Dark Matter and Dark Energy from the Solution of the Strong *CP* Problem

Roberto Mainini and Silvio A. Bonometto
Dipartimento di Fisica G. Occhialini, Università di Milano-Bicocca, Piazza della Scienza 3, 20126 Milano, Italy and INFN, Sezione di Milano, Italy
(Received 6 April 2004; published 15 September 2004)

The Peccei-Quinn (PQ) solution of the strong *CP* problem requires the existence of axions, which are viable candidates for dark matter. If the Nambu-Goldstone potential of the PQ model is replaced by a potential $V(|\Phi|)$ admitting a tracker solution, the scalar field $|\Phi|$ can account for dark energy, while the phase of Φ yields axion dark matter. If V is a supergravity (SUGRA) potential, the model essentially depends on a single parameter, the energy scale Λ. Once we set $\Lambda \simeq 10^{10}$ GeV at the quark-hadron transition, $|\Phi|$ naturally passes through values suitable to solve the strong *CP* problem, later growing to values providing fair amounts of dark matter and dark energy.

DOI: 10.1103/PhysRevLett.93.121301 PACS numbers: 98.80.Cq, 14.80.Mz, 95.35.+d

Introduction.—The solutions of the strong *CP* problem proposed by Peccei and Quinn (PQ) in 1977 [1] leads to one of the accepted models of dark matter (DM). PQ consider the Lagrangian term

$$\mathcal{L}_\theta = \frac{\alpha_s}{2\pi}\theta G \cdot \tilde{G}, \tag{1}$$

(α_s: strong coupling constant; G and $\tilde{G}$: gluon field tensor and its dual) yielding *CP* violations in strong interactions, and show that its effects are suppressed by making θ a dynamical variable, approaching zero in our cosmic era, its residual oscillations appearing as DM [2,3].

The θ dynamics is set by assuming that a complex field $\Phi = \phi e^{i\theta}/\sqrt{2}$ exists, whose evolution is ruled by a Nambu-Goldstone (NG) potential

$$V(|\Phi|) = \lambda[|\Phi|^2 - F_{PQ}^2]^2, \tag{2}$$

which is clearly U(1) invariant. At $T < F_{PQ}$ (the PQ energy scale, which shall be $\sim 10^{12}$ GeV), ϕ falls into the potential minimum so that the U(1) symmetry breaks as θ acquires different values in different horizons. When the chiral symmetry is also broken close to the quark-hadron transition, a further term must be added to the effective Lagrangian, arising because of instanton effects. This term reads

$$V_1 = \left[\sum_q \langle 0(T)|\bar{q}q|0(T)\rangle m_q\right](1 - \cos\theta). \tag{3}$$

At $T \simeq 0$, the square brackets approach $m_\pi^2 f_\pi^2$ (m_π, f_π: π-meson mass, decay constant).

The choice of a NG potential is the simplest possible. Here we explore the possibility of replacing it by a potential with a tracker solution [4,5]. Instead of taking a value $\simeq F_{PQ}$ soon, ϕ evolves over cosmological times. As in the PQ case, the potential shall involve a complex field Φ and be U(1) invariant. While ϕ rapidly settles on the tracker solution (apart of residual fluctuations) in almost any horizon, the symmetry is broken soon by the values taken by θ, which suffers no dynamical constraints and is therefore random, in different horizons. Later on, when a mass term arises because of the chiral symmetry break, dynamics becomes relevant also for the θ degree of freedom, as in the PQ case. Here this happens while ϕ still evolves over cosmological times. Finally, in the present epoch, ϕ accounts for dark energy (DE). Hence, besides yielding DM through its phase θ, the Φ field introduced to solve the strong *CP* problem accounts for DE through its modulus ϕ.

Within this model, DM and DE will be weakly coupled. If we take a generalization of the supergravity (SUGRA) potential [5] as tracking potential with an energy scale $\Lambda \sim 10^{10}$ GeV, we find reasonable values for today's DM and DE densities, while θ is driven to values even smaller than in the PQ case, so that *CP* is apparently conserved in strong interactions. In turn, Λ may be an indication of the scale where the soft breaking of supersymmetries occurred.

Lagrangian Theory.—The Lagrangian $\mathcal{L} = \sqrt{-g}\{g_{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - V(|\Phi|)\}$ can be rewritten in terms of ϕ and θ, adding also the term breaking the U(1) symmetry, as follows:

$$\mathcal{L} = \sqrt{-g}\Big\{\frac{1}{2}g_{\mu\nu}[\partial_\mu\phi\partial_\nu\phi + \phi^2\partial_\mu\theta\partial_\nu\theta] - V(\phi) - m^2(T,\phi)\phi^2(1-\cos\theta)\Big\}. \tag{4}$$

Here, $g_{\mu\nu}$ is the metric tensor. We shall assume that $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = a^2(d\tau^2 - \eta_{ij}dx_i dx_j)$, so that a is the scale factor, τ is the conformal time; Greek (Latin) indeces run from 0 to 3 (1 to 3); dots indicate differentiation in respect to τ. Around the energy scale Λ_{QCD} (quark-hadron transition), we shall take [6]

$$m(T,\phi) \simeq 0.1 m_o(\phi)\left(\frac{\Lambda_{QCD}}{T}\right)^{3.8}, \tag{5}$$

with $m_o(\phi) = m_\pi f_\pi/\phi$. At $T < \sim 0.3$–$0.2\Lambda_{QCD}$, $m(T,\phi)$ shall already approach its low-T behavior $m_o(\phi)$. The equations of motion then read

$$\ddot{\theta} + 2\left(\frac{\dot{a}}{a} + \frac{\dot{\phi}}{\phi}\right)\dot{\theta} + m^2 a^2 \sin\theta = 0, \quad (6)$$

$$\ddot{\phi} + 2\frac{\dot{a}}{a}\dot{\phi} + a^2 V'(\phi) = \phi\dot{\theta}^2, \quad (7)$$

and will be mostly used with $\sin\theta \simeq \theta$. Then, energy densities $\rho_{\theta,\phi} = \rho_{\theta,\phi;\mathrm{kin}} + \rho_{\theta,\phi;\mathrm{pot}}$ and pressures $p_{\theta,\phi} = \rho_{\theta,\phi;\mathrm{kin}} - \rho_{\theta,\phi;\mathrm{pot}}$, under the condition $\theta \ll 1$, are obtainable from

$$\rho_{\theta,\mathrm{kin}} = \frac{\phi^2}{2a^2}\dot{\theta}^2, \qquad \rho_{\theta,\mathrm{pot}} = \frac{m^2(T,\phi)}{2}\phi^2\theta, \qquad \rho_{\phi,\mathrm{kin}} = \frac{\dot{\phi}^2}{2a^2}, \qquad \rho_{\phi,\mathrm{pot}} = V(\phi). \quad (8)$$

The Case of the SUGRA Potential.—When θ undergoes many (nearly) harmonic oscillations within a Hubble time, $\langle\rho_{\theta,\mathrm{kin}}\rangle \simeq \langle\rho_{\theta,\mathrm{pot}}\rangle$ and $\langle p_\theta\rangle$ vanishes. Under such a condition, using Eqs. (6)–(8) it is easy to see that

$$\dot{\rho}_\theta + 3\frac{\dot{a}}{a}\rho_\theta = \frac{\dot{m}}{m}\rho_\theta, \qquad \dot{\rho}_\phi + 3\frac{\dot{a}}{a}(\rho_\phi + p_\phi) = -\frac{\dot{m}}{m}\rho_\theta. \quad (9)$$

When m is given by Eq. (5), $\dot{m}/m = -\dot{\phi}/\phi - 3.8\dot{T}/T$. At $T \simeq 0$, instead, $\dot{m}/m \simeq -\dot{\phi}/\phi$. Here below, the indices θ, ϕ will be replaced by DM, DE. Equation (9) clearly shows an exchange of energy between DM and DE. Let us notice that the former Eq. (9) can be formally integrated, yielding $\rho_{DM} \propto m/a^3$. In particular, this law holds at $T \ll \Lambda_{QCD}$, and then

$$\rho_{DM} a^3 \phi \simeq \mathrm{const}, \quad (10)$$

so that the usual behavior $\rho_{DM} \propto a^{-3}$ is modified by the energy outflow from DM to DE.

Let us now assume that the potential reads

$$V(\phi) = \frac{\Lambda^{\alpha+4}}{\phi^\alpha}\exp(4\pi\phi^2/m_p^2) \quad (11)$$

and does not depend on θ; in the radiation dominated era, it admits the tracker solution

$$\phi^{\alpha+2} = g_\alpha \Lambda^{\alpha+4} a^2 \tau^2, \quad (12)$$

with $g_\alpha = \alpha(\alpha+2)^2/4(\alpha+6)$. This solution holds until we approach the quark-hadron transition. Then, in Eq. (7), the term $\phi\dot{\theta}^2$, due to the DE-DM coupling, exceeds a^2V' and we enter a different tracking regime. This is shown in detail in Fig. 1, obtained for matter (baryon) density parameters $\Omega_m = 0.3$ ($\Omega_b = 0.03$) and $h = 0.7$ (Hubble constant in units of 100 km/s/Mpc). In particular, Fig. 1(a) shows the transition between these tracking regimes. Figure 1(b) then shows the low-z behavior ($1 + z = 1/a$), since DE density exceeds radiation and then gradually overcomes baryons (at $z \sim 10$) and DM (at $z \simeq 3$). Figure 1(c) is a landscape behavior of all components, down to $a = 1$. Notice, in particular, the a dependence of ρ_{DM}, occurring according to Eq. (10). In Fig. 2 we show the related behaviors of the density parameters Ω_i ($i = r, b, \theta, \phi$, i.e., radiation, baryons, DM, DE).

In general, once the density parameter Ω_{DE} (at $z = 0$) is assigned, a model with dynamical (coupled or un-

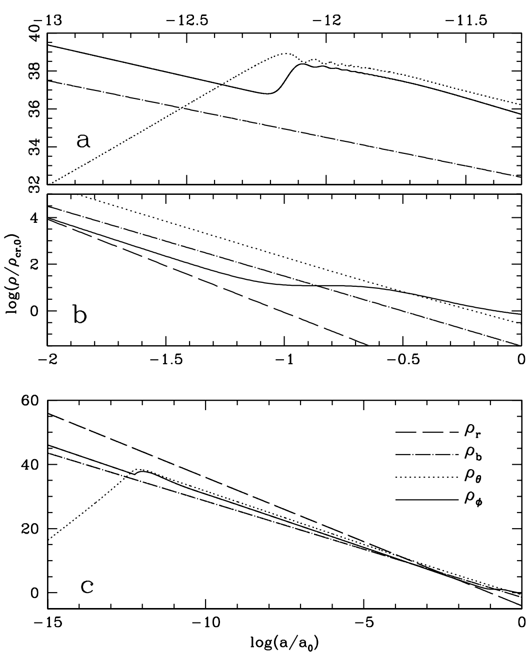

FIG. 1. Densities of the different components vs the scale factor a. Figure 1(a) magnifies the onset of the oscillation regime. Figure 1(b) shows the low-z behavior. Figure 1(c) is a landscape picture of the whole evolution. All abscissas are $\log(a/a_0)$.

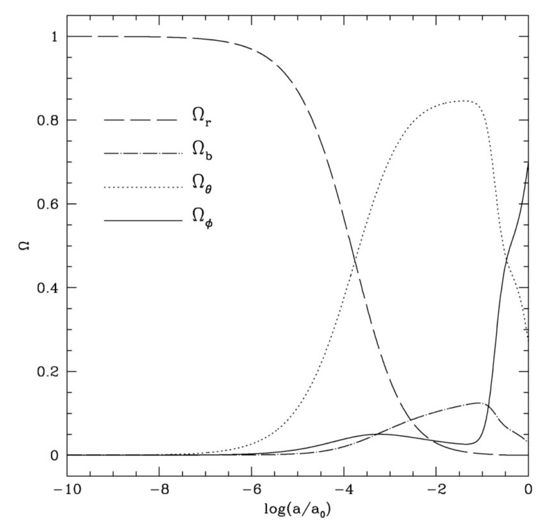

FIG. 2. Density parameters $\Omega_{r,b,\theta,\phi}$ (radiation, baryons, DM, DE) vs the scale factor a.

coupled) DE is not yet univocally determined. For instance, the potential (11) depends on the parameters α and Λ, and one of them can still be arbitrarily fixed. Other potentials show similar features.

In the present case, such arbitrariness no longer exists. Let us follow the behavior of ρ_{DM}, backwards in time, until the approximation $\theta \ll 1$ no longer applies. This moment must approximately coincide with the time when θ enters the oscillation regime. This occurs when

$$2(\dot{a}/a + \dot{\phi}/\phi) \simeq m(T, \phi)a \qquad (13)$$

(see Eq. (6)). At that time, according to Eq. (10), which is marginally valid up to there, and taking $\theta = 1$,

$$\rho_{DM} \simeq \rho_{o,DM} \frac{\phi_o}{\phi(a)} \frac{1}{a^3} \simeq m^2[T(a), \phi(a)]\phi^2(a). \qquad (14)$$

The system made by Eqs. (13) and (14) owing to Eq. (12), yields the scale factor a_h when fluctuations start and the value of Λ in the potential (11), as soon as $\rho_{o,DM}$ (the present density of DM) is assigned.

The plots shown in the previous section, drawn for $\Omega_{DM} = 0.27$, are obtained for $\Lambda \simeq 1.5 \times 10^{10}$ GeV, as is required by Eqs. (13) and (14). In this case, $a_h \sim 10^{-13}$. When Ω_{DM} goes from 0.2 to 0.4, $\log_{10}(\Lambda/\mathrm{GeV})$ (almost) linearly runs from 10.05 to 10.39 and a_h steadily lays at the eve of the quark-hadron transition. A model with DE and DM given by a single complex field based on SUGRA potential therefore bears a precise prediction on the scale Λ for the observational Ω_{DM} range. In turn, we can say that if the soft breaking of supersymmetries occurred at a scale slightly above 10^{10} GeV, $\Omega_{DE} \sim 0.3$ is a natural consequence.

Evolution of Inhomogeneities.—Besides predicting fair ratios between the world components, a viable model should also allow the formation of structures in the world. This matter will be treated in detail in a forthcoming paper. Let us however outline that the model treated here belongs to the class of coupled DE models treated by Amendola [7], with a time-dependent coupling. In fact, for small θ's, the r.h.s. of Eq. (7), after averaging over cosmological times, reads $C(\phi)\langle\rho_\theta\rangle a^2$ with $C(\phi) = 1/\phi$. Similarly, in Eq. (9), which is already averaged, the r.h.s. are $\pm C(\phi)\dot{\phi}\rho_\theta$ (C is the DE-DM coupling introduced in [7]). Let us also outline that Fig. 2 shows a ϕ-MDE phase, typical of this class of models, after matter-radiation equivalence, as the kinetic energy of DE is non-neglegible during the matter-dominated era.

By solving the fluctuation equations in [7] with the above $C(\phi)$, we find the behavior shown in Fig. 3 (top). Figure 3 (bottom) compares fluctuation evolutions in this model (solid curves) with those in an analogous cold dark matter model with a cosmological constant (ΛCDM) (dot-dashed curves) and in a coupled DE model with constant coupling $C = 0.25\sqrt{8\pi G} \simeq \langle C(\phi)\rangle$ (dashed curves). These plots show that the linear growth factor, from recombination to now, is significantly smaller than in coupled DE models with constant coupling and, more significantly, is quite close to ΛCDM. The essential differences from ΛCDM are that (i) objects should form earlier and (ii) baryon fluctuations keep below DM fluctuations until very recently.

Discussion.—The first evidences of DM date 70 years ago, but its nonbaryonic nature became compulsory in the 1970s, when big bang nucleosynthesis (BBNS) and cosmic microwave background radiation (CMBR) anisotropies were studied. DE is younger, but is now required both by supernova type Ia (SNIa) data [8], as well as by CMBR and deep galaxy data [9,10]. Axions have been candidate DM since the late 1970s, although various studies, as well as the occurrence of the SN 1987a, have finally constrained the PQ scale around values $F_{PQ} \sim 10^{12}$ GeV. Contributions to DM from topological singularities (cos-

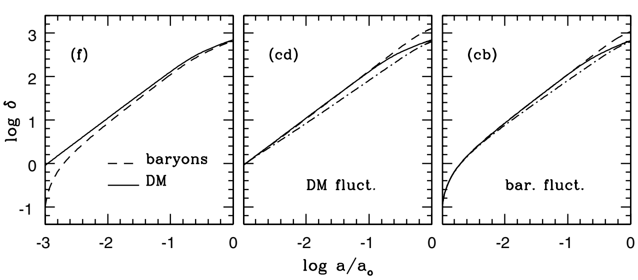

FIG. 3. Time evolution of DM and baryon fluctuations. The top figure shows DM and baryon fluctuation evolution in this model. The two bottom figures compare DM and baryon fluctuation evolutions in this model (solid curve), ΛCDM (dot-dashed line), coupled DE (dashed line).

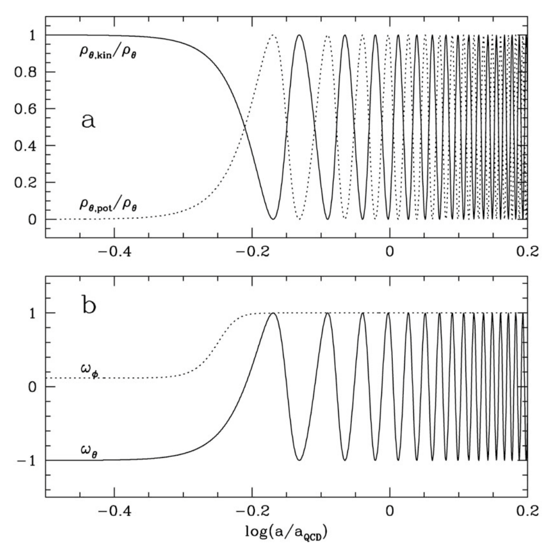

FIG. 4. The onset of coherent axion oscillations at the eve of the quark-hadron transition due to the increase of $m(T, \phi)$ causes the behaviors of $\rho_{\theta,\mathrm{pot}}$, $\rho_{\theta,\mathrm{kin}}$ (a) and $\omega_{\phi,\theta} = p_{\phi,\theta}/\rho_{\phi,\theta}$ (b) shown here.

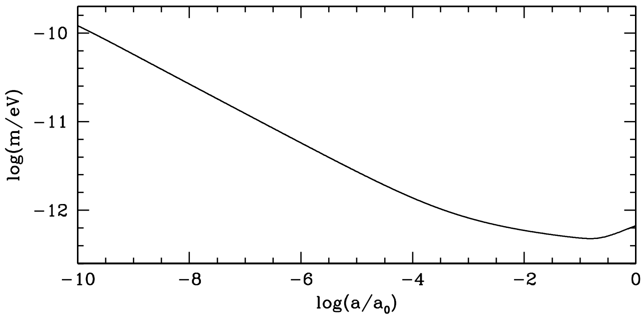

FIG. 5. ϕ variations cause a dependence of the effective axion mass on scale factor a, which is shown here.

mic string and walls) have also narrowed the constraints to F_{PQ} [11]. Here they were disregarded and could cause shifts in quantitative predictions. We shall deepen this point in further work.

The fact that scalar fields can yield both DM or DE by just changing an exponent in the potential stimulated the work of various authors. A potential like (11) was considered in *spintessence* models [12]. According to the choice of parameters, Φ yields either DM or DE.

On the contrary, in this Letter we deal with the possibility that Φ accounts for *both* DE and DM, and that the strong *CP* problem is simultaneously solved. As in the PQ model, the angle θ in Eq. (1) is turned into a dynamical variable, i.e., into the phase of a complex scalar field Φ, and is gradually driven to approach zero by our cosmic epoch. Residual θ oscillations yielding axions account for DM. The critical time for the onset of coherent axion oscillations occurs at the eve of the quark-hadron transition because of the rapid increase of $m(T, \phi)$. Here, ϕ replaces the constant F_{PQ} scale. This stage is illustrated by Figs. 4(a) and 4(b)where the behaviors of $\rho_{\theta,\rm pot}$, $\rho_{\theta,\rm kin}$, and $\omega_{\phi,\theta} = p_{\phi,\theta}/\rho_{\phi,\theta}$ are plotted.

The novel features of this model arise because the expectation value of ϕ is not a constant F_{PQ}, but evolves over cosmological times; in a sense, we let F_{PQ} evolve so to yield DE. Such evolution modifies the friction term in Eq. (6). The damping of θ oscillations is therefore greater and θ oscillations are smaller today. Further, accordingly to Eq. (5), the axion mass (or the oscillation frequency), which varies fairly rapidly during the formation of $\bar{q}q$ condensate, continues to evolve, over cosmological scales, due to the evolution of the ϕ field. The (low-)z dependence of $m_o(\phi)$ is shown in Fig. 5. We draw the reader's attention on the rebounce at $z \sim 10$, whose implications on halo formation could be critical [13].

Constraints on PQ axions came from $z = 0$ observations, which must be fulfilled by the same F_{PQ} scale, fulfilling also cosmological requirement. Here, ϕ attains values $\sim m_p$ today, so that most these constraints should be naturally satisfied. This matter, as well as the question of a direct axion detection, will be deepened in further work.

Let us outline that the choice of a SUGRA potential is arbitrary and could be replaced by other potentials, perhaps better approaching data at $z = 0$. However, using this potential allows to appreciate the conceptual echonomy in this approach. In the PQ approach, the F_{PQ} scale is assumed. Here, once Λ is set in a physically significant range around the quark-hadron transition ϕ naturally passes through values enabling to solve the strong *CP* problem and later naturally grows to values providing fair amounts of DM and DE.

Luciano Girardello, Antonio Masiero, and Federico Piazza are gratefully thanked for useful discussions.

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