improve, but, as this leads inevitably to an effective interaction which is more complicated than the simple Skyrme force, we postpone this to future investigations. In summary, the expressions we shall use for the field solutions are

$$g_\sigma\sigma_{\rm sol}(\vec{r}) = \frac{G_\sigma}{m_\sigma^2}\nabla^2\rho^{\rm cl} + G_\sigma\rho_s^{\rm cl} + \sum_{k\geq 1}(-d)^k(G_\sigma\rho^{\rm cl})^{k+1}, \tag{10}$$

$$g_\omega\omega_{\rm sol}(\vec{r}) = \frac{G_\omega}{m_\omega^2}\nabla^2\rho^{\rm cl} + G_\omega\rho^{\rm cl}. \tag{11}$$

The rest of the derivation amounts to substituting $g_\omega\omega_{\rm sol}$ and $g_\sigma\sigma_{\rm sol}$ into Eq. (9) for the energy. As usual the density and the scalar density to some power contain infinite terms corresponding to the self-interaction of the nucleon. Since our model is devised to describe the modification of the nucleon by the medium rather than the nucleon itself, we simply remove them. This amounts to the replacements $[\sum_i\delta(\vec{r} - \vec{R}_i)]^2 \to \sum_{i\neq j}\delta(\vec{r} - \vec{R}_i)\delta(\vec{r} - \vec{R}_j)$, which leads to the following many-body Hamiltonian, essentially equivalent to the QMC model:

$$H_{\rm QMC} = \sum_i\left[\frac{\vec{P}_i^2}{2M} + V_{\rm so}(i)\right] + \frac{G_\sigma}{2}\sum_{i\neq j}\frac{\vec{P}_i^2}{M^2}\delta(\vec{R}_{ij}) + \frac{G_\omega}{2}\sum_{i\neq j}\left[\delta(\vec{R}_{ij}) + \frac{1}{m_\omega^2}\nabla_i^2\delta(\vec{R}_{ij})\right] - \frac{G_\sigma}{2}\sum_{i\neq j}\left[\delta(\vec{R}_{ij}) + \frac{1}{m_\sigma^2}\nabla_i^2\delta(\vec{R}_{ij})\right] + \frac{dG_\sigma^2}{2}\sum_{i\neq j\neq k}\delta^2(ijk) - \frac{d^2G_\sigma^3}{2}\sum_{i\neq j\neq k\neq l}\delta^3(ijkl). \tag{12}$$

Here $\vec{R}_{ij} = \vec{R}_i - \vec{R}_j$ and ∇_i is the gradient with respect to $\vec{R}_i$. In Eq. (12) we have dropped the contact interactions involving more than 4-bodies, because their matrix elements vanish for antisymmetrized states. To shorten the equations, we used the notation $\delta^2(ijk)$ for $\delta(\vec{R}_{ij})\delta(\vec{R}_{jk})$ and analogously for $\delta^3(ijkl)$. For the spin-orbit interaction, we start from our previous result [3]:

$$\sum_i V_{\mathrm{so}}(i) = \sum_i \frac{1}{4M^{*2}(\vec{R}_i)} \vec{P}_i \times \nabla_i W(\vec{R}_i) \cdot \vec{\sigma}_i, \tag{13}$$

where $W(\vec{R}_i) = M^*(\vec{R}_i) + g_\omega \omega(\vec{R}_i)(1 - 2\mu_s)$, $\mu_s = 0.9$ is the isoscalar magnetic moment, and $\vec{\sigma}_i$ are the Pauli matrices. As this expression was derived as a first order approximation, it is consistent to evaluate it to the same order. So, on the right-hand side of Eq. (13), we replace M^* by M in the denominator, we approximate $M^* = M - g_\sigma \sigma$, and we use the leading approximations for the meson fields.

The final step is to quantize the classical Hamiltonian (12) by making the replacement $\vec{P}_i \rightarrow -i\nabla_i$. As usual we must deal with the ordering ambiguity which exists as soon as velocity dependent interactions are present. For the spin-orbit interaction there is no ambiguity because all orderings give the same matrix elements. The problem occurs only in the second term of Eq. (12). There are two possible Hermitian orderings when $\vec{P}_i$ becomes an operator acting on the right: $T_1 = [\vec{P}_i^2 \delta(\vec{R}_{ij}) + \delta(\vec{R}_{ij})\vec{P}_i^2]/2$ and $T_2 = \vec{P}_i \delta(\vec{R}_i - \vec{R}_j) \cdot \vec{P}_i$. However, using integration by parts and the commutation rules, one checks easily that the difference between the two orderings is of the form $\nabla_i^2 \delta(\vec{R}_{ij})$. Such an operator is already present in H_{QMC} in the third and fourth terms of Eq. (12), and we see that choosing one ordering or the other is equivalent to a change of the meson masses. In practice we tried both orderings and checked that this is equivalent to a 50 MeV change of m_σ. Since, in any case, we intend to study the sensitivity of our results to m_σ, choosing T_1 or T_2 is immaterial and for definiteness we adopt the form T_2.

To complete our effective Hamiltonian we now include the effect of the isovector ρ meson, which can be done by analogy with the ω meson. If we let $b^\alpha (\alpha = 1, 2, 3)$ be the time component of the field and τ^α be the isospin Pauli matrices, then the only changes are the replacement $g_\omega \omega(\vec{R}) \rightarrow g_\omega \omega(\vec{R}) + g_\rho \vec{b}(\vec{R}) \cdot \vec{\tau}/2$ in the expression (1) for the nucleon energy and $g_\omega \omega(\vec{R})(1 - 2\mu_s) \rightarrow g_\omega \omega(\vec{R}) \times (1 - 2\mu_s) + g_\rho(1 - 2\mu_v)\vec{b}(\vec{R}) \cdot \vec{\tau}/2$ in the expression (13) for the spin-orbit interaction, with g_ρ the free ρ-nucleon coupling constant and $\mu_v = 4.7$ the nucleon isovector magnetic moment. If we define $G_\rho = g_\rho^2/m_\rho^2$ with m_ρ the mass of the ρ meson, our quantum effective Hamiltonian finally takes the form

$$H_{\mathrm{QMC}} = \sum_i \frac{\overleftarrow{\nabla}_i \cdot \vec{\nabla}_i}{2M} + \frac{G_\sigma}{2M^2} \sum_{i \neq j} \overleftarrow{\nabla}_i \delta(\vec{R}_{ij}) \cdot \vec{\nabla}_i + \frac{1}{2} \sum_{i \neq j} [\nabla_i^2 \delta(\vec{R}_{ij})] \left[\frac{G_\omega}{m_\omega^2} - \frac{G_\sigma}{m_\sigma^2} + \frac{G_\rho}{m_\rho^2} \frac{\vec{\tau}_i \cdot \vec{\tau}_j}{4} \right] + \frac{1}{2} \sum_{i \neq j} \delta(\vec{R}_{ij}) \left[G_\omega - G_\sigma + G_\rho \frac{\vec{\tau}_i \cdot \vec{\tau}_j}{4} \right]$$
$$+ \frac{dG_\sigma^2}{2} \sum_{i \neq j \neq k} \delta^2(ijk) - \frac{d^2 G_\sigma^3}{2} \sum_{i \neq j \neq k \neq l} \delta^3(ijkl) + \frac{i}{4M^2} \sum_{i \neq j} A_{ij} \overleftarrow{\nabla}_i \delta(\vec{R}_{ij}) \times \vec{\nabla}_i \cdot \vec{\sigma}_i, \tag{14}$$

with $A_{ij} = G_\sigma + (2\mu_s - 1)G_\omega + (2\mu_v - 1)G_\rho \vec{\tau}_i \cdot \vec{\tau}_j/4$.

To fix the free parameters of the model, that is G_σ, G_ω, and G_ρ, we have computed, using the Hamiltonian (14), the volume and symmetry coefficients of the binding energy per nucleon of infinite nuclear matter: $E_B/A = a_1 + a_4(N - Z)^2/A^2$. We have used the experimental values $a_1 = -15.85$ MeV, $a_4 = 30$ MeV, and the saturation condition $\partial a_1/\partial \rho(\rho_0) = 0$, with $\rho_0 = 0.16\ \mathrm{fm}^{-3}$. In order to avoid the proliferation of tables, we show only the results corresponding to the bag radius $R_B = 0.8$ fm, which is realistic. We have used the physical masses, $m_\omega = 782$ MeV and $m_\rho = 770$ MeV, and we allow m_σ to take the values 500 and 600 MeV, which is a commonly accepted range. We get, in fm^2, $G_\sigma = 12.63$, $G_\omega = 9.62$, and $G_\rho = 9.68$ for $m_\sigma = 500$ MeV and $G_\sigma = 11.97$, $G_\omega = 8.1$, and $G_\rho = 6.46$ for $m_\sigma = 600$ MeV. These values are larger than in the Hartree approximation because the exchange terms tend to cancel the direct terms of the matrix elements, thereby forcing larger couplings to fit the data.

As a practical test of the capacity of our model to interpret a large body of nuclear data, we compare it with the effective Skyrme interaction. Since, in our formulation, the medium effects are summarized in the 3- and 4-body forces, we consider Skyrme forces of the same type, that is, without density dependent interactions. They are defined by a potential energy of the form

$$V = t_3 \sum_{i<j<k} \delta(\vec{R}_{ij})\delta(\vec{R}_{jk}) + \sum_{i<j} \Big\{ t_0(1 + x_0 P_\sigma)\delta(\vec{R}_{ij})$$
$$+ \frac{1}{4} t_2 \overleftarrow{\nabla}_{ij} \cdot \delta(\vec{R}_{ij}) \vec{\nabla}_{ij} - \frac{1}{8} t_1 [\delta(\vec{R}_{ij}) \vec{\nabla}_{ij}^2 + \overleftarrow{\nabla}_{ij}^2 \delta(\vec{R}_{ij})]$$
$$+ \frac{i}{4} W_0(\vec{\sigma}_i + \vec{\sigma}_j) \cdot \overleftarrow{\nabla}_{ij} \times \delta(\vec{R}_{ij}) \vec{\nabla}_{ij}^2 \Big\}, \tag{15}$$

with $\nabla_{ij} = \nabla_i - \nabla_j$. There is no 4-body force in Eq. (15) and we shall show its possible impact by setting its strength equal to zero in H_{QMC}. Since the spin exchange operator, P_σ, in V multiplies a contact interaction, its action on an antisymmetric state is equivalent to minus the isospin exchange operator. Comparison of Eq. (15) with the QMC Hamiltonian (14) allows one to identify

TABLE I. QMC predictions compared with the Skyrme force.

	QMC	QMC	SkIII	QMC($N = 3$)
m_σ(MeV)	500	600		600
t_0(MeV fm^3)	−1071	−1082	−1129	−1047
x_0	0.89	0.59	0.45	0.61
t_3(MeV fm^6)	16 620	14 926	14 000	12 513
M_{eff}/M	0.915	0.814	0.763	0.821
$5t_2 - 9t_1$(MeV fm^5)	−7622	−4330	−4030	−4036
W_0(MeV fm^5)	118	97	120	91
K(MeV)	327	327	355	364

$$t_0 = -G_\sigma + G_\omega - \frac{G_\rho}{4}, \quad t_3 = 3dG_\sigma^2, \quad x_0 = -\frac{G_\rho}{2t_0}. \tag{16}$$

For the other parameters, we cannot make a direct identification because, as our effective Hamiltonian is derived in the rest frame of the nucleus, its momentum dependent pieces violate Galilean invariance. This is irrelevant since it is devised for variational calculations where the trial state also violates Galilean invariance, but to make the identification we need to compare the respective Hartree-Fock Hamiltonians rather than the interactions themselves. To this end we make some simplifying assumptions which do not significantly damage the physics but avoid unnecessary technical complications. First, we restrict our considerations to doubly closed shell nuclei with $N = Z$. Second, we assume that one can neglect the difference between the radial wave functions of the single particle states with $j = l + 1/2$ and $j = l - 1/2$. This amounts to treating the spin-orbit interaction to first order, which is sufficient for our purposes. By comparing the Hartree-Fock Hamiltonian obtained from H_{QMC} and that of Ref. [7] corresponding to the Skyrme force, we obtain the relations

$$3t_1 + 5t_2 = \frac{8G_\sigma}{M^2} + 4\left(\frac{G_\omega}{m_\omega^2} - \frac{G_\sigma}{m_\sigma^2}\right) + 3\frac{G_\rho}{m_\rho^2}, \tag{17}$$

$$5t_2 - 9t_1 = \frac{2G_\sigma}{M^2} + 28\left(\frac{G_\omega}{m_\omega^2} - \frac{G_\sigma}{m_\sigma^2}\right) - 3\frac{G_\rho}{m_\rho^2}, \tag{18}$$

$$W_0 = \frac{1}{12M^2}\left[5G_\sigma + 5(2\mu_s - 1)G_\omega + \frac{3}{4}(2\mu_v - 1)G_\rho\right]. \tag{19}$$

In Table I we compare our results with the parameters of the force SkIII [8], which is considered a good representative of density independent effective interactions. We postpone a more extensive comparison to future work. We show the combinations $M_{\mathrm{eff}}/M = [1 + (3t_1 + 5t_2)M\rho_0/8]^{-1}$ and $5t_2 - 9t_1$, which are more pertinent than t_1, t_2 individually [7]. These combinations are quite sensitive to the σ mass. This is not unexpected since t_1, t_2 correspond to pieces of the Skyrme force which mock up the finite range of the interaction. Even bearing in mind that, since we use the parameters (a_1, a_4, ρ_0) as input, not all the numbers in Table I are predictions, we see that the level of agreement with SkIII, for $m_\sigma = 600$ MeV, is still impressive. An important point is that the spin-orbit strength, W_0, comes out with approximately the correct value, independent of the σ mass. The last column ($N = 3$) shows our results when we switch off the 4-body force. The main change is a decrease of the predicted 3-body force. Clearly this mocks up the effect of the attractive 4-body force which may then appear less important. However, this is misleading if we look at the compressibility of nuclear matter, $K = 9\rho^2\partial^2 a_1/\partial\rho^2$ (last row of Table I), which decreases by as much as 37MeV when we restore this 4-body force. The value we find, $K = 327$ MeV, is still a little too large with respect to the experimental range (200–300 MeV), but several simplifications made for this presentation can be eliminated in future work. Moreover, we have not yet included the long-range force of the pion. According to a preliminary calculation, it can reduce K by 20 MeV and M_{eff} by 70 MeV. This too will be investigated in future work [9].

In summary, we have demonstrated a remarkable agreement between the phenomenologically successful Skyrme force, SkIII, and the effective interaction corresponding to the quark meson coupling model—a result which suggests that the response of nucleon internal structure to the nuclear medium does indeed play a vital in nuclear structure.

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