Melting Pattern of Diquark Condensates in Quark Matter

K. Iida,[1] T. Matsuura,[2] M. Tachibana,[1] and T. Hatsuda[2]

[1]The Institute of Physical and Chemical Research (RIKEN), Wako, Saitama 351-0198, Japan

[2]Department of Physics, University of Tokyo, Tokyo 113-0033, Japan

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Thermal color superconducting phase transitions in high density three-flavor quark matter are investigated in the Ginzburg-Landau approach. The effects of nonzero strange quark mass, electric and color charge neutrality, and direct instantons are considered. Weak coupling calculations show that an interplay between the mass and electric neutrality effects near the critical temperature gives rise to three successive second-order phase transitions as the temperature increases: a modified color-flavor locked (mCFL) phase (*ud*, *ds*, and *us* pairings) → a *d*-quark superconducting (dSC) phase (*ud* and *ds* pairings) → an isoscalar pairing phase (*ud* pairing) → a normal phase (no pairing). The dSC phase is novel in the sense that while all eight gluons are Meissner screened as in the mCFL phase, three out of nine quark quasiparticles are always gapless.

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Unraveling the phase structure at high baryon density is one of the most challenging problems in quantum chromodynamics (QCD). Among others, color superconductivity in cold dense quark matter has been discussed from various viewpoints [1,2]. In relation to real systems, such as newly born compact stars in stellar collapse, it is important to study the color superconductivity not only as a function of the quark chemical potential μ but also as a function of the temperature T. This is because the possible presence of color superconducting quark matter affects the thermal evolution of the compact stars [3].

The purpose of this Letter is to investigate phase transitions in color superconducting quark matter with three flavors (*uds*) and three colors (RGB). We consider a realistic situation with a nonzero strange quark mass m_s, electric and color charge neutrality, and direct instantons effect. In weak coupling (m_s, $\Lambda_{\rm QCD} \ll \mu$), the effects of nonzero m_s and electric neutrality are particularly important since they induce multiple phase transitions in which the pattern of diquark pairing changes as T increases. We find a new phase, which we call *d*-quark superconductivity (dSC), as an interface between a modified color-flavor locked (mCFL) phase and an isoscalar two-flavor superconducting (2SC) phase. Our analysis is expected to give a plausible idea of what may occur in the strong coupling regime of physical interest continuously connected to the weak coupling regime.

Throughout this Letter, we adopt the Ginzburg-Landau (GL) approach near the transition temperatures, which was previously used to study the massless three-flavor case [4–6]. In a realistic situation, the GL potential acquires the following corrections. First of all, nonzero quark mass m_i ($i = u, d, s$) affects the GL potential through the quark propagator [7] in such a way as to lower the critical temperature of the ij pairing with a larger value of $(m_i^2 + m_j^2)/2$. Second, the chemical potential shift $\delta\mu_i$ ($i = u, d, s$) for electrically neutral and beta-equilibrated system with a gas of electrons affects the GL potential so as to lower the critical temperature of the ij pairing with a smaller value of $(\delta\mu_i + \delta\mu_j)/2$. In both cases, a change of the density of states at the Fermi surface plays a crucial role. Third, the instanton contribution gives an m_i dependence to the GL potential through the effective four-fermion interaction. The color neutrality makes a negligible correction to the GL potential near the transition temperatures as shown in [4].

We assume that diquark pairing in weak coupling takes place in the most attractive color-flavor antisymmetric channel with $J^P = 0^+$ [8]. Then, the pairing gap of a quark with (color, flavor) = (b, j) and that with (c, k) reads $\phi_{bcjk} = \epsilon_{abc}\epsilon_{ijk}(\mathbf{d}_a)^i$ [4]. The 3×3 matrix $(\mathbf{d}_a)^i$ transforms as a vector under $G = {\rm SU}(3)_C \times {\rm SU}(3)_{L+R} \times {\rm U}(1)_B$ and belongs to the $(3^*, 3^*)$ representation of ${\rm SU}(3)_C \times {\rm SU}(3)_{L+R}$.

The GL potential near the critical temperature T_c for a homogeneous system with $m_{u,d,s} = 0$ is invariant under ${\rm SU}(3)_C \times {\rm SU}(3)_{L+R} \times {\rm U}(1)_B$ and is given by [4,9]

$$S = \bar{\alpha}\sum_a |\mathbf{d}_a|^2 + \beta_1\Big(\sum_a |\mathbf{d}_a|^2\Big)^2 + \beta_2\sum_{ab}|\mathbf{d}_a^* \cdot \mathbf{d}_b|^2, \quad (1)$$

where $(\mathbf{d}_a)^i = (d_a^u, d_a^d, d_a^s)$ and the inner product is taken for flavor indices. In weak coupling [4],

$$\beta_{1,2} = \frac{7\zeta(3)}{8(\pi T_c)^2}N(\mu) \equiv \beta, \qquad \bar{\alpha} = 4N(\mu)t \equiv \alpha_0 t, \quad (2)$$

where $N(\mu) = \mu^2/2\pi^2$ is the density of states at the Fermi surface, and $t = (T - T_c)/T_c$. With the parameters in Eq. (2), one finds a second-order phase transition at $T = T_c$ from the color-flavor locked (CFL) phase ($d_a^i \propto \delta_a^i$) to the normal phase ($d_a^i = 0$) in mean-field theory [4].

Let us now consider the effect of nonzero quark masses in the quark propagator on the GL potential. Although we have derived a general formula for $m_i \neq 0$ [10], we assume $m_{u,d} \ll m_s$ and set $m_{u,d} = 0$ in the following discussions. We consider the high density regime, $\mu \gg m_s \neq 0$. Near T_c the leading effect of m_s is to modify the quadratic term in the GL potential (1). Since m_s affects only *us* and *ds* pairings, the correction reads

$$\epsilon \sum_a (|d_a^u|^2 + |d_a^d|^2) = \epsilon \sum_a (|\mathbf{d}_a|^2 - |d_a^s|^2), \tag{3}$$

which induces an asymmetry in the flavor structure of the CFL phase. The corrections to the quartic terms from m_s are subleading and can be neglected in the following analysis [11].

In weak coupling, ϵ can be calculated by taking m_s into account in the Nambu-Gor'kov quark propagator. Following Ref. [4], we expand the GL potential not only in d_a^i but also in m_s up to $\mathcal{O}(m_s^2)$, and obtain

$$\epsilon \simeq \alpha_0 \frac{m_s^2}{4\mu^2} \ln\left(\frac{\mu}{T_c}\right) \sim 2\alpha_0 \sigma. \tag{4}$$

Here the dimensionless parameter σ is defined as

$$\sigma = \left(\frac{3\pi^2}{8\sqrt{2}}\right) \frac{m_s^2}{g\mu^2}, \tag{5}$$

with g being the QCD coupling constant. Our weak coupling analysis of the GL potential near T_c is valid as long as $\sigma \ll 1$, $g \ll 1$, and $T_c \ll g\mu$, which are relevant at asymptotically high density. The last inequality is related to the color neutrality mentioned below. In Eq. (4) we have used a weak coupling relation, $\ln(T_c/\mu) \sim -3\pi^2/(\sqrt{2}g)$, which originates from the long-range color magnetic interaction [8]. Since the finite m_s decreases the density of states of the s quarks at the Fermi surface, ϵ becomes positive such that *ud* pairing is favored over *us* and *ds* pairings. Consequently, the CFL phase becomes asymmetric in flavor space and its critical temperature is lowered, leading to the appearance of the 2SC phase ($d_a^i \propto \delta^{is}$) just below T_c [7]. For general m_i, the density of states becomes smaller for larger value of $(m_i^2 + m_j^2)/2$. We also note that T_c itself is modified by m_s through the modification of the normal medium as $T_c(1 + \mathcal{O}(g\sigma))$.

We turn to discuss the effect of charge neutrality. Under beta equilibrium and charge neutrality, the electron chemical potential μ_e and $\delta\mu_i$ (the shift of the chemical potential from the average, μ) are related by $\delta\mu_i = -q_i\mu_e$ with q_i being the electric charge. From Ref. [4], one finds the correction to the GL potential due to $\delta\mu_i$ as

$$\eta\left(\frac{1}{3}\sum_a |\mathbf{d}_a|^2 - \sum_a |d_a^u|^2\right). \tag{6}$$

In weak coupling, one may regard normal quark matter and electrons as noninteracting Fermi gases and we have $\mu_e \simeq m_s^2/4\mu$. This estimate is valid in the vicinity of T_c since the corrections to μ_e from finite pairing gaps enter only the quartic terms of the GL potential. By combining this with the weak coupling expression for η given in Ref. [4], we obtain

$$\eta \simeq \alpha_0 \frac{m_s^2}{8\mu^2} \ln\left(\frac{\mu}{T_c}\right) \sim \alpha_0 \sigma. \tag{7}$$

Since $\eta > 0$, *ds* pairing is more favorable than *ud* and *us* pairings. This feature stems from the modification of the density of states by $\delta\mu_i$. The critical temperature for ij pairing is increased (decreased) when $\delta\mu_i + \delta\mu_j > 0 (< 0)$.

We consider color neutrality of the system as well. In contrast to the case at $T = 0$, however, it affects only the quartic terms in the GL potential through possible chemical potential differences among colors [4,12]. In weak coupling its magnitude is suppressed by $\mathcal{O}((T_c/g\mu)^2)$ compared to the leading quartic terms. Assuming $T_c \ll g\mu$ in weak coupling, color neutrality has no essential consequence to the phase transitions considered in this Letter. The charge neutrality is different from the color neutrality in the sense that the former shifts the quark chemical potentials even in the normal phase, while the latter works only when the pairing gap is finite.

The direct instanton at nonzero m_s, which induces an effective four-fermion interaction between u and d quarks [13], leads to a quadratic term in the GL potential, $\zeta \sum_a |d_a^s|^2$. An explicit weak coupling calculation shows that $\zeta \sim -\alpha_0 (m_s/\mu)(\Lambda_{\rm QCD}/\mu)^9 (1/g)^{14}$. The negative sign indicates that the instanton effect favors *ud* pairing as does Eq. (3), but the magnitude of ζ is highly suppressed at high densities. Therefore, we ignore instanton effects below.

Since the two effects of nonzero m_s, characterized by Eqs. (3) and (6), favor *ud* pairing and *ds* pairing, respectively, the finite temperature transition from the CFL to the normal phase at $m_s = 0$ is significantly modified. In fact, successive color-flavor unlockings take place instead of a simultaneous unlocking of all color-flavor combinations. To describe this *hierarchical thermal unlocking*, it is convenient to introduce a parametrization,

$$d_a^i = \mathrm{diag}(\Delta_1, \Delta_2, \Delta_3), \tag{8}$$

where $\Delta_{1,2,3}$ are assumed without loss of generality to be real. We also name the phases for later convenience as

$$\begin{aligned} &\Delta_{1,2,3} \neq 0 && : \text{mCFL}, \\ &\Delta_1 = 0, \quad \Delta_{2,3} \neq 0 && : \text{uSC}, \\ &\Delta_2 = 0, \quad \Delta_{1,3} \neq 0 && : \text{dSC}, \\ &\Delta_{1,2} = 0, \quad \Delta_3 \neq 0 && : \text{2SC}, \end{aligned} \tag{9}$$

where dSC (uSC) stands for superconductivity in which for d (u) quarks all three colors are involved in the pairing. In terms of the parametrization (8), the GL potential with corrections of $\mathcal{O}(m_s^2)$ to the quadratic term, Eqs. (3) and (6), reads

$$S = \bar{\alpha}'(\Delta_1^2 + \Delta_2^2 + \Delta_3^2) - \epsilon\Delta_3^2 - \eta\Delta_1^2 + \beta_1(\Delta_1^2 + \Delta_2^2 + \Delta_3^2)^2 + \beta_2(\Delta_1^4 + \Delta_2^4 + \Delta_3^4), \quad (10)$$

where $\bar{\alpha}' = \bar{\alpha} + \epsilon + \frac{\eta}{3}$.

We proceed to analyze the phase structure dictated by Eq. (10) with the weak coupling parameters (2), (4), and (7) up to leading order in g. In Figs. 1 and 2 the results obtained by solving the coupled algebraic equations, $\partial S/\partial\Delta_{1,2,3} = 0$, are summarized. Figure 1(a) shows the second-order phase transition, CFL $\rightarrow$ normal for $m_s = 0$. Figures 1(b) and 1(c) represent how the phase transitions and their critical temperatures bifurcate as we introduce (b) effects of a nonzero m_s in the quark propagator and then (c) effects of charge neutrality. In case (b), two second-order phase transitions arise: mCFL (with $\Delta_1 = \Delta_2$) $\rightarrow$ 2SC at $T = T_c^s \equiv (1 - 4\sigma)T_c$, and 2SC $\rightarrow$ normal at $T = T_c$. In case (c), there arise three successive second-order phase transitions, mCFL $\rightarrow$ dSC at $T = T_c^{\rm I}$, dSC $\rightarrow$ 2SC at $T = T_c^{\rm II}$, and 2SC $\rightarrow$ normal at $T = T_c^{\rm III}$. Shown in Fig. 2 is the T dependence of the gaps $\Delta_{1,2,3}$ for case (c). All the gaps are continuous functions of T, but their slopes are discontinuous at the critical points, which reflects the second-order nature of the transitions in the mean-field treatment of Eq. (10).

We may understand the bifurcation of the transition temperatures as follows. In the massless case (a), T_c is degenerate between the CFL and 2SC phases, the chemical potential is common to all three flavors and colors, and the CFL phase is more favorable than the 2SC phase below T_c. As one goes from (a) to (b), the density of states of the s quarks at the Fermi surface is reduced. Then the critical temperature for the CFL phase is lowered and the 2SC phase is allowed to appear at temperatures between T_c^s and T_c. As one goes from (b) to (c), the average chemical potential of ds (ud and us) quarks increases (decreases). Accordingly, the transition temperatures further change from T_c to $T_c^{\rm III}$ and from T_c^s to $T_c^{\rm I}$ and $T_c^{\rm II}$.

Now we examine in more detail how the color-flavor unlocking in case (c) proceeds with increasing T from the region below $T_c^{\rm I}$.

(i) Just below $T_c^{\rm I}$, we have a CFL-like phase, but the three gaps take different values, with an order $\Delta_3 > \Delta_1 > \Delta_2 \neq 0$ (the mCFL phase). The reason why this order is realized can be understood from the ϵ term and η term in Eq. (10): us pairing (Δ_2) is disfavored by both terms and, since $\epsilon > \eta(>0)$, ds pairing (Δ_1) is destabilized more effectively than ud pairing (Δ_3). The value of each gap in the mCFL phase reads $\Delta_i^2 = (\alpha_0/8\beta)((T_c - T)/T_c + c_i\sigma)$ with $c_{1,2,3} = (-4/3, -16/3, 8/3)$. The mCFL phase has only a global symmetry $\mathrm{U}(1)_{C+L+R} \times \mathrm{U}(1)_{C+L+R}$ in contrast to the global symmetry $\mathrm{SU}(3)_{C+L+R}$ in the CFL phase with $m_{u,d,s} = 0$. Generally, there are no gapless quark excitations in both mCFL and CFL phases. As T increases, the first unlocking transition, the unlocking of Δ_2 (the pairing between Bu and Rs quarks), takes place at

$$T_c^{\rm I} = (1 - (16/3)\sigma)T_c. \quad (11)$$

(ii) For $T_c^{\rm I} < T < T_c^{\rm II}$, $\Delta_2 = 0$ and $\Delta_i^2 = (\alpha_0/6\beta)((T_c - T)/T_c + c_i\sigma)$ with $c_{1,3} = (-7/3, 2/3)$. In this phase, we have only ud and ds pairings (the dSC phase), and there is a manifest symmetry, $U(1)_{C+L+R} \times U(1)_{C+L+R} \times U(1)_{C+V+B} \times U(1)_{C+V+B}$. By diagonalizing the squared 9×9 gap matrix in color-flavor space, we find three gapless quark excitations in the color-flavor combinations: Bu, Rs, and a linear combination of Ru and Bs. The second unlocking transition, the unlocking of Δ_1 (the pairing between Gs and Bd quarks), takes place at

$$T_c^{\rm II} = (1 - (7/3)\sigma)T_c. \quad (12)$$

(iii) For $T_c^{\rm II} < T < T_c^{\rm III}$, one finds the 2SC phase, which has only ud pairing with $\Delta_3^2 = (\alpha_0/4\beta)((T_c - T)/T_c - \frac{1}{3}\sigma)$. The 2SC phase has a symmetry $\mathrm{SU}(2)_C \times \mathrm{SU}(2)_{L+R} \times \mathrm{U}(1)_{C+B} \times \mathrm{U}(1)_{L+R+B}$ [6]. In this phase the

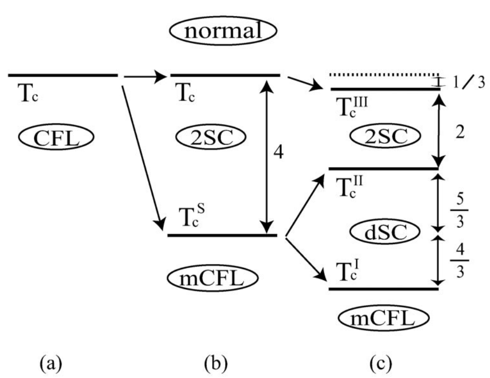

FIG. 1. Transition temperatures of the three-flavor color superconductor in weak coupling: (a) all quarks are massless; (b) nonzero m_s in the quark propagator is considered; (c) electric charge neutrality is further imposed. The numbers attached to the arrows are in units of σT_c.

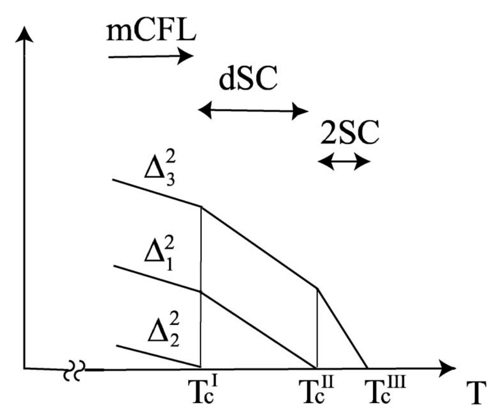

FIG. 2. A schematic illustration of the gaps squared as a function of T.

TABLE I. The symmetry, the quark modes always gapless, and the number of Meissner screened gluons in the mCFL, dSC, and 2SC phases. The gapless quark mode (Ru, Bs) denotes the linear combination of Ru and Bs quarks.

	Symmetry	Gapless quark modes	Number of massive gluons
mCFL	$[U(1)]^2$	None	8
dSC	$[U(1)]^4$	Bu, Rs (Ru, Bs)	8
2SC	$[SU(2)]^2 \times [U(1)]^2$	Bu, Bd, Bs Rs, Gs	5

s quark and B quark excitations are always gapless. The third unlocking transition, the unlocking of Δ_3 (the pairing between Rd and Gu quarks), occurs at

$$T_c^{\rm III} = (1 - (1/3)\sigma)T_c. \tag{13}$$

Above $T_c^{\rm III}$, the system is in the normal phase.

In Table I, we summarize the symmetry and the gapless quark modes in each phase discussed above. The number of gluons having nonzero Meissner masses, which is related to the remaining color symmetry, is also shown [10]. We note that more gapless quark modes may appear if the system is in the close vicinity of $T_c^{\rm I}$, $T_c^{\rm II}$, and $T_c^{\rm III}$ where Δ_2, Δ_1, and Δ_3 are less than $\mathcal{O}(m_s^2/\mu)$ [14]. We also note that near the lower end of the density region where the present GL analysis is valid, it is possible that all the gaps are smaller than $\mathcal{O}(m_s^2/\mu)$ between $T_c^{\rm I}$ and $T_c^{\rm III}$.

So far, we have studied the phase transitions in the mean-field level. In weak coupling, thermally fluctuating gauge fields could change the order of the transitions described in Figs. 1 and 2 [6]. We recapitulate some of the results here with the detailed account left for future publication [10]. First, the second-order transition, mCFL → dSC, remains second order even in the presence of the thermal gluon fluctuations. This is because all eight gluons are Meissner screened at $T = T_c^{\rm I}$ and thus cannot induce a cubic term with respect to the order parameter in the GL potential. On the other hand, the transitions, dSC → 2SC and 2SC → normal, become weak first order since some gluons, which are massless in the high temperature phase, become Meissner screened in the low temperature phase (Table I).

In summary, we have investigated color-flavor unlockings at finite temperatures taking into account the strange quark mass and charge neutrality in the GL approach at high density. We find three successive unlocking transitions, mCFL → dSC → 2SC → normal. In the novel dSC phase, all eight gluons are Meissner screened and the three quark excitations are always gapless. The question of how the phase structure we obtained near T_c is connected to that at $T = 0$, is an interesting open problem. At $T = 0$, color-flavor unlocking due to the Fermi momentum *mismatch* between paired quarks is expected at $\mu \sim m_s^2/T_c$ [15]. This unlocking is different in mechanism from our unlocking near T_c where the Fermi momentum *average* is important. Clarification of the color-flavor unlocking lines in a wide range of the T-μ plane requires analysis with both mechanisms.

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