Pattern of Breakdown of Laminar Flow into Turbulent Spots

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The breakdown into turbulent spots is the least understood stage of the laminar-turbulent transition process. With cellular-automaton stochastic simulations and stability analysis, we show that the pattern of breakdown in boundary-layer flow bears a connection to laminar instability and may be reconstructed using macroscopic properties of the transition zone, such as persistence times and transitional intermittency. We propose experimental tests of our ideas.

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The transition to turbulence in a boundary layer (Fig. 1) has several features which distinguish it from the internal shear flows [1–3] receiving attention recently: the Reynolds number $R = U\delta/\nu$, where U, $\delta(x)$, and ν are the free-stream velocity, boundary-layer thickness, and kinematic viscosity, respectively, varies with streamwise position x. As x increases, laminar flow is destabilized progressively, leading to a moving, spatially periodic vorticity pattern (Fig. 2). At larger x, we enter the *transition zone* where flat, arrowhead shaped *turbulent spots* first appear [5], and travel downstream, widening, lengthening, and merging to fill the entire boundary layer. Spots are the building blocks of boundary-layer turbulence, and their statistics and dynamics contain the physics of the crossover from unstable laminar to fully turbulent behavior. Numerical simulations and experiments have followed the creation and downstream propagation of single spots in great detail, but the relation of turbulence to boundary-layer instability has not been understood, in part because spot birth *statistics*, involving the detection of tiny patches of turbulence as they are born as a function of space and time, is difficult to obtain numerically or experimentally.

In this Letter we question the conventional wisdom that spot birth is completely random in time and uniformly distributed in the spanwise direction. We ask, Does the regular pattern observed in the precursor to transition to turbulence give rise to any local regularity in the birth of turbulent spots? As a direct experimental verification is difficult, can we connect spot birth distribution to macroscopic measures in the transition zone? To answer these we perform stochastic cellular-automaton simulations of spot breakdown (birth) and propagation in the transition zone. Our main finding is that spot birth is clearly related to the pattern of laminar instability in decelerating flows, while in the flow over a flat plate the connection is less clear. We explain the experimentally observed behavior of turbulent intermittency, $\gamma(x)$ (defined as the fraction of time the flow is turbulent). We establish that macroscopic measures like γ, the burst rate B, and the persistence time W of laminar flow can tell us whether the breakdown is regular (the term is used here to mean that there is a local pattern in the breakdown) or random and, if regular, what the two-dimensional pattern is. Our predictions (Figs. 4–7) on how these quantities behave are (i) directly testable in experiment and (ii) of value in inferring spot birth scenarios from experimental results.

In shear flows, the threshold Reynolds number for transition to turbulence [2] and the mechanisms triggering instability depend on the strength q of background noise. For example, at low q (less than 0.2% of the kinetic energy of the flow), a linear instability of the laminar flow results in growing Tollmien-Schlichting (TS) waves [6]. The secondary instability of these waves, denoted here by T2, gives rise to a pattern of aligned (harmonic) or, more frequently, staggered [subharmonic; see Fig. 2(a)] vorticity crests [6,7]. At high q, the interaction of several modes leads to large transient growth (due to the non-normality of the stability operator), often resulting in streamwise streaks [4,8,9]. The secondary instability (S2) of these streaks gives rise to patterns such as that shown in Fig. 2(b) [4,10]. So far as we know, a connection between this stage and the nature of the transition zone has never been made.

Current wisdom, largely phenomenological, on turbulent spots may be summarized as follows: a turbulent spot is an almost two-dimensional arrowhead shaped object (parallel to the wall), which, for a given pressure gradient, remains self-similar as it grows [11,12]. These features are similar at any level of q. We define x_t, upstream of which no turbulent spots are born, as the location of onset of transition to turbulence. If U is the free-stream velocity at x_t, ζ the half angle subtended by the spot at the streamwise location of its origin, and U_h and U_r the respective speeds with which its head and rear convect, then the

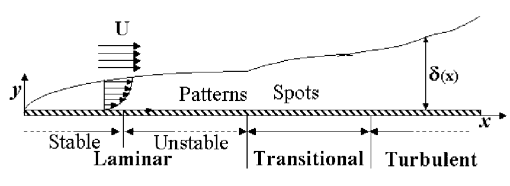

FIG. 1. The laminar-turbulent transition process on a boundary layer in the flow past a semi-infinite flat plate.

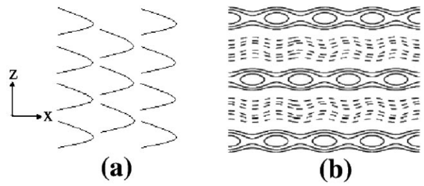

FIG. 2. Examples of two-dimensional patterns of disturbance vorticity observed in a destabilized laminar boundary layer; z is the spanwise coordinate. (a) T2: a subharmonic secondary instability of Tollmien-Schlichting waves; (b) S2: secondary instability of streamwise streaks [4].

nondimensional spot propagation parameter, which accounts for both streamwise and lateral growth,

$$\sigma \equiv [U_r^{-1} - U_h^{-1}]U \tan\zeta, \tag{1}$$

is constant for a given pressure gradient [11]. The orientation of the flow past a solid object can generate a streamwise variation in the free-stream velocity. For the flow past a tilted flat plate, which is under consideration here, $U \simeq x^m$ (appropriately nondimensionalized), where m is a constant defining the pressure gradient. A negative m denotes an adverse pressure gradient, i.e., decelerating flow. On a flat plate, $U_h \simeq 0.9U$, $U_r \simeq 0.5U$, and $\zeta \simeq 10°$ (see, e.g., [11]), while in strong adverse pressure gradients ($m = -0.06$), $U_h \simeq 0.9U$, $U_r \simeq 0.4U$ [13], and $\zeta \simeq 20°$ [14]. Given a particular disturbance environment, the nature and amplification of instabilities, and the consequent breakdown into spots, depend on Reynolds number $\mathrm{Re}(x)$ alone. It is thus expected that most spots are born within a short neighborhood downstream of x_t; i.e., we may assume concentrated breakdown around x_t [15]. This is accepted as a good model for spot breakdown, being borne out repeatedly by indirect inferences made from experiment.

We realize that the pattern of spot breakdown will manifest itself in the scenarios for spot merger: if spots are more closely spaced in the spanwise direction z than in x, lateral mergers would be more frequent than longitudinal. It can be easily worked out that transition zone characteristics depend crucially on the merger scenario; e.g., if lateral mergers rather than longitudinal ones were to be predominant, the loss of area covered by turbulence would be greater, so this scenario would result in a slower realization of full turbulence, *for the same mean spot birth rate.* We therefore study different scenarios of spot birth: (i) mostly regular, with the pattern in (x, z, t) related to laminar instability; (ii) random, with spots appearing according to a Poisson distribution in time, and uniformly in the spanwise coordinate z; and (iii) combinations of the two. We now construct our stochastic model and show how our results are obtained.

The transition zone (as viewed from above) is discretized into an $L_x \times L_z$ (200×400) rectangular grid in x and z, respectively (Fig. 3). Each site is assigned an integer $\chi(X, Z, T)$, equal to 0 if the flow there is laminar and 1 if it is turbulent. (Uppercase represents discretized quantities.) All spots are born at $X = 1$, corresponding to $x = x_t$ (distribution over a small streamwise distance is seen not to affect results significantly). Each spot is approximated to be triangular, but the precise shape is found to be immaterial. During each time interval ΔT, the head of a spot moves forward by two grid locations, while its rear moves by one location. Simultaneously, the spot spreads laterally on each side by one spanwise grid location, so the aspect ratio $\Delta Z/\Delta X$ of the grid corresponds to the tangent of the half angle ζ of lateral spot propagation. These are the simplest discrete approximations of the spot growth described above. Spot statistics are obtained over 5–20 $\times 10^6$ time steps (after achieving a statistically stationary state) by prescribing a particular rule for spot generation. Edge effects are avoided by using a periodic boundary condition in Z.

In regular breakdown, spot birth is periodic in the spanwise direction and staggered in time. During the first time interval, one spot is born at every $Z = kN_z + i$, $k = 0, 1, 2 \cdots L_z/N_z$, $0 \leq i \leq N_z$. After N_t time steps, spots form at $Z = kN_z + j$ for harmonic and $Z = (k + 1/2)N_z + j$ for subharmonic breakdown, where $j = i + 1$. The spot formation rate n is thus $= L_z/(N_z \Delta Z N_t \Delta T)$ per unit spanwise length per unit time. This prescription corresponds to spots forming at the crests of an oblique pattern of spanwise and streamwise wave numbers $\beta = 2\pi/N_z\Delta Z$ and $\alpha = 2\pi/N_t\Delta T v$, respectively, where v is the streamwise velocity of the wave crest. For T2 breakdown, we conduct a secondary instability analysis in the standard manner [16], as described in detail in [17]. From the obliqueness of the most unstable secondary mode, the most likely ratio of N_z/N_t is found to be about 2.5 for flat-plate flow and 4.9 at $m = -0.06$. The Reynolds numbers used (600 and 220, respectively, based on boundary-layer momentum thickness) are consistent with experimental transition onset. Because of amplitude modulations and other reasons, the ratio β/α of the vorticity crests are related to, but not completely specified by, the corresponding ratio of the most unstable secondary mode; we use instability computations only to provide an estimate. Note that the pattern we speak of is a purely localized entity in time and space. Instabilities are sensitive to randomness in the environment [18], so averaged over long times, breakdown is uniform in the spanwise

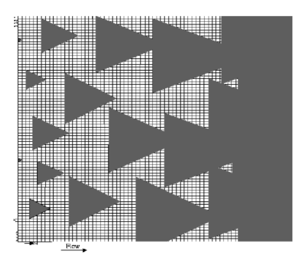

FIG. 3. Schematic of the simulation with random spot birth: according to a Poisson distribution in time, and uniformly distributed in z.

direction and random in time. In the simulations, the phase of the oblique wave is randomized with a small probability (1%), and a fraction (5%) of the spots are generated randomly; results are insensitive to doubling or halving these numbers.

The transitional intermittency obtained from the simulations with a regular subharmonic pattern as estimated from T2 is shown in Fig. 4(a) to closely follow experimental results in a highly decelerating flow. The simulations at other similar pressure gradients (not shown) match corresponding experiments equally well. For random spot generation, the intermittency γ is given by

$$\gamma = 1 - \exp[-n\sigma(x - x_t)^2/U], \tag{2}$$

n being the mean spot birth rate per unit time per unit spanwise length. For convenience of data presentation, we define an intermittency parameter $F \equiv \sqrt{-\ln(1-\gamma)}$ which varies linearly with x in (2) [15]. Shown in Fig. 4(b) are simulation results in terms of F for various birth scenarios. In the early stages, when the spots are too small to "see" each other, the intermittency is independent of the nature of breakdown, as expected. At higher x, for the same mean breakdown rate and identical spot growth, the character of the intermittency is seen to be very different, telling us that spot merger plays a large role in transition to turbulence. This is a new insight, since existing literature takes the mean rate of birth to *define* the transition zone. The downstream increase in slope usually seen in experiments on decelerating boundary layers, unexplained until now, is seen to be characteristic of dominant longitudinal merger, where the head of one spot merges with the rear of the one ahead.

For a flat plate, on the other hand, experiments at low q seem closer to a random birth scenario [Fig. 5(a)]. Since the instability is less oblique, regular breakdown, too, leads to a combination of lateral and streamwise merger similar to random spot birth. Also, the scope for randomness is higher since (i) we find the obliqueness to be a strong function of background disturbance amplitude, unlike in decelerating flow, and (ii) the instability is weaker, and the unstable region longer [14,22]. At higher levels of free-stream disturbance, the S2 route takes place [21] for $m = 0$; N_z/N_t is estimated from [23] to be 5. Simulations with this ratio show an excellent agreement with the measurements [21] at a free-stream turbulence of 1.5% [Fig. 5(b)]. We confirm that less stringent constraints of concentrated breakdown, e.g., as shown in Fig. 5(a), do not change the qualitative behavior for any case. In decelerating flows it was conjectured by [24] that rapid transition is due to breakdown once every TS cycle. Our position is that the breakdown is dictated by a secondary pattern, e.g., T2 or S2 (not the TS which lose their identity well upstream); we explain not only the extent of the transition zone, but the qualitative behavior within it.

The spot birth scenario can be reconstructed with the help of several other easily measurable quantities, such as the burst rate B, defined here as the number of switchovers from laminar to turbulent flow per unit time at a given location; and the persistence time distribution of laminar flow. For a Poisson process, we can derive that $B \propto (1 - \gamma)[-\ln(1-\gamma)]^{1/2}$. Our simulations with a random breakdown give rise to burst rates which agree well with this relationship (Fig. 6). Any deviation from this behavior is a sign that spot breakdown is not random: the more the regularity, the more symmetric the distribution with respect to $\gamma = 0.5$. The probability density function of the persistence time W, defined here as the extent of time that the flow continuously remains laminar at a given x, is plotted in Fig. 7. Strings of zeros between two 1's are measured in the middle of the span to avoid repeated input from a given spot. With random spot birth, an exponential decay is expected and obtained, whereas in the case of regular breakdown a large modulation and a flat portion are evident in the early and middle stages of

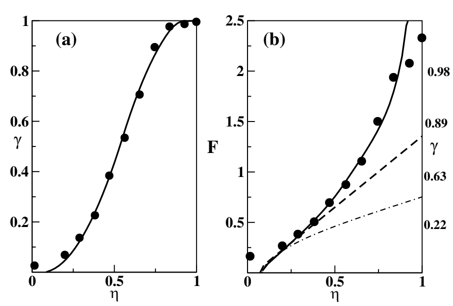

FIG. 4. (a) Intermittency in a decelerating flow, $m = -0.06$. η is streamwise distance scaled by transition zone length. Solid line: stochastic simulation, with $N_z = 49, N_t = 10$, similar to the dominant secondary mode; symbols: experiment [19]. (b) The parameter F for data in (a). Also shown are the simulation results of random breakdown (long dashes), and of a regular breakdown where lateral merger would be dominant, $N_z = 10, N_t = 40$ (dot-dashed line). Experimental error is likely to be significant at either end of the transition zone, since switches from laminar to turbulent flow are difficult to distinguish. Also, when $\gamma \sim 1$, F depends sensitively on γ, so errors appear amplified.

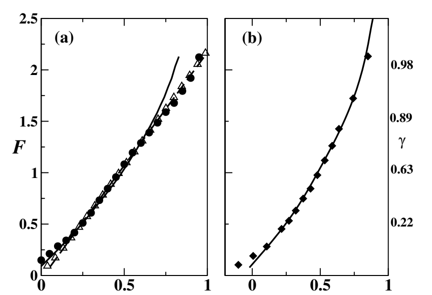

FIG. 5. Intermittency in flat-plate flow. (a) T2 route: circles, experiment [20]; solid line, regular breakdown; $N_z/N_t = 2.5$; long dashes, random breakdown; triangles, random spot birth, distributed over $X = 1$ to 10. (b) S2 route: symbols, experiment [21]; line, regular breakdown; $N_z/N_t = 5$.

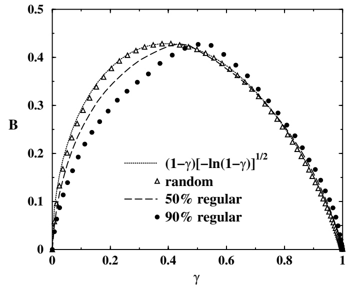

FIG. 6. Variation of the burst rate with the intermittency for different scenarios of spot birth. The regular pattern conforms to subharmonic T2.

transition, respectively. The distribution and its downstream variation are distinctive (neither resembling a power law nor a Poisson process), and can be used to define spot birth effectively. The other persistence time, of turbulent portions (being independent of statistics), is uninteresting.

To summarize, stochastic cellular-automaton simulations of spot generation and propagation in transitional boundary layers have been conducted. In conjunction with experimental measurements, the new approach can help one to understand the transition to turbulence. In highly decelerating flows, a locally regular pattern of breakdown into turbulent spots as dictated by our computations of secondary instability gives rise to transitional intermittency behavior as observed (and unexplained up to now), whereas measurements in flat-plate flow seem more consistent with random breakdown. The scenario of merger of turbulent spots is shown to be an important player in the laminar-turbulent transition process. Remarkably, the pattern of spot birth may be inferred from macroscopic characteristics of the transition zone. To test our predictions, we advocate experimental measurements of the distributions of B, W, and γ in the flow past plates subjected to different, *especially adverse*, pressure gradients. In this work, we have used Navier-Stokes equations for computing secondary instability, and a phenomenological model of spot growth thereafter. Direct solutions of the Navier-Stokes equations, increasingly feasible at Reynolds numbers large enough for transitional flow, will prove illuminating, especially with respect to spot propagation, even if entire statistics are not possible to obtain.

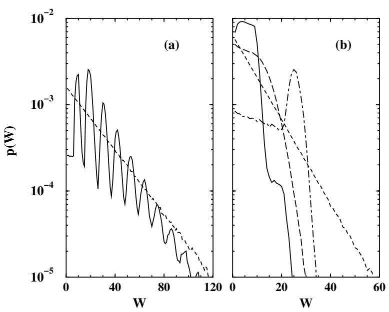

FIG. 7. Probability density function of persistence time of laminar flow at (a) $\gamma = 0.1$ and (b) $\gamma = 0.5$. Dashed line, random breakdown; solid line, 90% regular (subharmonic T2); long dashes, 50% regular. Thin dot-dashed line, predominant lateral merger as in Fig. 4(b).

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