Partonic Calculation of the Two-Photon Exchange Contribution to Elastic Electron-Proton Scattering at Large Momentum Transfer

Y.-C. Chen,[1] A. Afanasev,[2] S. J. Brodsky,[3] C. E. Carlson,[2,4] and M. Vanderhaeghen[2,4]

[1]*Department of Physics, National Taiwan University, Taipei 10617, Taiwan*
[2]*Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA*
[3]*SLAC, Stanford University, Stanford, California 94309, USA*
[4]*Department of Physics, College of William and Mary, Williamsburg, Virginia 23187, USA*
(Received 4 March 2004; published 14 September 2004)

We estimate the two-photon exchange contribution to elastic electron-proton scattering at large momentum transfer through the scattering off a parton in the proton. We relate the two-photon exchange amplitude to the generalized parton distributions which appear in hard exclusive processes. We find that when taking the polarization transfer determinations of the form factors as input, adding in the 2-photon correction does reproduce the Rosenbluth cross sections.

DOI: 10.1103/PhysRevLett.93.122301 PACS numbers: 25.30.Bf, 13.40.Gp, 24.85.+p

There are currently two experimental methods to extract the ratio of electric (G_{Ep}) to magnetic (G_{Mp}) proton form factors: unpolarized measurements employing the Rosenbluth separation technique and experiments using a polarized electron beam measuring the ratio of the (in-plane) polarization components of the recoiling proton parallel and perpendicular to its momentum. Recent experiments at Jefferson Lab (JLab) [1,2] confirm the earlier Rosenbluth data [3], which are at variance with the JLab measurements of G_{Ep}/G_{Mp} at larger Q^2 using the polarization transfer technique [4,5]. This discrepancy casts doubt on electron scattering as a precision tool and needs to be sorted out in detail.

Given that no flaws in either experimental technique have been identified to date, the most likely explanation is that 2γ exchange effects are responsible for the discrepancy. The general structure of two- and multi-photon exchange contributions to the elastic electron-proton scattering observables has recently been studied [6]. It was found in that work that the 2γ exchange contribution to the unpolarized cross section can be kinematically enhanced at larger Q^2 compared with the $(G_{Ep})^2$ term, while the 2γ exchange contribution to the polarization measurements need not affect the results in a significant way. An explicit model calculation of the 2γ exchange effects was performed recently in [7], where the contribution to the 2γ exchange amplitude was calculated for a nucleon intermediate state. This calculation found that the 2γ exchange correction with the intermediate nucleon has the proper sign and magnitude to partially resolve the discrepancy. Here, we report the first model calculation of the elastic electron-nucleon scattering at large momentum transfer through the scattering off partons in a nucleon. We relate the two-photon exchange amplitude to generalized parton distributions (GPDs), which also enter in other wide angle scattering processes.

To describe the elastic electron-nucleon scattering

$$l(k) + N(p) \to l(k') + N(p'), \tag{1}$$

we adopt the definitions: $P = (p + p')/2$, $K = (k + k')/2$, $q = k - k' = p' - p$, and choose $Q^2 = -q^2$ and $\nu = K \cdot P$ as the independent kinematical invariants. Neglecting the electron mass, the T-matrix for elastic electron-nucleon scattering can be expressed through three independent Lorentz structures as [6]:

$$T_{h,\lambda'_N\lambda_N} = \frac{e^2}{Q^2}\bar{u}(k', h)\gamma_\mu u(k, h) \times \bar{u}(p', \lambda'_N)\Big(\tilde{G}_M \gamma^\mu - \tilde{F}_2 \frac{P^\mu}{M} + \tilde{F}_3 \frac{\gamma \cdot K P^\mu}{M^2}\Big)u(p, \lambda_N), \tag{2}$$

where $h = \pm 1/2$ is the electron helicity and λ_N (λ'_N) are the helicities of the incoming (outgoing) nucleon. Furthermore, $\tilde{G}_M$, $\tilde{F}_2$, and $\tilde{F}_3$ are complex functions of ν and Q^2, and we introduced the factor e^2/Q^2 for convenience, where $e = \sqrt{4\pi/137}$ is the proton charge, and M is the nucleon mass. To separate the one- and two-photon exchange contributions, we introduce the decompositions: $\tilde{G}_M = G_M + \delta\tilde{G}_M$ and $\tilde{F}_2 = F_2 + \delta\tilde{F}_2$, where G_M (F_2) are the proton magnetic (Pauli) form factors, respectively, defined from the matrix element of the electromagnetic current. The amplitudes $\tilde{F}_3, \delta\tilde{G}_M$ and $\delta\tilde{F}_2$ originate from processes involving at least the exchange of two photons and are of order e^2 (relative to the factor e^2 in (2)). The reduced cross section for elastic electron-nucleon scattering, including corrections up to order e^2 is given by [6]:

$$\sigma_R = G_M^2 + \frac{\varepsilon}{\tau}G_E^2 + 2G_M \mathcal{R}\Big(\delta\tilde{G}_M + \varepsilon\frac{\nu}{M^2}\tilde{F}_3\Big) + 2\frac{\varepsilon}{\tau}G_E \mathcal{R}\Big(\delta\tilde{G}_E + \frac{\nu}{M^2}\tilde{F}_3\Big) + O(e^4), \tag{3}$$

where $\mathcal{R}$ denotes the real part, $\tau \equiv Q^2/(4M^2)$, $\varepsilon \equiv [\nu^2 - M^4\tau(1+\tau)]/[\nu^2 + M^4\tau(1+\tau)]$, and $\tilde{G}_E \equiv \tilde{G}_M - (1+\tau)\tilde{F}_2 = G_E + \delta\tilde{G}_E$. G_E is the proton electric form factor and $\delta\tilde{G}_E$ is the 2γ exchange correction.

An observable which is directly proportional to the 2γ exchange is the analyzing power, or recoil proton polarization, normal to the scattering plane [8]. These are equal by T-invariance. This single spin asymmetry is:

$$A_n = \sqrt{\frac{2\varepsilon(1+\varepsilon)}{\tau}}\frac{1}{\sigma_R}\left\{-G_M \mathcal{I}\left(\delta\tilde{G}_E + \frac{\nu}{M^2}\tilde{F}_3\right) + G_E \mathcal{I}\left[\delta\tilde{G}_M + \left(\frac{2\varepsilon}{1+\varepsilon}\right)\frac{\nu}{M^2}\tilde{F}_3\right]\right\}, \quad (4)$$

where $\mathcal{I}$ denotes the imaginary part.

To estimate the 2γ contribution to $\delta\tilde{G}_M$, $\delta\tilde{G}_E$, and $\tilde{F}_3$ at large Q^2, we consider in this Letter a partonic calculation as shown in Fig. 1. As a first step, we calculate elastic electron-quark scattering with massless quarks: $l(k) + q(p_q) \rightarrow l(k') + q(p'_q)$. The Mandelstam invariants are given by $\hat{s} \equiv (k+p_q)^2$, Q^2, and $\hat{u} \equiv (k - p'_q)^2$, satisfying $\hat{s} + \hat{u} = Q^2$. The T-matrix for the 2γ part of the electron-quark hard scattering process can be written as:

$$H^{\text{hard}}_{h,\lambda} = \frac{e^2}{Q^2}\bar{u}(k', h)\gamma_\mu u(k, h) \times \bar{u}(p'_q, \lambda)(e_q^2 \tilde{f}_1 \gamma^\mu + e_q^2 \tilde{f}_3 \gamma \cdot K P_q^\mu) u(p_q, \lambda), \quad (5)$$

where $P_q \equiv (p_q + p'_q)/2$, e_q is the fractional quark charge (for a flavor q), and the quark helicity $\lambda = \pm 1/2$ is conserved in the hard scattering process.

To calculate the hard amplitudes $H^{\text{hard}}_{h,\lambda}$, we consider the 2γ exchange direct and crossed box diagrams of Fig. 2. The 2γ exchange contribution to the elastic electron scattering off elementary spin 1/2 particles has been calculated before. Early references include Refs. [9,10], which we have verified explicitly. For further use, we separate $\tilde{f}_1$ into a soft and hard part, i.e., $\tilde{f}_1 = \tilde{f}_1^{\text{soft}} + \tilde{f}_1^{\text{hard}}$, using the procedure of [11]. The soft part corresponds with the situation where one of the photons in Fig. 2 carries zero four-momentum and is obtained by replacing the other photon's four-momentum by q in the numerator and in its propagator in the loop integral, yielding:

$$\mathcal{R}(\tilde{f}_1^{\text{soft}}) = \frac{e^2}{4\pi^2}\left\{\ln\left(\frac{\lambda^2}{\sqrt{-\hat{s}\,\hat{u}}}\right)\ln\left|\frac{\hat{s}}{\hat{u}}\right| + \frac{\pi^2}{2}\right\}, \quad (6)$$

$$\mathcal{R}(\tilde{f}_1^{\text{hard}}) = \frac{e^2}{4\pi^2}\left\{\frac{1}{2}\ln\left|\frac{\hat{s}}{\hat{u}}\right| + \frac{Q^2}{4}\left[\frac{1}{\hat{u}}\ln^2\left|\frac{\hat{s}}{Q^2}\right| - \frac{1}{\hat{s}}\ln^2\left|\frac{\hat{u}}{Q^2}\right| - \frac{1}{\hat{s}}\pi^2\right]\right\}, \quad (7)$$

where $\tilde{f}_1^{\text{soft}}$, which contains a term proportional to $\ln\lambda^2$ (λ is an infinitesimal photon mass), is IR divergent. The real part of $\tilde{f}_3$ from Fig. 2 is IR finite, and is given by:

$$\mathcal{R}(\tilde{f}_3) = \frac{e^2}{4\pi^2}\frac{1}{\hat{s}\,\hat{u}}\left\{\hat{s}\ln\left|\frac{\hat{s}}{Q^2}\right| + \hat{u}\ln\left|\frac{\hat{u}}{Q^2}\right| + \frac{\hat{s}-\hat{u}}{2} \times \left[\frac{\hat{s}}{\hat{u}}\ln^2\left|\frac{\hat{s}}{Q^2}\right| - \frac{\hat{u}}{\hat{s}}\ln^2\left|\frac{\hat{u}}{Q^2}\right| - \frac{\hat{u}}{\hat{s}}\pi^2\right]\right\}. \quad (8)$$

The π^2 terms in Eqs. (6)–(8) guarantee crossing symmetry of the scattering amplitude at the quark-level and are absent in the e^+e^- annihilation channel.

The imaginary parts of $\tilde{f}_1$ and $\tilde{f}_3$ originate solely from the direct 2γ exchange box diagram of Fig. 2 and are:

$$\mathcal{I}(\tilde{f}_1^{\text{soft}}) = -\frac{e^2}{4\pi}\ln\left(\frac{\lambda^2}{\hat{s}}\right), \quad (9)$$

$$\mathcal{I}(\tilde{f}_1^{\text{hard}}) = -\frac{e^2}{4\pi}\left\{\frac{Q^2}{2\hat{u}}\ln\left(\frac{\hat{s}}{Q^2}\right) + \frac{1}{2}\right\}, \quad (10)$$

$$\mathcal{I}(\tilde{f}_3) = -\frac{e^2}{4\pi}\frac{1}{\hat{u}}\left\{\frac{\hat{s}-\hat{u}}{\hat{u}}\ln\left(\frac{\hat{s}}{Q^2}\right) + 1\right\}. \quad (11)$$

We next discuss how to embed the quarks in the nucleon. We begin by discussing the soft contributions. The soft parts involve long wavelength photons, which see the proton as a whole. There are also soft contributions when the photons interact with different quarks. One can show that the IR contributions from these processes, added to the IR contributions from the handbag diagrams, give the same result as the IR contributions calculated with just a nucleon intermediate state, satisfying the low energy theorem. We extend this and take the full summed soft contribution to be the same as on a nucleon.

For the real parts, the IR divergence arising from the direct and crossed box diagrams is canceled when adding the bremsstrahlung interference contribution with a soft-photon emitted from the electron and proton. This pro-

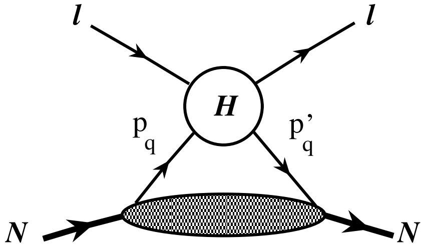

FIG. 1. Handbag diagram for the elastic lepton-nucleon scattering at large momentum transfers. In the hard scattering process H, the lepton scatters from quarks with momenta p_q and p'_q. The lower blob represents the GPDs of the nucleon.

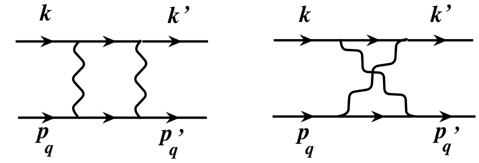

FIG. 2. Direct and crossed box diagrams for H in Fig. 1.

vides a radiative correction term proportional to the target charge Z, which may be written as:

$$\sigma_{R,\rm soft} = \sigma_{1\gamma}(1 + \delta_{2\gamma}^{soft} + \delta_{\rm brems}^{ep}), \tag{12}$$

where $\sigma_{1\gamma}$ is the 1γ exchange cross section. In Eq. (12), the soft-photon part of the nucleon box diagram is given by

$$\delta_{2\gamma}^{\rm soft} = \frac{e^2}{2\pi^2}\left\{\ln\left(\frac{\lambda^2}{\sqrt{(s-M^2)|u-M^2|}}\right)\ln\left|\frac{s-M^2}{u-M^2}\right| - L\left(\frac{s-M^2}{s}\right) - \frac{1}{2}\ln^2\left(\frac{s-M^2}{s}\right) + \mathcal{R}\left[L\left(\frac{u-M^2}{u}\right)\right] + \frac{1}{2}\ln^2\left(\frac{u-M^2}{u}\right) + \frac{\pi^2}{2}\right\}, \tag{13}$$

where L is the Spence function. The bremsstrahlung contribution $\delta_{\rm brems}^{ep}$ we take from Ref. [12] [see their Eq. (4.14)]. When comparing with elastic ep cross section data, which are usually radiatively corrected using the Mo and Tsai procedure [13], we only have to consider the difference between our above IR finite $\delta_{2\gamma}^{\rm soft} + \delta_{\rm brems}^{ep}$ and the expression for the Z-dependent radiative correction in [13]. This difference is predominantly given by a constant shift proportional to $\pi^2/2$ in Eq. (13). Shifting the $\pi^2/2$ term from the soft to the hard radiative correction, Eq. (7), makes a negligible numerical effect on the *total* correction to the Rosenbluth cross section. (Ref. [7], in contrast to us, reported an ε-dependent difference of their soft term from Ref. [13]. This is due to different allocation of finite parts between soft and hard terms, but has negligible impact to our *total* result.)

For the hard part, a partonic calculation can be reliably performed provided that all kinematical variables, $s = (p+k)^2$, $-u = -(p-k')^2$, and Q^2, are large compared to a typical hadronic scale. In the following, we shall choose M^2 as the lower limit on $s, -u, Q^2$. This part of the amplitude is a convolution between an electron-quark hard scattering and a soft nucleon matrix element. We choose a frame where $q^+ = 0$, introducing light cone variables $a^\pm \propto (a^0 \pm a^3)$, with $P^3 > 0$. The $+$ momentum fractions of electrons and partons are defined as $\eta = K^+/P^+$ and $x = P_q^+/P^+$, respectively. At large Q^2, we can neglect the intrinsic transverse momenta of the active quarks. The Mandelstam invariants for the hard process are then given by $\hat{s} = Q^2(x+\eta)^2/(4x\eta)$ and $\hat{u} = -Q^2(x-\eta)^2/(4x\eta)$. We extend the handbag formalism [14], used in wide angle Compton scattering [15,16], to the 2γ exchange process in elastic ep scattering, and keep the x dependence in the hard scattering amplitude. This yields the T-matrix for the process (1) as:

$$\begin{aligned} T^{\rm hard}_{h,\lambda'_N\lambda_N} &= \int_{-1}^{1}\frac{dx}{x}\sum_q \frac{1}{2}[H^{\rm hard}_{h,+\frac{1}{2}} + H^{\rm hard}_{h,-\frac{1}{2}}] \\ &\times\left[H^q(x,0,q^2)\bar{u}(p',\lambda'_N)\gamma\cdot n u(p,\lambda_N) + E^q(x,0,q^2)\bar{u}(p',\lambda'_N)\frac{i\sigma^{\mu\nu}n_\mu q_\nu}{2M}u(p,\lambda_N)\right] \\ &+ \int_{-1}^{1}\frac{dx}{x}\sum_q \frac{1}{2}[H^{\rm hard}_{h,+\frac{1}{2}} - H^{\rm hard}_{h,-\frac{1}{2}}]{\rm sgn}(x) \\ &\times\tilde{H}^q(x,0,q^2)\bar{u}(p',\lambda'_N)\gamma\cdot n\gamma_5 u(p,\lambda_N), \end{aligned} \tag{14}$$

where $H^{\rm hard}$ is evaluated using $\tilde{f}_1^{\rm hard}$ and $\tilde{f}_3$ and where $n^\mu = 2/\sqrt{M^4 - su}(-\eta P^\mu + K^\mu)$ is a Sudakov four-vector ($n^2 = 0$). Furthermore, $H^q, E^q, \tilde{H}^q$ are the GPDs for a quark q in the nucleon. From Eqs. (2), (5), and (14), the hard 2γ exchange parts to $\delta\tilde{G}_M$, $\delta\tilde{G}_E$, $\tilde{F}_3$ are obtained as:

$$\delta\tilde{G}_M^{\rm hard} = C, \quad \delta\tilde{G}_E^{\rm hard} = -\left(\frac{1+\varepsilon}{2\varepsilon}\right)(A - C) + \sqrt{\frac{1+\varepsilon}{2\varepsilon}}B,$$

$$\tilde{F}_3 = \frac{M^2}{\nu}\left(\frac{1+\varepsilon}{2\varepsilon}\right)(A - C), \tag{15}$$

with

$$\begin{aligned} A &\equiv \int_{-1}^{1}\frac{dx}{x}\frac{[(\hat{s}-\hat{u})\tilde{f}_1^{\rm hard} - \hat{s}\,\hat{u}\,\tilde{f}_3]}{s-u}\sum_q e_q^2(H^q + E^q), \\ B &\equiv \int_{-1}^{1}\frac{dx}{x}\frac{[(\hat{s}-\hat{u})\tilde{f}_1^{\rm hard} - \hat{s}\,\hat{u}\,\tilde{f}_3]}{(s-u)}\sum_q e_q^2(H^q - \tau E^q), \\ C &\equiv \int_{-1}^{1}\frac{dx}{x}\tilde{f}_1^{\rm hard}{\rm sgn}(x)\sum_q e_q^2\tilde{H}^q. \end{aligned} \tag{16}$$

To estimate the amplitudes of Eq. (15), we need to specify a model for the GPDs. Following Refs. [15,16], we use an unfactorized (valence) model for the GPDs H, E, and $\tilde{H}$ in terms of a forward parton distribution and a Gaussian factor in x and q^2 (see Eq. (68) in second Ref. [16] with transverse size parameter $a = 0.8$ GeV^{-1}). For more details on the model description see Ref. [17].

In Fig. 3, we display the effect of 2γ exchange on the cross sections. For the form factors, we use the G_{Ep}/G_{Mp} ratio as extracted from the polarization transfer experiments [5]. For G_{Mp}, we adopt the parametrization of [18] (scaled by a factor 0.995, as discussed further on). Figure 3 illustrates that the values of G_{Ep} as extracted from the polarization data are inconsistent with the slopes one extracts from a linear fit to the Rosenbluth data in the Q^2 range where data from both methods exist. By adding the 2γ correction, one first observes that the Rosenbluth plot becomes slightly nonlinear, in particular, at the largest ε values. Furthermore, one sees that over most of the ε range, the slope is indeed steeper in agreement

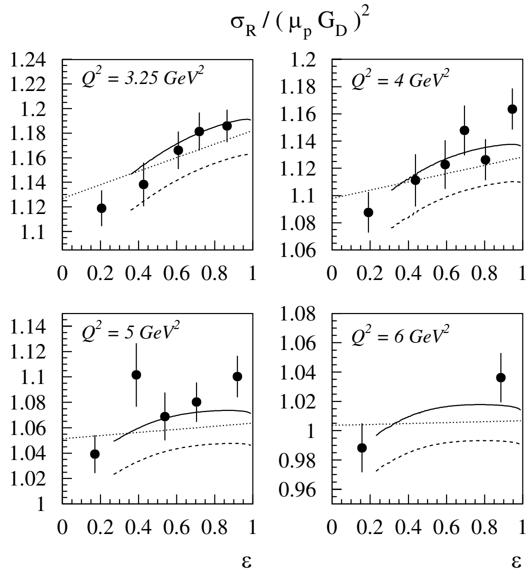

FIG. 3. Rosenbluth plots for elastic ep scattering: σ_R divided by $(\mu_p G_D)^2$, with $G_D = (1 + Q^2/0.71)^{-2}$. Dotted curves: Born approximation using G_{Ep}/G_{Mp} from polarization data [4,5]. Dashed curves: results when adding the GPD calculation for the hard 2γ exchange correction, for the kinematical range $s, -u > M^2$. Full curves are the total results including, in addition, the soft 2γ exchange correction relative to Mo and Tsai. The data are from Ref. [3].

with the Rosenbluth data. We checked that this feature is largely independent of the specific model used for the GPDs. Third, in order to fit the data when including the 2γ exchange correction, one has to slightly decrease the value of G_{Mp} of [18] (by a factor 0.995). This change in G_{Mp} is just the simplest estimate of how the additional radiative corrections would affect the extraction of G_{Mp}. We see that including the 2γ exchange allows the reconciliation of both polarization transfer and Rosenbluth data. It is clearly worthwhile to do a global reanalysis of all large Q^2 elastic data including the 2γ correction, which is however beyond the scope of this Letter.

The real part of the 2γ exchange amplitude can be accessed directly as the deviation from unity of the ratio of e^+/e^- elastic scattering. Our calculation gives an e^+/e^- ratio of about 0.98 in the range $Q^2 = 2-5$ GeV^2 and at large ε values, about 1σ below the data of Ref. [19]. At smaller values of ε, where no data exist at moderately large Q^2, our prediction for the e^+/e^- ratio rises and becomes larger than 1 around $\varepsilon = 0.35$. It will be an important test of the calculation to check this trend.

A test of the imaginary part of the 2γ exchange amplitudes is obtained using A_n, shown in Fig. 4.

In summary, we estimated the 2γ exchange contribution to elastic ep scattering at large Q^2 in a partonic model calculation, and expressed this amplitude in terms of nucleon GPDs. We found that the 2γ exchange contribution quantitatively resolves the existing discrepancy between Rosenbluth and polarization transfer experiments

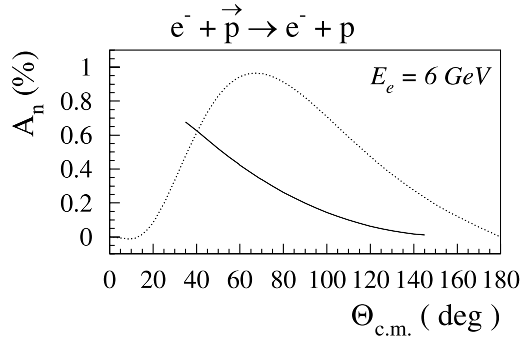

FIG. 4. Proton normal spin asymmetry for elastic ep scattering as function of the center of mass ($c.m.$) scattering angle. The GPD calculation is shown by the solid curve, bounded by the kinematic range where $-u, Q^2 > M^2$. For comparison, the elastic contribution is shown by the dotted curve [8].

This work was supported by the Taiwanese NSC under contract No. 92-2112-M002-049 (Y. C. C.), by the NSF under grant No. PHY-0245056 (C. E. C.), and by the U.S DOE under contracts No. DE-AC05-84ER40150 (A. A., M. V.), DE-AC03-76SF00515 (S. J. B.), and DE-FG02-04ER41302 (M. V.). M. V. also thanks P. Guichon for helpful discussions.

[1] M. E. Christy *et al.*, Phys. Rev. C **70**, 015206 (2004).
[2] JLab E01-001 Coll., J. Arrington, nucl-ex/0312017.
[3] L. Andivahis *et al.*, Phys. Rev. D **50**, 5491 (1994).
[4] M. K. Jones *et al.*, Phys. Rev. Lett. **84**, 1398 (2000).
[5] O. Gayou *et al.*, Phys. Rev. Lett. **88**, 092301 (2002).
[6] P. Guichon and M. Vanderhaeghen, Phys. Rev. Lett. **91**, 142303 (2003).
[7] P. G. Blunden, W. Melnitchouk, and J. A. Tjon, Phys. Rev. Lett. **91**, 142304 (2003).
[8] A. De Rujula, J. M. Kaplan, and E. de Rafael, Nucl. Phys. **B35**, 365 (1971).
[9] P. Van Nieuwenhuizen, Nucl. Phys. **B28**, 429 (1971).
[10] I. B. Khriplovich, Sov. J. Nucl. Phys. **17**, 298 (1973).
[11] G. Grammer, Jr. and D. R. Yennie, Phys. Rev. D **8**, 4332 (1973).
[12] L. C. Maximon and J. A. Tjon, Phys. Rev. C **62**, 054320 (2000).
[13] L. W. Mo and Y.-S. Tsai, Rev. Mod. Phys. **41**, 205 (1969).
[14] S. J. Brodsky, F. E. Close, and J. F. Gunion, Phys. Rev. D **5**, 1384 (1972).
[15] A. V. Radyushkin, Phys. Rev. D **58**, 114008 (1998).
[16] M. Diehl, Th. Feldmann, R. Jakob, and P. Kroll, Phys. Lett. B **460**, 204 (1999); Eur. Phys. J. C **8**, 409 (1999).
[17] M. Gorchtein, P. A. M. Guichon, and M. Vanderhaeghen, Nucl. Phys. **A741**, 234 (2004).
[18] E. J. Brash *et al.*, Phys. Rev. C **65**, 051001 (2002).
[19] J. Mar *et al.*, Phys. Rev. Lett. **21**, 482 (1968).