Insulating Behavior of a Trapped Ideal Fermi Gas

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We investigate theoretically and experimentally the center-of-mass motion of an ideal Fermi gas in a combined periodic and harmonic potential. We find a crossover from a conducting to an insulating regime as the Fermi energy moves from the first Bloch band into the band gap of the lattice. The conducting regime is characterized by an oscillation of the cloud about the potential minimum, while in the insulating case the center of mass remains on one side of the potential.

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Recent experiments have shown great promise for studies of novel regimes of superfluidity with trapped fermions [1]. In this context, the combination of trapped cold fermions and optical lattices [2] is very promising. On the one hand, it represents the natural analogy with superconductivity of electrons in crystal lattices, while, on the other, it might be an additional tool to manipulate the interaction properties of the system [3], and to investigate the superfluid phase. Atomic Fermi gases in lattices are interesting also in view of observing novel quantum phases that have been predicted for interacting Fermi [4] and Fermi-Bose systems [5].

In this work we investigate a Fermi gas of noninteracting atoms trapped in a harmonic field and subjected to a one-dimensional optical lattice. We study both theoretically and experimentally the center-of-mass dynamics of the system after a sudden displacement of its equilibrium position along the lattice. We find that the statistical distribution makes the dynamical properties of such a system highly nontrivial. The most dramatic new feature is that the system can exhibit a conducting or an insulating behavior, depending on whether the Fermi energy lies in the first Bloch band of the lattice or in the gap region. In the first case the cloud oscillates symmetrically in the harmonic potential, while in the second case it remains trapped on one side of the potential. By tuning the width of the lattice band we observe the full crossover between the two different regimes.

Semiclassical model.—Let us first introduce our theoretical model to describe the system. Neglecting the interatomic collisions, the many-body Hamiltonian can be simply written as a sum of single particle Hamiltonians:

$$H_0 = \left(\frac{p_z^2}{2m} + \frac{1}{2}m\omega_z^2 z^2 + sE_R \sin^2(2\pi z/\lambda)\right) + \left(\frac{p_x^2 + p_y^2}{2m} + \frac{1}{2}m\omega_\perp^2(x^2 + y^2)\right), \tag{1}$$

where s is a tunable dimensionsless parameter, $E_R = \frac{\hbar^2}{2m}(\frac{2\pi}{\lambda})^2$ is the recoil energy, and λ is the wavelength of the laser creating the lattice. Since the harmonic oscillator length is typically much larger than the lattice spacing $d = \lambda/2$, we can study the system in the semiclassical approximation. We first concentrate on the z component of the Hamiltonian (1). In the limit of a vanishing harmonic field ($\omega_z \to 0$), the eigenfunctions of the stationary Schrödinger equation are Bloch waves. The eigenenergies $\varepsilon_n(p_z)$ have the typical Bloch band structure as a function of the quasimomentum p_z, defined in the first Brillouin zone ($-\pi\hbar/d \le p_z \le +\pi\hbar/d$), and of band index n [in the following we consider only the lowest energy Bloch band $n = 1$, hereafter called $\varepsilon(p_z)$]. In a semiclassical approach, we can incorporate the effects of the periodic potential modifying the kinetic part of the Hamiltonian. This corresponds to replace, in Eq. (1), the sum of the kinetic energy term along z and the lattice potential by the dispersion $\varepsilon(p_z)$. We rewrite Eq. (1) as

$$H_0 = \varepsilon(p_z) + \frac{1}{2}m\omega_z^2 z^2 + \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2}m\omega_\perp^2(x^2 + y^2). \tag{2}$$

The effects of a sudden small displacement z_0 of the center of the harmonic trap can be included adding the perturbative term $H_{\rm pert}(z, t) = m\omega_z^2\Theta(t)z_0 z$, where $\Theta(t)$ is the unit step function. A first insight into the behavior of the system is provided by the study of the linear dynamics in a 1D configuration.

One-dimensional analysis.—The physics becomes particularly clear in the semiclassical phase space, as shown in Fig. 1. The lines are isoenergetic single particle orbits; they belong to two different classes, separated by the dashed orbit, which has an energy 2δ, corresponding to the width of the first Bloch band. The orbits are closed when their energy is in the band, i.e., the particles oscillate around the trap minimum. The orbits are instead open when their energy is in the gap, because the quasimomentum is reversed at the band edges. Particles in these orbits oscillate on the sides of the harmonic potential, performing the analogous of Bloch oscillations in a linear potential. The single particle oscillation frequency is given by

$$\omega_0(E) = \pi\left(\int_{z_1(E)}^{z_2(E)} \frac{d\zeta}{[\partial\varepsilon(P_z)/\partial P_z]_{P_z=P_z(\zeta,E)}}\right)^{-1}, \quad (3)$$

where $z_1(E)$ and $z_2(E)$ are, respectively, the maximum and the minimum positions reached by the particle of energy E. The quantity (3) depends strongly on energy, vanishing for $E = 2\delta$.

As shown in Fig. 1(a), the Fermi gas at $T = 0$ uniformly fills the phase-space region with energy below E_F. A sudden displacement of the center of the harmonic potential corresponds to a shift of the center of the phase space [see Fig. 1(b)]. The blue region contains particles that are still in equilibrium in the new configuration of the trapping field. The red and yellow regions, containing particles moving on closed and open orbits, respectively, are instead out of equilibrium and give rise to a collective dipole motion. The phase-space region opens and melts during the dynamics as a consequence of the energy dependence of the single particle oscillation frequency [see Figs. 1(c) and 1(d)], yet leaving constant the phase-space volume (because of the Liouville theorem) and therefore preserving the Pauli principle. On the one hand, the red orbits dephase on a longer time scale with respect to the yellow ones. Therefore, the relaxation and the frequency of the oscillation mode are dominated by the particles moving around the center of the phase space, in the red region. On the other hand, the yellow orbits remain open, trapping the center of mass of the system on one side of the harmonic potential. Notice that in this 1D configuration, the damping of the oscillation disappears in the linear limit (small initial displacement) at $T = 0$.

Three-dimensional analysis.—The semiclassical phase-space distribution $f(\vec{r}, \vec{p}, t)$ is governed by the semiclassical Liouville equation

$$\frac{\partial f}{\partial t} + \frac{\partial H}{\partial \vec{p}}\frac{\partial f}{\partial \vec{r}} - \frac{\partial H}{\partial \vec{r}}\frac{\partial f}{\partial \vec{p}} = 0, \quad (4)$$

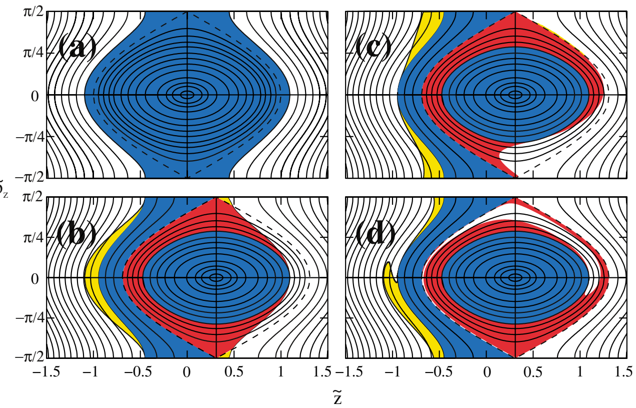

FIG. 1 (color). Phase trajectories for a trapped 1D Fermi gas in a lattice at $T = 0$, just before and after the displacement of the trap [(a) and (b), respectively], and their dynamical evolution [(c) and (d)]. The ordinate and abscissa are in units of $\tilde{p}_z \equiv p_z d/2\hbar$ and $\tilde{z} \equiv \sqrt{m\omega_z^2 z^2/4\delta}$, respectively.

where $H = H_0 + H_{\rm pert}$. Such an equation correctly describes the quantum dynamics up to orders of $O(\hbar^2)$ [6]. For small displacements, we can linearize the distribution function: $f(\vec{r}, \vec{p}, t) = f_0(\vec{r}, \vec{p}) + g(\vec{r}, \vec{p}, t)$, where $f_0(\vec{r}, \vec{p}) = \{\exp[(H_0(\vec{r}, \vec{p}) - \mu(T))/T] + 1\}^{-1}$ is the stationary fermionic solution of Eq. (4) for temperature T relative to H_0, and $g(\vec{r}, \vec{p}, t)$ is a small time-dependent correction induced by the perturbation $H_{\rm pert}(z, t)$. The dipole oscillations evolve according to [7]

$$\frac{\langle\hat{z}(t)\rangle}{z_0} = -1 + \frac{2m\omega_z^2}{\pi}\int_0^{+\infty} d\omega\left(\frac{1-\cos(\omega t)}{\omega}\right)\mathrm{Im}(\chi(\omega)), \quad (5)$$

where $\mathrm{Im}(\chi(\omega))$ is the imaginary part of the dipole response function $\chi(\omega)$. In order to calculate it, we solve Eq. (4) in the linear regime by the general method illustrated in [8], by using the energy of motion in the z direction $E_z = \varepsilon(p_z) + \frac{1}{2}m\omega_z^2 z^2$ as an independent variable instead of p_z. We obtain

$$\mathrm{Im}(\chi(\omega)) = -\frac{1}{(\hbar\omega_\perp)^2}\frac{\pi}{\hbar}\sum_{n=1}^{+\infty}\int_0^{+\infty} dE_z\delta[\omega - n\omega_0(E_z)]n\left[\int_0^{+\infty} dE_\perp E_\perp\frac{\partial f_0(E_z+E_\perp, T)}{\partial E_z}\right]Q^2(n, E_z), \quad (6)$$

where

$$Q(n, E_z) = -\frac{1}{n\pi}\int_{z_1(E_z)}^{z_2(E_z)} dz \sin\left(n\pi\frac{\omega_0(E_z)}{\omega_0(E_z, z)}\right), \quad (7)$$

$\omega_0(E_z)$ is given by Eq. (3), and $\omega_0(E_z, z)$ is obtained from Eq. (3) by replacing $z_2(E_z)$ with z. Notice that, because of the 3D nature of the system, all values $E_z \le E_F$ contribute to the response function (6) even at zero temperature, where the integral inside the square brackets becomes $(E_z - E_F)\Theta(E_F - E_z)$. As a consequence, a damping due to dephasing is in general present also in the linear limit, different from the 1D case. In tight binding approximation (large lattice height, corresponding to small tunneling between the potential barriers) the first band is $\varepsilon(p_z) = 2\delta\sin^2(p_z\lambda/4\hbar)$ where $\delta \equiv \delta(s)$ [9] is a decreasing function of the lattice height. With this choice for the dispersion, the frequency $\omega_0(E_z)$ can be expressed in terms of elliptic integrals [10], and we can calculate analytically the quantities of interest. In regimes out of tight binding we use a parametrization of the energy band [11].

Experiment.—We realize experimentally the system by using a Fermi gas of ^{40}K atoms. The sample is prepared in the $F = 9/2$, $m_F = 9/2$ state and sympathetically cooled with bosonic rubidium [12] to quantum degeneracy in a magnetic trap with frequencies $\omega_z = 2\pi \times 24\ \mathrm{s}^{-1}$ and $\omega_\perp = 2\pi \times 275\ \mathrm{s}^{-1}$. Subsequently a one-dimensional optical lattice is adiabatically superimposed along the weak axis of the trap (z direction) [2]. The lattice is created by a laser beam in standing-wave configuration, with wavelength $\lambda = 863$ nm. The lattice height can be adjusted in the range $U = 0.1 - 8E_R$, where $E_R/k_B = 317$ nK. The typical Fermi gas is composed by 3×10^4 atoms at a temperature that can be varied between $0.2T_F$ and T_F, where the Fermi temperature is $T_F \approx 330$ nK. The system is brought out of equilibrium by displacing the magnetic trap minimum along the lattice. The typical displacement is $z_0 = 15\ \mu$m, much smaller than the $1/e^2$ radius of the cloud (110 μm) [13]. After a variable evolution time in the trap, the atoms are released from the combined potential. We detect the position of the center of mass of the cloud by absorption imaging after a ballistic expansion of 8 ms.

Results.—In Fig. 2 we show both the theoretical prediction (solid line) and the experimental observation (solid circles) of the dipole oscillation of a Fermi gas at $T = 0.3T_F$ in the presence of a lattice with $s = 3$. For comparison, we show also the dipole motion in the absence of the periodic potential (dashed line and open circles), which consists in an undamped harmonic oscillation at the trap frequency ω_z. We observe a dramatic change of the dipole oscillation in the presence of the lattice, which is well described by the 3D model. The appearance of an offset in the oscillations is due to the significant fraction of particles moving along open orbits: for the given parameters the Fermi energy is indeed larger than the bandwidth $2\delta \approx 0.4E_F$. This fraction behaves macroscopically as an insulator, because its center of mass does not move under the harmonic force but stays trapped on one side. The fraction of the gas occupying closed orbits can instead oscillate in the harmonic potential, and has therefore a conducting nature. A damping appears as expected because of the dephasing between different orbits. Besides, the oscillation frequency is reduced because of the larger effective mass of the atoms in the lattice. The observed phenomena only weakly depend on the gas temperature, at least in the region $0.2 - 1\ T_F$ that we have explored so far in both experiment and theory [14]. We observe just a slow increase of both offset and damping for increasing temperatures, as expected because of the broader energy distribution of the gas. We instead find a much stronger dependence on the width of the first energy band. This effect is shown by a series of measurements performed by keeping the atom number and temperature of the Fermi gas constant (at 0.3 T_F), and varying the lattice height. In this way it was possible to move the Fermi energy within the energy gap between the first and the second bands of the lattice. In Figs. 3 and 4 we plot the measured dependencies of the offset, damping rate, and oscillation frequency, and we find a good agreement with the theoretical calculations. Figure 3 shows the crossover from a conducting behavior in low lattices to an almost completely insulating behavior in higher lattices. Here we have plotted the relative oscillation offset defined as z_{osc}/z_0, where z_{osc} is the center of oscillation of the system in the lattice, as a function of the lattice height. Note how the relative offset, which represents the insulating fraction of the Fermi gas, stays small as long as $2\delta < E_F$, and then raises quite rapidly toward unity. Since in the present experiment $E_F \approx E_R$, an insulating fraction appears already with low lattices; the theory, however, shows that the threshold for the insulation moves to higher lattices in case of smaller Fermi energies (dashed line in Fig. 3). The disagreement between experiment and

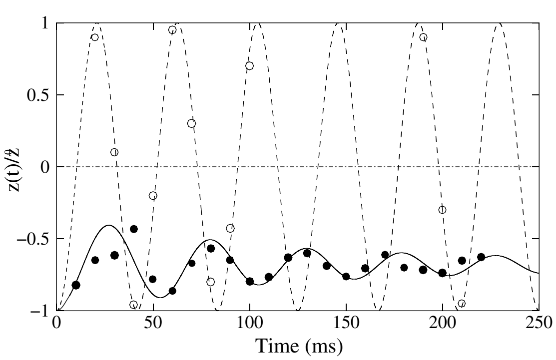

FIG. 2. Dipole oscillations of the Fermi gas of ^{40}K atoms at $T = 0.3T_F$ in the presence (filled circles and full line) and in the absence (empty circles and dotted line) of a lattice with height $s = 3$. The lines are the theoretical predictions, and the circles are the experimental results. The horizontal dot-dashed line represents the trap minimum.

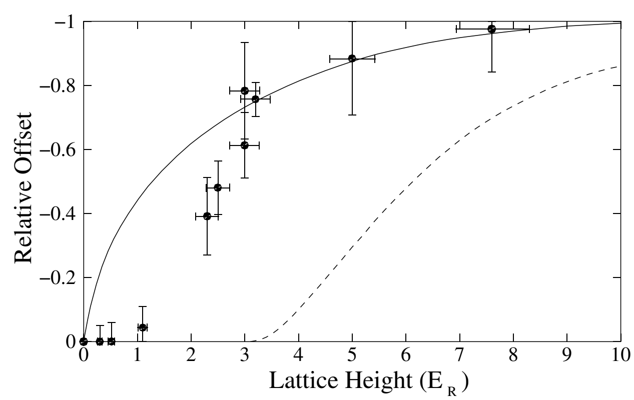

FIG. 3. Relative offset of the oscillations of a Fermi gas in the lattice (normalized to the initial displacement) as a function of the lattice height. The circles are the experimental data and the continuous line is the theoretical prediction for 3×10^4 atoms at $T = 100$ nK. The dashed line is the prediction for a Fermi gas of 2.5×10^3 atoms at $T = 0$.

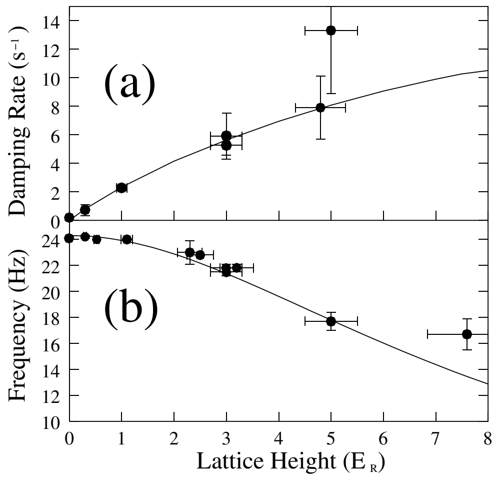

FIG. 4. (a) Comparison between theory (line) and experiments (circles) for the damping rate of the dipole oscillations of the Fermi gas as a function of the lattice height. (b) Oscillation frequency of the Fermi gas as a function of the lattice height. The line is the expectation for a particle oscillating at the band bottom.

theory at low lattice heights, $s < 3$, can arise from the population of higher bands due to the finite temperature and/or Landau-Zener tunneling, which are not included in the theoretical model. Increasing s, the energy gap between bands increases, and the single band calculations become more realistic. In Fig. 4 we show the observed features of the conducting fraction of the gas. The damping rate of the oscillation shown in Fig. 4(a) also increases with the lattice height, because of an increased dispersion of the oscillation frequencies of atoms in closed orbits [Eq. (3)]. As shown in Fig. 4(b), the oscillation frequency of this component of the gas is close to that expected for a particle at the bottom of the band $\omega_0(E_z = 0)$. Actually, the theoretical analysis shows that this value is strictly reached only asymptotically during the oscillation, because initially closed orbits with different frequencies contribute to the dipole motion.

We note that the insulating behavior of the Fermi gas persists even at $T = 0$ as long as E_F lies in the band gap. This is a consequence of the Pauli principle which keeps the energy distribution broad. An analogous insulating phenomenon can be observed also in uncondensed Bose gases at temperatures larger than the bandwidth. Indeed, also in this case a significative part of the single particle orbits will have energy in the gap region giving rise to insulation. However, we have observed [15] that in the case of bosons, or even in the case of fermions admixed with bosons, interatomic collisions quench the open orbits and eventually bring the system into the equilibrium position. When approaching $T = 0$, bosons condense and tend to show a purely conductive behavior [16], in contrast to fermions.

Conclusion.—In this Letter we have investigated the dynamics of a noninteracting Fermi gas in the presence of a combined periodic and harmonic potential, comparing the experimental observation with a semiclassical theory. We have shown that the system crosses from a conducting to an insulating behavior by tuning the Fermi energy relatively to the first energy band of the lattice. The present work provides a basis for further investigation of interacting Fermi gases in lattices, in both normal and superfluid phases, including possible superfluid Josephson-like oscillations [17], in analogy to what is already observed in Bose-Einstein condensates [16].

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[1] S. Jochim *et al.*, Science **302**, 2101 (2003); M. Greiner *et al.*, Nature (London) **426**, 537 (2003); M.W. Zwierlein *et al.*, Phys. Rev. Lett. **91**, 250401 (2003); C. A. Regal *et al.*, Phys. Rev. Lett. **92**, 040403 (2004); M.W. Zwierlein *et al.*, Phys. Rev. Lett. **92**, 120403 (2004); T. Bourdel *et al.*, cond-mat/0403091 [Phys. Rev. Lett. (to be published)]; J. Kinast *et al.*, Phys. Rev. Lett. **92**, 150402 (2004); M. Bartenstein *et al.*, Phys. Rev. Lett. **92**, 203201 (2004).
[2] G. Modugno *et al.*, Phys. Rev. A **68**, 011601(R) (2003).
[3] W. Hofstetter *et al.*, Phys. Rev. Lett. **89**, 220407 (2002).
[4] M. Rigol *et al.*, Phys. Rev. Lett. **91**, 130403 (2003).
[5] A. Albus *et al.*, Phys. Rev. A **68**, 023606 (2003); H. P. Büchler *et al.*, Phys. Rev. Lett. **90**, 130401 (2003); M. Lewenstein *et al.*, Phys. Rev. Lett. **92**, 050401 (2004).
[6] E. P. Wigner, Phys. Rev. **40**, 749 (1932).
[7] L. Pitaevskii and S. Stringari, *Bose-Einstein Condensation* (Oxford Science Publications, Oxford, 2003).
[8] D. M. Brink, A. Dellafiore, and M. Di Toro, Nucl. Phys. **A456**, 205 (1986).
[9] M. Krämer *et al.*, Eur. Phys. J. D **27**, 247 (2003).
[10] M. Abramowitz and I. A. Stegun, *Book of Mathematical Functions* (National Bureau of Standards, Washington, DC, 1984).
[11] For small values of the lattice height we fit the band dispersion with $\varepsilon(P_z) = c\frac{P_z^2}{2m}[\exp(a(|\frac{P_z d}{2\hbar}| - b)) + 1]^{-1}$, where a, b, c are free parameters.
[12] G. Roati *et al.*, Phys. Rev. Lett. **89**, 150403 (2002).
[13] We have checked that the observed phenomenology does not depend on the displacement up to $z_0 = 35\ \mu$m, justifying the use of a linear approach in the theory.
[14] An extension of the analysis to higher temperatures requires one to take into account the thermal occupation of the excited energy bands.
[15] H. Ott *et al.*, Phys. Rev. Lett. **92**, 160601 (2004).
[16] F. S. Cataliotti *et al.*, Science **293**, 749 (2001).
[17] M. Wouters *et al.*, cond-mat/0312154 [Phys. Rev. A (to be published)].