Renormalization Flow of QED

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We investigate textbook QED in the framework of the exact renormalization group. In the strong-coupling region, we study the influence of fluctuation-induced photonic and fermionic self-interactions on the nonperturbative running of the gauge coupling. Our findings confirm the triviality hypothesis of complete charge screening if the ultraviolet cutoff is sent to infinity. Though the Landau pole does not belong to the physical coupling domain owing to spontaneous chiral-symmetry-breaking (χSB), the theory predicts a scale of maximal UV extension of the same order as the Landau pole scale. In addition, we verify that the χSB phase of the theory which is characterized by a light fermion and a Goldstone boson also has a trivial Yukawa coupling.

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Though quantum field theory celebrates its greatest triumph with quantum electrodynamics (QED), the high-energy behavior of QED remains a sore spot, since it is inaccessible to the otherwise successful perturbative concepts. For instance, keeping the renormalized coupling e_R fixed, small-coupling perturbation theory predicts its own failure in the ultraviolet (UV) in the form of the Landau pole singularity:

$$\frac{1}{e_R^2} - \frac{1}{e_\Lambda^2} = \beta_0 \ln\frac{\Lambda}{m_R}, \quad \beta_0 = \frac{N_f}{6\pi^2}. \tag{1}$$

The coupling e_Λ at the UV cutoff Λ diverges for $\Lambda \to \Lambda_L = m_R \exp[1/(\beta_0 e_R^2)]$. It was early realized [1] that this behavior can signal the failure of QED as a fundamental quantum field theory which should be valid on all length scales. From a different viewpoint, keeping the initial UV coupling e_Λ fixed, the renormalized coupling e_R vanishes in the limit $\Lambda \to \infty$, resulting in a free, or "trivial," theory with complete charge screening.

Already in the dawning of the renormalization group (RG), a possible alternative scenario was discussed [2] in which an interacting UV-stable fixed point of the RG transformation, $e_\Lambda^2 \to e_*^2 \in (0, \infty)$ for $\Lambda \to \infty$, facilitates a finite UV completion of QED ("asymptotic safety" [3]). However, no sign of such a fixed point has been found so far. On the contrary, nonperturbative lattice simulations have provided evidence for triviality [4] (see also [5]). Moreover, careful extrapolation of raw lattice data shows that the Landau pole singularity is outside the physical parameter space owing to the onset of spontaneous chiral-symmetry-breaking (χSB) [4]. This strong-coupling phenomenon of χSB has also been observed in analytical studies using truncated Dyson-Schwinger equations (DSE) in a quenched approximation [6].

In addition to the fundamental character of this problem as a matter of principle, the high-energy fate of QED or other standard-model building blocks and its further extensions can give us direct bounds on the scale where new physical phenomena may be expected. In particular Landau pole singularities of the type of Eq. (1) are used to constrain properties of hypothetical particles, such as the Higgs scalar in the standard model [7]. Our work is moreover motivated by the recent observation that a hypothetically nontrivial U(1) sector of the standard-model with a UV-stable fixed point has the potential to solve the hierarchy problem of the Higgs sector [8].

In this Letter, we report on nonperturbative results obtained from the RG flow equation for the effective average action Γ_k [9]. We work in Euclidean spacetime continuum where our methods can easily bridge a wide range of scales, allow for the full implementation of chiral symmetry as well as a simple inclusion of bare masses (explicit χSB terms), and furnish unquenched calculations.

The effective average action is a free-energy functional that interpolates between the initial UV action $\Gamma_{k=\Lambda}$ and the full quantum action $\Gamma_{k\to 0}$. The infrared (IR) regulator scale k separates the fluctuations with momenta $p^2 \gtrsim k^2$, the effect of which has already been included in Γ_k, from those with smaller momenta which have not yet been integrated out. The full RG trajectory is given by the solution to the flow equation [$t = \ln(k/\Lambda)$],

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{STr}\, \partial_t R_k (\Gamma_k^{(2)}[\phi] + R_k)^{-1}, \tag{2}$$

where $\Gamma_k^{(2)}$ denotes the second functional derivative with respect to the fields $\phi = (A_\mu, \bar\psi, \psi)$, and the regulator function R_k implements the infrared regularization at $p^2 \simeq k^2$. Effectively, Eq. (2) is a smooth realization of the Wilsonian momentum-shell integration, being dominated by momenta $p^2 \simeq k^2$.

On microscopic scales, QED is defined by the action

$$S_\Lambda = \int_x \left(\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar\psi i \not{D}[A] \psi + \bar\psi \gamma_5 m_\Lambda \psi \right), \tag{3}$$

which involves the microscopic UV parameters e_Λ and m_Λ, and $D_\mu[A] = \partial_\mu - i e_\Lambda A_\mu$. Further possible gauge-invariant interactions are RG irrelevant by power counting. Invoking the universality hypothesis, the IR physics should only depend on the parameters occurring in Eq. (3). This hypothesis can nevertheless be questioned: since the coupling increases towards the UV, higher operators can acquire large anomalous dimensions that spoil naive power counting and enlarge the set of RG relevant operators, offering new routes to UV completion. These operators may in turn exert a strong influence on the running gauge coupling and potentially induce an interacting fixed point. In order to test this scenario quantitatively, we study how the running of the gauge coupling can be affected by photonic self-interactions of the type

$$\Gamma_{k,A} = \int_x W(\theta) = \int_x \Big(W_1 \theta + \frac{W_2}{2}\theta^2 + \frac{W_3}{3!}\theta^3 + \ldots \Big), \quad (4)$$

where $\theta = (1/4) F_{\mu\nu} F_{\mu\nu}$. Thus, we include infinitely many fluctuation-induced photon operators in our truncation of Γ_k ($W_1 \equiv Z_F$ denotes the wave function renormalization of the photon). Of course, there are further tensor structures involving, e.g., the dual field strength that can contribute to the flow, but we do not expect their influence on the running coupling to be qualitatively different from those of Eq. (4). Moreover, our truncation neglects the momentum dependence of the couplings W_i. Since it is natural to assume that their strength will drop off with increasing external momenta, we expect that momentum dependencies imply a weaker influence on the gauge coupling than is estimated by Eq. (4). Note that this argument could be invalidated by the occurrence of yet unknown photonic bound states giving rise to momentum poles in the couplings W_i.

Fluctuations induce not only photonic but also fermionic self-interactions, the lowest order of which we include in the fermionic part of the truncation,

$$\Gamma_{k,\psi} = \int_x \Big[\bar\psi (i Z_\psi \partial\!\!\!/ + Z_1 e_\Lambda A\!\!\!/ + Z_\psi m \gamma_5)\psi + \frac{1}{2}(Z_- \bar\lambda_- (V-A) + Z_+ \bar\lambda_+ (V+A)) \Big], \quad (5)$$

where $(V \pm A) := (\bar\psi \gamma_\mu \psi)^2 \mp (\bar\psi \gamma_\mu \gamma_5 \psi)^2$. These fermionic interactions do not only influence the running of the gauge coupling but are also essential for the approach to χSB in reminiscence of the Nambu–Jona-Lasinio (NJL) model. The k-dependent dimensionless running couplings are related to the bare couplings e_Λ, $\bar\lambda_\pm$ by

$$e = \frac{e_\Lambda Z_1}{Z_F^{1/2} Z_\psi}, \qquad \lambda_\pm = \frac{Z_\pm k^2 \bar\lambda_\pm}{Z_\psi^2}. \quad (6)$$

QED initial conditions for the flow are defined by $Z_F, Z_\psi, Z_1|_\Lambda = 1$ and $W_{i>1}, \bar\lambda_\pm|_\Lambda = 0$.

Inserting this truncation into Eq. (2), we obtain the β functions for e, $\lambda_\pm$, m, Z_F, W_i, and Z_ψ, once the regulator R_k is specified. Of central interest is the photon anomalous dimension $\eta_F = -\partial_t \ln Z_F$ which contains the photon self-interaction contributions to the β_{e^2} function, $\beta_{e^2} = \eta_F e^2 + \ldots$ [cf. Eq. (9) below]. In order to deal with the photon sector of Eq. (4), we use techniques developed in [10] that employ background-field-dependent and chiral-symmetry-preserving regulators of the form

$$R_k^\psi(i\bar{D\!\!\!/}) = Z_\psi i\bar{D\!\!\!/}\, r_{\rm F}[(i\bar{D\!\!\!/})^2/k^2], \qquad R_k^A(\bar\Gamma^{(2)}_{k,A}) = \bar\Gamma^{(2)}_{k,A}\, r[\bar\Gamma^{(2)}_{k,A}/(Z_F k^2)], \quad (7)$$

where the bar indicates background-field dependence, and $r(y)$, $r_{\rm F}(y)$ denote dimensionless regulator shape functions. As a result, we arrive at an asymptotic series for η_F to all orders of the coupling,

$$\eta_F = \sum_{n=1}^\infty a_n(r; m, \eta_\psi)\Big(\frac{e^2}{16\pi^2}\Big)^n = \frac{N_f}{6\pi^2} e^2 + \mathcal{O}(e^4, m^2 e^2),$$

where the coefficients a_n depend functionally on the regulator shape functions r, r_F. Here, the structure of the all-order result arises from the feedback of the flow of the W_i's on η_F, whereas the global shape of the function $W(\theta)$ has been neglected [10]. To one-loop, we obtain the correct universal β_{e^2} function coefficient, since $\beta_{e^2} = \eta_F e^2 + \ldots$. To higher order, the result is explicitly regulator dependent as it should be, since only the existence of zeros of the β_{e^2} function and their critical exponents are universal [11]. Now, QED could evade triviality if a UV-stable fixed point in β_{e^2} and η_F existed for all regulators. By contrast, our results show that $\eta_F(e_*^2) = 0$ has only the solution $e_*^2 = 0$ for *all* regulator shape functions r, r_F. In fact, for all physically admissible regulators a lower bound $0 < \eta_F^{\text{1-loop}}/2 \le \eta_F[r]$ exists for all values $e^2 > 0$. In the strong-coupling regime, this lower bound is satisfied by Litim's optimized regulator [12],

$$r_F(y) = \frac{1}{\sqrt{y}}(1-\sqrt{y})\theta(1-y), \qquad r(y) = \frac{1}{y}(1-y)\theta(1-y),$$

for which the all-order anomalous dimension yields a simple integral representation,

$$\eta_F = \frac{e^2 N_f}{6\pi^2}\frac{1-\eta_\psi}{1+m^2/k^2}[1 - I(e^2)], \qquad I(e^2) = \frac{1}{\pi^2}\int_0^\infty ds\, s^2 K_2(2\sqrt{s}) \mathrm{Li}_2(e^{-4\pi^2/e\sqrt{3/s}}), \quad (8)$$

involving a modified Bessel function K_2 and the polylogarithm Li. In the strong-coupling limit, $e^2 \to \infty$, the integral goes to $I(e^2) \to 1/2$, such that the strong-coupling limit finally approaches half the one-loop result [13]. Moreover, the explicit electron mass dependence illustrates the threshold behavior: once the IR scale k drops below the electron mass scale, fluctuations become strongly suppressed and the flow essentially stops.

The fermionic self-interactions also contribute directly to the β_{e^2} function. The detailed form can be read off from a Ward-Takahashi identity as demonstrated in [8],

$$\partial_t e^2 \equiv \beta_{e^2} = \eta_F e^2 + 2e^2 \frac{\sum_{i=\pm} c_i \partial_t \lambda_i}{1 + \sum_{i=\pm} c_i \lambda_i}, \tag{9}$$

where $c_+ = N_f/(4\pi)^2$, $c_- = (N_f + 1)/(4\pi)^2$ for the optimized regulator. From this representation, it is apparent that if η_F does not induce a UV-stable interacting fixed point, no such fixed point can be induced at all, since $\partial_t \lambda_\pm \to 0$ at a global fixed point. (Explicit representations of the $\lambda_\pm$ flows can be found in [8].) As one of our main results, we therefore exclude such a fixed point for the resolution of the triviality problem. We have confirmed that even higher-order fermionic and fermion-gauge field interactions cannot modify the qualitative structure of Eq. (9). The gauge coupling hence is generally not bounded from above for increasing UV cutoff.

In order to deal with the phenomenon of χSB that we expect for strong coupling, we use partial bosonization techniques as developed in [14] in order to study the formation of the chiral condensate and a dynamical fermion mass. Moreover, we can treat dynamical as well as explicit fermion masses on the same footing by translating the fermion self-interactions as well as the fermion mass into a bosonic sector of the form

$$\Gamma_{k,\phi} = \int Z_\phi |\partial_\mu \phi|^2 + \mathrm{U}(\phi) + \bar{h}(\bar{\psi}_R \psi_L \phi - \bar{\psi}_L \psi_R \phi^*). \tag{10}$$

Here we concentrate on the scalar boson in the s channel. We truncate the scalar potential to the simple form

$$\mathrm{U}(\phi) = \bar{m}_\phi^2 \phi^* \phi + \frac{1}{2} \bar{\lambda}_\phi (\phi^* \phi)^2 - \frac{1}{2} \bar{\nu}(\phi + \phi^*), \tag{11}$$

where the $\bar{\nu}$ term breaks chiral symmetry explicitly and thus carries the information about an explicit electron mass; if $\bar{\nu} = 0$ vanishes at any scale it vanishes at all scales by chiral symmetry (massless QED). Spontaneous χSB is monitored by the sign of $\bar{m}_\phi^2$, negative values indicating an induced chiral condensate.

Following the techniques of [14], we trade the four-fermion interactions and the electron mass for the parameters occurring in Eq. (10), such that, for instance, the resulting electron mass can be deduced from

$$m = \bar{h}|\phi_0|/Z_\psi, \tag{12}$$

where ϕ_0 denotes the minimum of the potential (11). We would like to stress that the fermion-boson translation employed here is a highly efficient technique for controlling the (approximate) chiral symmetry together with its explicit breaking by the mass; no fine tuning of the bare mass is necessary and there is no proliferation of symmetry-breaking operators as in a purely fermionic formulation. Together with the β functions for the bosonic sector (see [15]), we can evaluate the RG trajectory of the complete system for a variety of initial conditions. Although the number of parameters has seemingly increased, the system remains solely determined by the choice of the gauge coupling and the electron mass, owing to the existence of an IR stable "bound-state" fixed point [14,15]. This is a manifestation of universality: the physics at large distance scales is independent of the details of the microscopic interactions.

For the quantitative analysis, we work in the Landau gauge which is a fixed point of the RG, and we concentrate on the $N_f = 1$ case where the "chiral" symmetry is given by $\mathrm{U}_F(1) \times \mathrm{U}_A(1)$, i.e., fermion-number and axial U(1)'s with χSB corresponding to the breaking of $\mathrm{U}_A(1)$. At zero bare mass, $m_\Lambda = 0$, i.e., without explicit χSB, our analysis reveals two phases separated by a critical coupling e_{cr}^2. For $e_\Lambda^2 \le e_{\mathrm{cr}}^2$, chiral-symmetry is preserved and the electron remains massless. For $e_\Lambda^2 > e_{\mathrm{cr}}^2$, χSB renders the electron massive and a Goldstone boson arises from the ϕ field. Switching on an explicit electron mass, the transition between the two phases turns into a crossover with the light mode of the ϕ field interpolating between a positronium bound state and a pseudo-Goldstone boson.

In our truncation, the value of the critical coupling is $e_{\mathrm{cr}}^2 = 38.41$. For comparison, we also mention the result for e_{cr}^2 in the quenched approximation, $e_{\mathrm{cr,q}}^2 \simeq 14.81$, which is in reasonable agreement with the quenched DSE result [6] in the Landau gauge, $e_{\mathrm{cr},q\mathrm{DSE}}^2 = 4\pi^2/3 \simeq 13.16$. Note that our approximation includes nonladder diagrams such that gauge dependences are reduced [16]. The relation $e_{\mathrm{cr}}^2 > e_{\mathrm{cr},q}^2$ results from the fact that unquenched fluctuations imply charge screening; therefore larger bare couplings are necessary for χSB.

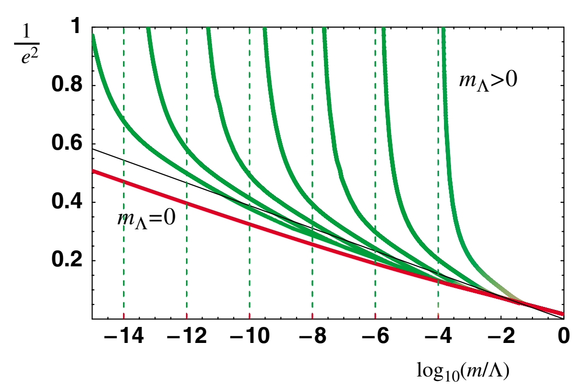

FIG. 1 (color online). Map of the bare couplings $[\log_{10}(m_\Lambda/\Lambda), 1/e_\Lambda^2]$ to the plane of renormalized couplings $[\log_{10}(m_R/\Lambda), 1/e_R^2]$. The dashed vertical lines denote lines of constant bare mass in the bare coupling plane which are mapped onto the solid lines in the renormalized coupling plane [sub- and supercritical values of the bare coupling are denoted by green (light gray) and red (dark gray), respectively]. The solid red line is the line of vanishing bare mass (the thin black line, its 1-loop counterpart). Its preimage is a vertical line at $-\infty$. Note that the region below this line is inaccessible, i.e., for a certain fixed value of the renormalized coupling we have a minimal renormalized mass in units of the cutoff Λ. Hence it is impossible to send $\Lambda \to \infty$ while keeping both renormalized mass and coupling fixed. This demonstrates triviality.

In Fig. 1, we plot the resulting renormalized values of the gauge coupling and electron mass,

$$e_R = \lim_{k\to 0} e, \quad m_R = \lim_{k\to 0} m, \tag{13}$$

as functions of the bare parameters. Shown are lines of constant bare mass m_Λ. For finite m_Λ, the curves exhibit a linear regime and a pole. This displays the crossover behavior from a χSB dominated mass at strong-coupling (linear regime) to an explicit mass term at weak coupling; the limiting pole corresponds to $m_R \simeq m_\Lambda$ for weak coupling. If we attempt to move the cutoff to infinity but keep m_R fixed, we need to take the limit $m_R/\Lambda \to 0$ which can only be approached on the curve $m_\Lambda/\Lambda \to 0$. In this limit $\ln(m_R/\Lambda) \to -\infty$, the renormalized coupling goes to $e_R \to 0$. This is the manifestation of triviality: the whole range of bare couplings $0 \le e_\Lambda^2 \le e_{\mathrm{cr}}^2$ for m_R fixed is mapped onto a single point $e_R^2 = 0$. For a nontrivial theory, at least one curve would have to intersect the $1/e_R^2$ axis at some finite e_R^2 for $m_R/\Lambda \to 0$.

On the other hand, if we want to keep $e_R > 0$ fixed, we are forced to accept a finite value for $m_R/\Lambda > 0$. Fixing the electron mass to its physical value also determines the absolute value of the cutoff, once the bare mass is fixed. The maximal cutoff value is obtained for vanishing bare mass $m_\Lambda/\Lambda \to 0$, and we find $\Lambda_{\max}^{m_\Lambda=0} \sim 10^{278\pm 8}$ GeV for QED parameters. Yet, the limit $m_\Lambda \to 0$ does not correspond to "ordinary" QED, since the electron mass is then fully generated by χSB, and a massless Goldstone boson arises. In order to rediscover ordinary QED in the IR with given e_R and m_R, we have to choose a sufficiently large bare mass m_Λ in order to lift the pseudo-Goldstone boson to a positronium state with mass $\simeq 2m_R$. This implies a small reduction of the maximal UV scale.

For given renormalized mass and coupling, we observe that the maximum possible bare coupling e_Λ occurs for $m_\Lambda \to 0$, which is a supercritical but still finite number. This fact describes the absence of the Landau pole singularity: for given physical IR parameters, large bare coupling values are inaccessible owing to χSB, in agreement with [4].

We would like to stress that the maximal UV scale is regulator dependent. Considering QED as being embedded in an underlying theory, the latter should become visible at this scale. In this sense, the regulator dependence corresponds to the physical threshold behavior towards the underlying theory.

Next we check whether QED can evade triviality in an unusual way: we fine tune the system onto e_{cr}^2 from above with $m_R/\Lambda \to 0$, such that the IR spectrum consists of a light fermion, a free photon (since $e_R \to 0$), and a Goldstone boson with Yukawa coupling to the fermion. In other words, QED with χSB could have a Yukawa theory as low-energy limit. However, we have confirmed explicitly that this Yukawa coupling is also trivial in the limit of $\Lambda \to \infty$ in much the same way as the gauge coupling, which agrees with lattice evidence [17].

We would finally like to point to open questions of the present investigation. First, our truncation of the fermion sector is organized as a derivative expansion. This is justified if the fermion anomalous dimension η_ψ remains small. In the Landau gauge, we have confirmed that this is indeed the case even at strong-coupling, so our truncation is self-consistent. Nevertheless, as is visible in Eq. (8), a potentially large fermion anomalous dimension could strongly modify the UV behavior. Even though this may not happen in the QED universality class, a fermionic system with large η_ψ and strong UV momentum dependences can offer new routes to UV completion of interacting QFT's. Second, it seems worthwhile to extend our studies to theories with strong NJL-like interactions. The UV flow of systems with strong gauge and four-fermion couplings still is unknown territory, the exploration of which is dedicated to future work.

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