Nonequilibrium Dynamics and Thermodynamics of a Degenerate Fermi Gas Across a Feshbach Resonance

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We consider a two-species degenerate Fermi gas coupled by a diatomic Feshbach resonance. We show that the resulting superfluid can exhibit a form of coherent BEC-to-BCS oscillations in response to a nonadiabatic change in the system's parameters, such as, for example, a sudden shift in the position of the Feshbach resonance. In the narrow resonance limit, the resulting solitonlike collisionless dynamics can be calculated analytically. In equilibrium, the thermodynamics can be accurately computed across the full range of BCS-BEC crossover, with corrections controlled by the ratio of the resonance width to the Fermi energy.

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Advances to control interactions in trapped degenerate gases by tuning through a Feshbach resonance (FR), led to observation of molecular Bose-Einstein condensation (BEC) in bosonic [1] and fermionic atomic gases [2,3]. Such tunability allows studies of these systems in previously unexplored regimes, observing paired fermionic condensates along the crossover between the BCS regime of weakly-paired, strongly overlapping Cooper pairs, and the BEC regime of tightly bound, weakly-interacting diatomic molecules. An even more exciting possibility, unavailable in other related (e.g., superfluids in condensed matter) systems, is the nonadiabatic switching of system's parameters, thereby allowing access to highly coherent and nonequilibrium quantum states of matter. For bosonic ^{85}Rb atoms, this was recently realized in experiments by Donley *et al.* [1]. Using short magnetic field pulses that briefly bring the system close to a nearby FR, they observed coherent oscillations in the atomic condensate corresponding to Rabi oscillations between atomic and molecular condensates [4].

In this Letter we study a zero-temperature collisionless dynamics of a two-species degenerate atomic Fermi gas near a FR that can be tuned through the Fermi sea. The goal is to understand the evolution of the system following a nonadiabatic change in an externally-controlled system's parameter, such as the detuning of the FR, ω_0, relative to the Fermi energy, ϵ_F.

Our main result is the demonstration that such a system exhibits collective coherent oscillations of the Fermi gas between BCS- and BEC-like paired superfluid states. This dynamics is a paired-fermions analog of the atomic-to-molecular condensate Rabi oscillations observed by Donley *et al.* in trapped bosonic gases. In the narrow FR, we calculate the frequency and amplitude of these oscillations and find their analytic form. Our work directly builds on the recent discovery by Barankov *et al.* [5] of integrable dynamics in the BCS model following a sudden change in the negative s-wave scattering length. The model we study and its analysis apply across the full BCS-BEC crossover. For large positive detuning ($\omega_0/2 \gg \epsilon_F$), our results reduce to those of Ref. [5]. In the new complementary regime, $\omega_0/2 < \epsilon_F$, the atoms with energy near $\omega_0/2$ can periodically interconvert with molecules. The resulting hole-burning (Fig. 1) in the atomic momentum distribution should in principle be observable in the time-of-flight experiments.

We also analyze the thermodynamics along the full range of the BCS-BEC crossover [6,7]. We show that for a FR that is narrow compared to the Fermi energy, and a background scattering length a_{bg} that is short compared to the interatomic spacing, $n^{-1/3}$, low-temperature thermodynamics can be accurately computed analytically [8].

Our starting point is the Hamiltonian [9]

$$H = \sum_{p,\sigma} \epsilon_p \hat{a}^\dagger_{p\sigma}\hat{a}_{p\sigma} + \sum_p \left(\epsilon_0 + \frac{\epsilon_p}{2}\right)\hat{b}^\dagger_p \hat{b}_p + \sum_{p,q} \frac{g}{\sqrt{V}}(\hat{b}_q \hat{a}^\dagger_{p+q\uparrow}\hat{a}^\dagger_{-p\downarrow} + \hat{b}^\dagger_q \hat{a}_{-p\downarrow}\hat{a}_{p+q\uparrow}) \quad (1)$$

describing fermionic atoms, created by $\hat{a}^\dagger_{p\sigma}$ with momentum p, "spin" $\sigma = \uparrow, \downarrow$, and kinetic energy $\epsilon_p = p^2/2m$, that are coupled to diatomic molecular (resonant) states created by $\hat{b}^\dagger_q$. The position and the width (molecular lifetime) of the FR are, respectively, controlled by the

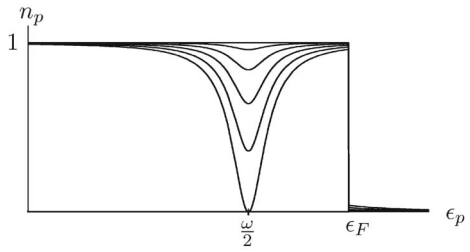

FIG. 1. Atomic momentum distribution n_p displaying oscillations between a Fermi sea and Bose-condensed molecules. The atomic depletion in n_p of width $\Delta \approx \gamma\sqrt{\omega_0}$ appears at the molecular energy determined by the detuning ω_0.

bare detuning energy ϵ_0 and coupling g, the former easily experimentally tunable by a magnetic field. We have neglected nonresonant atom-atom and molecule-molecule interactions that we expect near a FR to be subdominant to the resonant scattering.

In what follows, we focus on molecules with zero center of mass momentum, $\hat{b}_0$, neglecting molecules $\hat{b}_{q\neq 0}$ excited above the molecular condensate. In equilibrium, this is justified at low temperatures and weak interactions. However, for nonequilibrium dynamics, the validity of this approximation is a more delicate issue, as one might expect such excitations to be induced by a nonadiabatic shift in the FR. Nevertheless, physically we expect that for an initially homogeneous condensate [9] and weak interactions, the dynamics will be dominated by $b_0(t)$. However, on sufficiently long time scales, the $b_{q\neq 0}$ excitations and particle collisions should decohere and damp out the collective BEC-BCS oscillations studied here, allowing a slow relaxation to a new equilibrium state for the shifted FR. Determining this relaxational dynamics is beyond the scope of the present work. We thus replace $\hat{b}_q$ in Eq. (1) by $\hat{b}_0\delta_{\mathbf{q},0} \equiv \hat{b}\delta_{\mathbf{q},0}$. Expected macroscopic occupation ($\langle \hat{b}^\dagger \hat{b}\rangle \gg 1$) of the molecular level ϵ_0 allows us to neglect quantum molecular fluctuations and replace operators $\hat{b}(t)$, $\hat{b}^\dagger(t)$ by the c-numbers $b(t)$, $b^*(t)$.

We now look for time-dependent fermionic operators in terms of reference, time-independent fermions $\hat{\alpha}_{p\uparrow}$, $\hat{\alpha}_{-p\downarrow}$, related to $\hat{a}_{p,\sigma}(t)$ through the Bogoliubov amplitudes $u_p(t)$, $v_p(t)$ by $\hat{a}_{p\uparrow} = u_p^*\hat{\alpha}_{p\uparrow} + v_p\hat{\alpha}^\dagger_{-p\downarrow}$, $\hat{a}_{-p\downarrow} = u_p^*\hat{\alpha}_{-p\downarrow} - v_p\hat{\alpha}^\dagger_{p\uparrow}$ with a constraint $u_p^*u_p + v_p^*v_p = 1$, that ensures fermionic anticommutation relations. The Heisenberg dynamics is then encoded in the equations of motion for $u_p(t)$, $v_p(t)$, and $b(t)$

$$i\partial_t u_p = -\epsilon_p u_p + \frac{gb^*}{\sqrt{V}}v_p,\ i\partial_t v_p = \epsilon_p v_p + \frac{gb}{\sqrt{V}}u_p, \quad (2)$$

$$i\partial_t b = \epsilon_0 b + \frac{g}{\sqrt{V}}\sum_p u_p^* v_p. \quad (3)$$

The dynamical evolution Eqs. (2) and (3) preserve pair correlations of fermions. Accordingly, we choose the initial state to be of the BCS type, $|\Omega(0)\rangle = \prod_p [u_p(0) + v_p(0)a^\dagger_{p\uparrow}a^\dagger_{-p\downarrow}]|0\rangle$. As a result of the time evolution, the fermion wave function preserves the same form with time-dependent factors $u_p(t)$ and $v_p(t)$.

Following Ref. [5], we utilize Anderson's spin analogy for the BCS problem [10] and look for a vector representation of these equations in terms of the variables $S_p = S_p^x + iS_p^y = 2v_p^*u_p$, $S_p^z = v_pv_p^* - u_pu_p^*$ that satisfy $|\mathbf{S}_p|^2 = S_p^*S_p + S_p^zS_p^z = 1$. Eq. (2) becomes

$$\partial_t S_p = 2i\epsilon_p S_p - \frac{2ig}{\sqrt{V}}b^*S_p^z, \quad \partial_t S_p^z = \frac{ig}{\sqrt{V}}(b^*S_p^* - bS_p), \quad (4)$$

and has a form of Bloch equations for a spin precessing in an effective (p-dependent) field, whose azimuthal dynamics is in turn self-consistently determined by molecular ($b(t)$) evolution, Eq. (3). It is now straightforward to check that these equations can be solved by an ansatz [5]

$$\begin{aligned} S_p(t) &= e^{i\omega t}\Big[-(2\epsilon_p - \omega)C_p\Omega(t) + iC_p\dot{\Omega}(t)\Big], \\ S_p^z(t) &= C_p\Omega^2(t) - D_p, \quad \frac{gb(t)}{\sqrt{V}} = \Omega(t)e^{-i\omega t}, \end{aligned} \quad (5)$$

provided that the real function $\Omega(t)$ satisfies

$$\dot{\Omega}^2 + (\Omega^2 - \Delta_-^2)(\Omega^2 - \Delta_+^2) = 0, \quad (6)$$

and that C_p and D_p satisfy

$$\frac{D_p^2 - 1}{C_p^2} = \Delta_-^2\Delta_+^2, \qquad 2\frac{D_p}{C_p} = (2\epsilon_p - \omega)^2 + \Delta_-^2 + \Delta_+^2. \quad (7)$$

Here Δ_+, Δ_-, and ω are parameters characterizing the periodic instantonlike solution $\Omega(t)$ expressible in terms of an elliptic integral. It follows from Eq. (6) that $b(t)/\sqrt{V}$ oscillates between the values Δ_-/g and Δ_+/g (we choose $\Delta_- < \Delta_+$) with a period of oscillations given by

$$T = \frac{2}{\Delta_+}K\left(1 - \frac{\Delta_-^2}{\Delta_+^2}\right). \quad (8)$$

$K(m)$ is the complete elliptic integral of the first kind.

This ansatz is compatible with Eq. (3) provided that

$$\epsilon_0 - \omega = \frac{g^2}{2}\int \frac{d^3p}{(2\pi)^3}(2\epsilon_p - \omega)C_p, \quad (9)$$

$$1 = -\frac{g^2}{2}\int \frac{d^3p}{(2\pi)^3}C_p. \quad (10)$$

Equations (9) and (10) determine ω and Δ_+ in terms of the experimentally controlled (initial conditions) parameters ϵ_0 and Δ_-. As we will see below, the first of these equations is a nonequlibrium generalization of the BCS gap equation. The second one simply reflects the conservation of the total particle number, $\frac{dN}{dt} = 0$, with

$$N = 2b^*b + V\int \frac{d^3p}{(2\pi)^3}(S_p^z + 1). \quad (11)$$

We note that Eq. (7) only determines C_p and D_p up to a p-dependent sign that one would expect to be fixed by the initial fermion momentum distribution, encoded in $|\Omega(0)\rangle$. With the exception of a filled Fermi-sea initial condition, $\Omega(0) = \Delta_- = 0$, the solution encoded in C_p and D_p, Eq. (7), does *not* correspond to initial conditions

($u_p(0)$ and $v_p(0)$) characteristic of a ground state of the Hamiltonian (1) for any value of bare detuning ϵ_0. Nevertheless, we fix the sign of C_p to most closely match $n_p = \frac{1}{2}(S_p^z + 1)$ to the initial fermion momentum distribution. For a large positive detuning, this corresponds to a Fermi-Dirac function with discontinuity at the chemical potential μ that separates the holelike and particlelike states. Combining this criterion with Eq. (7), we find

$$C_p = \frac{2\ \mathrm{sgn}(\epsilon_p - \mu)}{\sqrt{[(2\epsilon_p - \omega)^2 + \Delta_-^2 + \Delta_+^2]^2 - 4\Delta_-^2\Delta_+^2}}, \quad (12)$$

where, as in equilibrium problem, μ is implicitly determined by the conserved total particle number N, Eq. (11), that reduces to $N = V\int\frac{d^3p}{(2\pi)^3}(1 - D_p)$.

Equation (9) involves a divergent integral, arising from an unphysical aspect of the model Eq. (1), that the atomic modes with an arbitrarily large energy interact with the molecular ones with equal strength g. In a more realistic model, the momentum dependence of the coupling g would cut off this divergence. As usual, our ignorance of this high energy physics can be buried in a (UV cutoff-dependent) relation between the parameter ϵ_0 appearing in H and the position of the physical FR, ω_0.

To see this, we calculate the two-atom scattering amplitude within the model Hamiltonian Eq. (1). It is completely determined by the self-energy of the molecules,

$$\Sigma(E) = \int\frac{d\omega}{2\pi}\frac{d^3p}{(2\pi)^3}\frac{ig^2}{(E - \omega - \epsilon_p + i0)(\omega - \epsilon_p + i0)} = -i\frac{g^2}{4\pi}m^{\frac{3}{2}}\sqrt{E} - g^2\int\frac{d^3p}{(2\pi)^3}\frac{m}{p^2}, \quad (13)$$

that diverges in exactly the same way as the integral in Eq. (9). Since $\Sigma(E)$ enters the retarded molecular propagator $G_b(E) = [E - \epsilon_0 - \Sigma(E)]^{-1}$ in combination with ϵ_0, we can trade in ϵ_0 for the physical FR detuning ω_0 according to $\omega_0 = \epsilon_0 - g^2\int\frac{d^3p}{(2\pi)^3}\frac{m}{p^2}$. This leads to the two-atom scattering amplitude $f = \frac{-\gamma/\sqrt{m}}{E - \omega_0 + i\gamma\sqrt{E}}$ that is consistent with the generic form based on unitarity [11] and is identical to that of the Fano-Anderson model (equivalent to our model at the two-body scattering level) [12]. We see that the scattering phase changes by π as the energy changes from below to above the physical (renormalized) FR, ω_0, with finite width for positive detuning, $\omega_0 > 0$, controlled by $\gamma = g^2m^{3/2}/4\pi$. In contrast for negative detuning, $\omega_0 < 0$, the scattering amplitude has a real pole at negative energy (a bound state in the open channel) that corresponds to a real molecular state [13].

With above renormalization of detuning, the nonequilibrium gap Eq. (9) becomes

$$\omega_0 - \omega = \frac{g^2}{2}\int\frac{d^3p}{(2\pi)^3}\left[(2\epsilon_p - \omega)C_p - \frac{2m}{p^2}\right], \quad (14)$$

where all the integrals are now convergent. This, together with the atom-conservation condition, (10), the total atom-number Eq. (11), the detuning ω_0, and the initial molecular density Δ_-, determines the condensate frequency ω and the molecular density maximum Δ_+ that controls the period of oscillations, in accordance with Eq. (8). Although the complete solution requires a numerical evaluation of the integrals, here we will focus on two analytically tractable regimes: (i) $\Delta_+ - \Delta_- \ll \Delta_+$ that corresponds to small oscillations of the condensate about the BCS-BEC ground state, and (ii) $\Delta_- \ll \Delta_+$ that corresponds to the evolution of the filled Fermi sea, following a large downward shift in detuning.

Let us first consider the regime of small oscillations $\Delta_+ - \Delta_- \ll \Delta_+$ about a BCS-BEC ground state. Limiting our analysis to $\omega > 0$, we find that Eq. (10) constrains ω to be close to 2μ. This, together with the condition Eq. (14), gives (at $\Delta_- = \Delta_+ = \Delta$ and $\mu = \omega/2$)

$$\omega_0 - 2\mu = \frac{g^2}{2}\int\frac{d^3p}{(2\pi)^3}\left[\frac{1}{\sqrt{(\epsilon_p - \mu)^2 + \Delta^2}} - \frac{1}{\epsilon_p}\right]. \quad (15)$$

This coincides with the BCS-BEC gap equation that can be derived in the equilibrium treatment of this problem [14]. It relates the condensate density Δ/g to the FR ω_0, with μ determined by Eq. (11). Simple analysis of these equations shows that for a large positive detuning, $\omega_0 \gg \epsilon_F$, molecules are strongly suppressed, leading μ to "stick" to ϵ_F, and to a conventional atomic BCS ground state, with $\Delta(\omega_0) \approx 8e^{-2}\epsilon_F e^{-(\omega_0 - 2\epsilon_F)/g^2\nu(\epsilon_F)}$ ($\nu(\epsilon) = m^{\frac{3}{2}}\epsilon^{\frac{1}{2}}/\sqrt{2}\pi^2$ is the atomic density of states). In this far off-resonance BCS regime, the accuracy of the mean-field treatment is controlled by the ratio of the width of the FR to Fermi energy, namely, by the dimensionless parameter γ^2/ϵ_F [8].

As the detuning ω_0 is lowered toward and below ϵ_F, the chemical potential begins to track the detuning, $\mu(\omega_0) \approx \omega_0/2 - \mathcal{O}[g^2\nu(\omega_0/2)]$, with atoms from states between ϵ_F and $\mu(\omega_0)$ converting into Bose-condensed molecules. The density of these tightly bound molecules that coexist with BCS's Fermi sea determine the gap, which displays a rounded mean-field behavior $\Delta_{\mathrm{equil}}(\omega_0) \approx \sqrt{\frac{2^{5/2}}{3\pi}\gamma[\epsilon_F^{3/2} - \mu(\omega_0)^{3/2}]}$. In the $g \to 0$ limit, ω_0 crossing of ϵ_F is a genuine quantum transition, with an upper-critical dimension of $d_{\mathrm{uc}} = 2$, and is therefore mean-field in 3D [15]. A finite atom-molecule coupling g rounds the transition into a smooth crossover near ϵ_F, that for small g (i.e., narrow FR, $\gamma^2 \ll \epsilon_F$) is therefore also accurately described by the mean-field theory summarized by Eqs. (11) and (15). Clearly, no additional anomalies appear when the 2-body FR ($\omega_0 = 0$) is crossed, since by that point nearly all atoms are bound up into well-ordered Bose-condensed molecules. In this $\mu < 0$ BEC regime, the remaining dilute fermionic atoms

are paired but are nondegenerate, and therefore realize a "strongly-coupled" BCS state [16]. This picture of the ground state (easily generalizable to a finite temperature [14]) is consistent with recent observations by Regal, *et al.* [2], who find that molecules appear at about 0.5 Gauss above the experimentally determined 2-body FR (in our interpretation corresponding to $\omega_0 \approx 2\epsilon_F$).

The solution, Eq. (5), then describes small oscillations about this equilibrium BCS-BEC state, with the period $T = \pi/\Delta$ given by Eq. (8). Because $\omega \approx 2\mu$, for a sufficiently small oscillations, the momentum distribution $n_p(t) = \frac{1}{2}[S_p^z(t) + 1]$ only changes near the Fermi surface, $\epsilon_p = \mu$.

The other interesting limit, $\Delta_- \to 0$, $\Delta_+ = \Delta$ describes oscillations between a Fermi sea (a normal "metal," with $\mu = \epsilon_F$) of atoms and bosonic molecules, following a nonadiabatic shift of detuning from ∞ down to ω_0. In this regime, Eqs. (10) and (14) reduce to

$$\omega_0 - \omega = \frac{g^2}{2}\int \frac{d^3p}{(2\pi)^3}\left[\frac{\left(\epsilon_p - \frac{\omega}{2}\right)\mathrm{sgn}\left(\epsilon_p - \epsilon_F\right)}{\left(\epsilon_p - \frac{\omega}{2}\right)^2 + \frac{\Delta^2}{4}} - \frac{1}{\epsilon_p}\right],$$

$$1 = -\frac{g^2}{4}\int \frac{d^3p}{(2\pi)^3}\frac{\mathrm{sgn}(\epsilon_p - \epsilon_F)}{\left(\epsilon_p - \frac{\omega}{2}\right)^2 + \frac{\Delta^2}{4}}. \qquad (16)$$

Simple integration of the atom-number conservation (second) equation above gives

$$\Delta \approx g^2\nu(\omega/2)\tan^{-1}[(2\epsilon_F - \omega)/\Delta]. \qquad (17)$$

The behavior then depends qualitatively on whether the detuning ω_0 is shifted to the BCS ($\omega_0/2 \gg \epsilon_F$) or to the BCS-BEC ($0 < \omega_0/2 < \epsilon_F$) regime, i.e., above or below the Fermi surface. In the former case, Eq. (17) reduces to $2\epsilon_F - \omega \approx \Delta^2/g^2\nu(\omega/2) \ll \Delta$, with an exponentially suppressed BCS-like gap $\Delta \propto e^{-\frac{\omega_0 - 2\epsilon_F}{g^2\nu(\epsilon_F)}}$ given by the gap (first) equation in Eq. (16). Hence in this regime, for narrow FR, $\omega \approx 2\epsilon_F$ and oscillations are confined to the vicinity of the Fermi surface.

In contrast, for the detuning shift *below* the Fermi surface, $0 < \omega_0/2 < \epsilon_F$, it is now Eq. (17) that determines the molecular density. It gives $\Delta \approx g^2\nu(\omega/2)/2 = \gamma\sqrt{\omega} \times \mathcal{O}(1)$, while the gap equation gives $\omega_0 - \omega = g^2\nu(\epsilon_F) \times \mathcal{O}(1) \ll \omega_0$, enforcing the molecular energy ω to stick to the detuning ω_0. We note that although $\Delta \approx \gamma\sqrt{\omega_0}$ is much larger (scales as g^2 rather than exponentially suppressed in $1/g^2$) than for the detuning into the BCS regime, it is much smaller than the equilibrium $\Delta_{\mathrm{equil}}(\omega_0)$, found as the solution to the equilibrium gap equation Eq. (15) in this regime. This suppression of the condensate oscillations is due to energy conservation between the atoms in the Fermi sea and the molecules. For a narrow FR resonance, $\gamma\sqrt{\omega_0} \ll \omega_0 < \epsilon_F$ it is only a small fraction of atoms in the Fermi sea that are in the immediate vicinity (set by the resonance width $\gamma\sqrt{\omega_0}$) of ω_0 that can resonantly bind into molecules. The resulting resonant atom-molecule interconversion leads to a narrow oscillatory depletion of the Fermi sea, illustrated in Fig. 1, with a period of oscillations that diverges as $T \propto \frac{2}{\Delta_+}\log(\frac{\Delta_+}{\Delta_-})$ in the limit of the "normal" Fermi sea ($\Delta_- \to 0$) as the initial condition.

Analysis of the gap and atom-conservation equations shows that the amplitude of the atom-molecule oscillations vanishes as the molecular energy $\omega \approx \omega_0$ approaches zero. This is again enforced by the energy conservation that, in the absence of other degrees of freedom (e.g., molecules above a condensate, $b_{q\neq 0}$), forbids conversion of molecules at negative energy ω_0 into atoms at positive energy ϵ_p. We leave analysis of the dynamics that incorporates these additional degrees of freedom to future research.

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[1] E. A. Donley *et al.*, Nature (London) **417**, 529 (2002).
[2] C. A. Regal *et al.*, Phys. Rev. Lett. **92**, 040403 (2004).
[3] M.W. Zwierlein *et al.*, Phys. Rev. Lett. **92**, 120403 (2004).
[4] M. Holland and S. J. J. M. F. Kokkelmans, Phys. Rev. Lett. **89**, 180401 (2002).
[5] R. A. Barankov *et al.*, cond-mat/0312053.
[6] M. Holland *et al.*, Phys. Rev. Lett. **87**, 120406 (2001).
[7] E. Timmermans *et al.*, Phys. Lett. A **285**, 228 (2001).
[8] It appears that current experimental systems are described by a *wide* FR, limiting direct quantitative applicability of our analysis. We thank A. Bulgac for emphasizing this point to us.
[9] We focus on a homogenous system of volume V, leaving extension to the case of the trap for future work.
[10] P.W. Anderson, Phys. Rev. **112**, 1900 (1958).
[11] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Butterworth-Heinemann, Oxford, 1981).
[12] U. Fano, Phys. Rev. **124**, 1866 (1961).
[13] We note here that although at vanishing E the scattering amplitude diverges as $1/\omega_0$, for a narrow resonance, $\gamma \ll \sqrt{\epsilon_F}$, a dilute system is still weakly-interacting and therefore amenable to a controlled perturbative treatment (in contrast to claims in the literature).
[14] A.V. Andreev, V. Gurarie, and L. Radzihovsky (unpublished).
[15] D. I. Uzunov, Phys. Lett. A **87**, 11 (1981).
[16] N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000).
[17] R. A. Barankov *et al.*, Phys. Rev. Lett. **93**, 130403 (2004).