Vacuum Čerenkov Radiation

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Within the classical Maxwell-Chern-Simons limit of the standard-model extension, the emission of light by uniformly moving charges is studied confirming the possibility of a Čerenkov-type effect. In this context, the exact radiation rate for charged magnetic point dipoles is determined and found in agreement with a phase-space estimate under certain assumptions.

The Čerenkov effect—emission of radiation from charges moving at or above the phase speed of light—is experimentally and theoretically well established in conventional macroscopic media [1], and its importance for modern particle detectors in high-energy and cosmic-ray physics can hardly be overstated. The reasons behind the recent revival of interest in this subject are twofold.

First, conventional-physics investigations, such as observations at CERN involving lead ions [2] and experiments in exotic condensed-matter systems [3], have found unexpected features of the Čerenkov effect. They include nonstandard kinematical radiation conditions, backward photon emission, and backward-pointing Čerenkov cones. Some of these issues have been studied theoretically [4].

Second, many candidate models underlying established physics predict Lorentz-breaking vacua [5], in which modified light speeds, for instance, offer the possibility of Lorentz tests via a Čerenkov-type mechanism called "vacuum Čerenkov radiation." At presently attainable energies, this and other Lorentz-breaking effects are described by the standard-model extension (SME) [6]. Candidate underlying models include strings [7], spacetime foam [8], noncommutative geometry [9], varying scalars [10], random-dynamics models [11], multiverses [12], and brane worlds [13]. Numerous analyses of Lorentz breaking in mesons, baryons, electrons, photons, muons, neutrinos, and the Higgs sector have been performed within the SME [5]. Although of substantial importance for Lorentz-violation studies [13,14], a detailed investigation of vacuum Čerenkov radiation is currently still lacking.

The present work is primarily intended to fill this gap. However, we expect our analysis to remain applicable also for conventional macroscopic media. In particular, our study provides a new conceptual perspective on Čerenkov radiation augmenting the usual physics picture: our fully relativistic Lagrangian allows us to work in the charge's rest frame, where fields typically behave like $r^{-1} \exp(-\sqrt{p \cdot p}\, r)$ at large distances r from the source. Here, p^μ satisfies the plane-wave dispersion relation, and the metric has signature -2. Conventional massive fields $p \cdot p = m^2$ lead to the Yukawa potential $r^{-1} \exp(-mr)$,

and the massless limit gives the standard r^{-1} behavior. In these cases, the energy-momentum tensor vanishes rapidly for $r \to \infty$ precluding energy-momentum flux to infinity. However, Lorentz-violating vacua and macroscopic media permit $p \cdot p < 0$ resulting in oscillatory far fields and thus the possibility of radiation. We show that this idea leads to the standard radiation conditions and facilitates the determination of the exact emission rate for charged magnetic dipoles within our dispersive and anisotropic model. To our knowledge, this is in many respects the first such result.

The present analysis employs the classical Maxwell-Chern-Simons limit of the SME given by the Lagrangian

$$\mathcal{L}_{\text{MCS}} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + k_\mu A_\nu \tilde{F}^{\mu\nu} - A_\mu j^\mu. \qquad (1)$$

Here, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ denotes the conventional electromagnetic field-strength tensor and $\tilde{F}^{\mu\nu} = \tfrac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ its dual, as usual. We have included an external source $j^\mu = (\varrho, \vec{j})$ and adopted natural units $c = \hbar = 1$. The spacetime-constant nondynamical $k^\mu = (k^0, \vec{k})$ violates Lorentz, PT, and CPT invariance [6,15]. Although tightly constrained observationally [16], this model has been studied extensively in the literature [6,16,17].

The potential A^μ obeys the equation of motion

$$(\Box\eta^{\mu\nu} - \partial^\mu\partial^\nu - 2\varepsilon^{\mu\nu\rho\sigma}k_\rho\partial_\sigma)A_\nu = j^\mu. \qquad (2)$$

Paralleling the conventional case, current conservation $\partial_\mu j^\mu = 0$ is required for consistency. Equation (2) gives the following modified Coulomb and Ampère laws:

$$\vec{\nabla} \cdot \vec{E} - 2\vec{k} \cdot \vec{B} = \varrho,$$
$$-\dot{\vec{E}} + \vec{\nabla} \times \vec{B} - 2k_0\vec{B} + 2\vec{k} \times \vec{E} = \vec{j}. \qquad (3)$$

The field-potential relationship is unaltered, so that the homogeneous Maxwell equations remain unchanged. Gauge invariance of physics is evident from Eqs. (3), and any of the usual conditions on A^μ, like Lorentz or Coulomb gauge, can be imposed [6]. Equation (2) implies that for $j^\mu \neq 0$ the energy-momentum tensor

$$\Theta^{\mu\nu} = -F^{\mu\alpha}F^{\nu}{}_{\alpha} + \tfrac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta} - k^{\nu}\tilde{F}^{\mu\alpha}A_{\alpha} \qquad (4)$$

is, in general, not conserved, as expected:

$$\partial_{\mu}\Theta^{\mu\nu} = j_{\mu}F^{\mu\nu}. \qquad (5)$$

Although $\Theta^{\mu\nu}$ depends on A^{μ}, the physical 4-momentum remains gauge invariant [16].

Up to homogeneous solutions, Eq. (2) is solved by

$$A^{\mu}(x) = \int_{C_{\omega}} \frac{d^{4}p}{(2\pi)^{4}} G^{\mu\nu}\hat{j}_{\nu} \exp(-ip \cdot x), \qquad (6)$$

where

$$G^{\mu\nu} \equiv -\frac{p^{2}\eta^{\mu\nu} + 2i\varepsilon^{\mu\nu\rho\sigma}k_{\rho}p_{\sigma} + 4k^{\mu}k^{\nu}}{p^{4} + 4p^{2}k^{2} - 4(p \cdot k)^{2}}. \qquad (7)$$

Here, $x^{\mu} = (t, \vec{r})$ is the spacetime-position vector and $p^{\mu} = (\omega, \vec{p})$ the Fourier-space wave vector. A caret denotes the four-dimensional Fourier transform. The poles of the integrand in Eq. (6) give the dispersion relation

$$p^{4} + 4p^{2}k^{2} - 4(p \cdot k)^{2} = 0. \qquad (8)$$

To ensure causal propagation, the ω-integration contour C_{ω} must pass above all poles on the real-ω axis, as usual. This is best implemented by replacing $\omega \to \omega + i\varepsilon$ in each ω in the denominator of the integrand in Eq. (6). The infinitesimal positive parameter ε is taken to approach zero after the integration. Note, however, that for time-like k^{μ}, poles on the imaginary-ω axis occur, so that causality is violated [16,17]. In what follows, we therefore focus on the spacelike- and lightlike-k^{μ} cases.

The current distribution describing the particle should be time independent in the particle's rest frame, so that $\hat{j}^{\mu}(p^{\mu}) = 2\pi\delta(\omega)\tilde{j}^{\mu}(\vec{p})$, where the tilde denotes the three-dimensional Fourier transform. Then, Eq. (6) takes the form

$$A^{\mu}(\vec{r}) = \int \frac{d^{3}\vec{p}}{(2\pi)^{3}} \frac{N^{\mu\nu}(\vec{p})\tilde{j}_{\nu}(\vec{p}) \exp(i\vec{p} \cdot \vec{r})}{\vec{p}^{4} - 4\vec{p}^{2}k^{2} - 4(\vec{p} \cdot \vec{k} - i\varepsilon k_{0})^{2}}, \qquad (9)$$

where $N^{\mu\nu}(\vec{p}) \equiv \vec{p}^{2}\eta^{\mu\nu} - 2i\varepsilon^{\mu\nu\rho s}k_{\rho}p_{s} - 4k^{\mu}k^{\nu}$, and Latin indices run from 1 to 3. Evaluation of the $|\vec{p}|$-type integral with complex-analysis methods gives certain residues of the integrand in the complex $|\vec{p}|$ plane, which typically contain the factor $\exp(i\vec{p}_{0} \cdot \vec{r})$. Here, $\vec{p}_{0}$ denotes the location of a pole. The remaining angular integrations correspond to averaging the residues over all directions, so that the qualitative behavior of the integral (9) is determined by the residues. In particular, A^{μ} decreases exponentially with increasing r for $\mathrm{Im}(\vec{p}_{0}) \neq \vec{0}$, while $\mathrm{Re}(\vec{p}_{0}) \neq \vec{0}$ leads to oscillations with distance. As mentioned earlier, energy transport to spatial infinity requires nondecaying oscillatory fields. *Thus, one expects vacuum Čerenkov radiation only when there are real $p^{\mu} = (0, \vec{p})$ satisfying the plane-wave dispersion relation in the source's rest frame.* In a general inertial frame, this condition

reads $p'^{\mu} = (\vec{\beta} \cdot \vec{p}', \vec{p}')$, where $\vec{\beta}$ denotes the velocity of the particle. This is seen to be equivalent to the conventional phase speed condition $c'_{ph} = |\omega'|/|\vec{p}'| \leq |\vec{\beta}|$. Note that spacelike plane-wave vectors are not necessarily associated with positivity and stability problems [10].

The usual method for calculating the radiation rate—extraction of the r^{-2} piece of the modified Poynting vector and integration over a spherical surface—is challenging because the determination of the far fields is hampered by the complexity of the integral (9). For further progress, an ansatz for the current $j^{\mu} = J^{\mu}$ describing the particle is advantageous. The most general form of $J^{\mu}(x)$ consistent with current conservation and the presumed time independence in the particle's rest frame is

$$J^{\mu}(\vec{r}) = (\rho(\vec{r}), \vec{\nabla} \times \vec{f}(\vec{r})), \qquad (10)$$

where $\rho(\vec{r})$ is the source's charge density and $f(\vec{r})$ is an arbitrary vector field. Moreover, we require both $\rho(\vec{r})$ and $f(\vec{r})$ to vanish rapidly outside the finite volume V_{0} associated with the particle. We can therefore drop various boundary terms in the subsequent manipulations, if the integration volume V is chosen large enough.

Spatial integration of Eq. (5) yields

$$\int_{\sigma} d\sigma^{l}\Theta_{l\nu} = \int_{V} d^{3}\vec{r}J^{\mu}F_{\mu\nu} - \frac{\partial}{\partial t} \int_{V} d^{3}\vec{r}\Theta_{0\nu}, \qquad (11)$$

where σ is the boundary of V, and $d\sigma^{l}$ the corresponding surface element with outward orientation. The energy-momentum flux $\dot{P}_{\nu} \equiv \int_{\sigma} d\sigma^{l}\Theta_{l\nu}$ through the surface σ is therefore caused by the source $J^{\mu}(x)$ in the enclosed volume V and the decrease in the field's 4-momentum in V, as usual. Using the Maxwell equation $\vec{\nabla} \times \vec{E} = 0$ and the zeroth component of Eq. (11) one obtains the following expression for the radiated energy:

$$\int_{\sigma} d\vec{\sigma} \cdot \vec{S} = -\int_{\sigma} d\vec{\sigma} \cdot (\vec{f} \times \vec{E}), \qquad (12)$$

where a modified Poynting vector $\Theta_{l0} \equiv S_{l} = -S^{l}$ has been defined. Since $\vec{f}$ goes to zero on the boundary of a large volume, the energy flux to infinity vanishes. This feature is model independent: in the particle's rest frame, J^{μ} is spatially localized, and time-translation invariance holds. The resulting energy conservation implies zero energy flux through any closed surface. *Thus, the net radiated energy vanishes in the rest frame of a localized static source.* Note that any nonzero 4-momentum radiated by such a source is therefore necessarily spacelike.

The 3-momentum-emission rate is obtained similarly:

$$\dot{\vec{P}} = \int_{V} d^{3}\vec{r}J_{\mu}\vec{\nabla}A^{\mu}. \qquad (13)$$

Employing Eq. (9) and the Fourier expansion of J^{μ} yields

$$\dot{\vec{P}} = i \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{\tilde{J}^\mu(-\vec{p})N_{\mu\nu}(\vec{p})\tilde{J}^\nu(\vec{p})}{\vec{p}^4 - 4\vec{p}^2k^2 - 4(\vec{p}\cdot\vec{k} - i\varepsilon k_0)^2}\vec{p} \quad (14)$$

in the limit $V \to \infty$. Note that the integrand is odd in $\vec{p}$, so that $\dot{\vec{P}}$ vanishes if singularities are absent. However, this symmetry argument fails for integrands with poles at real $\vec{p} = \vec{p}_0$, consistent with our radiation condition. The dispersion relation (8) indeed admits such solutions opening a doorway for nonzero 3-momentum emission.

We now focus on the current distribution $\rho = q\delta(\vec{r})$ and $\vec{J} = -\vec{\mu} \times \vec{\nabla}\delta(\vec{r})$, which describes a pointlike charge q with magnetic dipole moment $\vec{\mu}$. The use of a suitable regulation of the delta function then permits a closed-form evaluation of the integral in Eq. (14). This gives the exact rest-frame rate of 3-momentum radiation for a charged magnetic point dipole:

$$\dot{\vec{P}} = -\frac{\text{sgn}(k_0)}{12\pi}\frac{k_0^5}{|\vec{k}|^5}\{[3q^2\vec{k}^2/k_0^2 + 6q\vec{k}\cdot\vec{\mu} - \vec{\mu}^2k_0^2$$
$$+ 5(\vec{k}\cdot\vec{\mu})^2k_0^2/\vec{k}^2 + 10(\vec{k}\times\vec{\mu})^2]k_0\vec{k}$$
$$- 2[q\vec{k}^2/k_0^2 + \vec{k}\cdot\vec{\mu}]k_0^3\vec{\mu}\}. \quad (15)$$

A nonzero flux in the above static case might appear counterintuitive. However, similar situations arise in conventional physics as well. For instance, constant nonparallel $\vec{E}$ and $\vec{B}$ fields are associated with a finite Poynting flux $\vec{S} = \vec{E} \times \vec{B}$. Although suppressed by four powers of k^μ, the rate (15) remains nonvanishing in the zero-charge limit $q \to 0$. Ordinary refractive indices typically require a minimal speed of the charge for the emission of Čerenkov light. This no longer holds true in the present context, as can be seen in the case for lightlike k^μ and $\vec{\mu} = \vec{0}$. *Thus, vacuum Čerenkov radiation need not necessarily be a threshold effect.*

The radiation rate in the laboratory frame is often more useful. To avoid unwieldy expressions, we consider the special case of vanishing $\vec{\mu}$ and spacelike k^μ. We further choose the laboratory such that $k_0' = 0$ and $\vec{k}' \neq \vec{0}$. Then, suppressing the primes, Eq. (15) becomes

$$\dot{P}_\mu = \frac{q^2}{4\pi}\frac{\gamma^3(\vec{\beta}\cdot\vec{k})^4}{\vec{k}^2 + \gamma^2(\vec{\beta}\cdot\vec{k})^2}K_\mu, \quad (16)$$

where

$$K^\mu \equiv \frac{\text{sgn}(\vec{\beta}\cdot\vec{k})}{\sqrt{\vec{k}^2 + \gamma^2(\vec{\beta}\cdot\vec{k})^2}}\left(\vec{k} + \frac{\gamma^2(\vec{\beta}\cdot\vec{k})}{\gamma^2(\vec{\beta}\cdot\vec{k})}\vec{\beta}\right). \quad (17)$$

Here, $\vec{\beta}$ is the 3-velocity of the charge in the laboratory and $\gamma = (1 - \vec{\beta}^2)^{-1/2}$. The overdot now denotes differentiation with respect to laboratory time. For particle 3-velocities perpendicular to $\vec{k}$, radiation is absent. In all other cases, both $\dot{P}^0$ and the projection of $\dot{\vec{P}}$ onto $\vec{\beta}$ are positive, so that conventional charges are decelerated

in our laboratory frame. Note, however, that as a consequence of the anisotropic vacuum, the net emitted 3-momentum is typically not aligned with the charge's velocity. The backreaction of the radiation on the charge will then in general lead to a curved trajectory for the particle. Regardless of anisotropies, 4-momentum loss implies nongeodesic motion. *Vacuum Čerenkov radiation is therefore always associated with equivalence-principle violations.*

Most cosmic-ray analyses of Lorentz violation are based on purely kinematical models, so that it is interesting to study, whether a modified dispersion relation by itself permits a sensible estimate for the Čerenkov rate. In quantum theory, the Čerenkov effect corresponds to the decay of a charge P_a into itself P_b and a photon P_c. In the center-of-mass frame, the rate for this process obeys

$$d\Gamma = \frac{|\mathcal{M}_{a\to b,c}|^2}{2m}(2\pi)^4\delta^{(4)}(p_a^\mu - p_b^\mu - p_c^\mu)d\Pi_b d\Pi_c, \quad (18)$$

where the transition amplitude $\mathcal{M}_{a\to b,c}$ contains information about the dynamics of the decay. The remaining factors describe the kinematics of the process. They include phase-space elements $d\Pi_s$ and various 4-momenta $p_s^\mu = (E_s, \vec{p}_s)$, where $s \in \{a, b, c\}$ refers to the corresponding particle. In what follows, we consider a conventional charge q with $p_a^2 = p_b^2 = m^2$. To facilitate a transparent comparison with the classical result (15), we further assume photon 4-momenta p_c^μ obeying the dispersion relation (8), select a lightlike k^μ parameter, and take the static-source limit $m \to \infty$.

An order-of-magnitude estimate for the transition amplitude is $|\mathcal{M}_{a\to b,c}|^2 \sim q^2m^2$ [18], where the spinor normalization implicit in Eq. (18) has been used. The phase-space element $d\Pi_b$ is determined by the conventional relation $2E_b(\vec{p})d\Pi_b = (2\pi)^{-3}d^3\vec{p}$. The construction of the invariant phase-space element $d\Pi_c$ for the photon requires more care due to the presence of Lorentz breaking. Coordinate independence requires

$$d\Pi_c = \frac{d^3\vec{p}_c}{(2\pi)^3 2|\vec{p}_c + \text{sgn}(k^0)\vec{k}|} \quad (19)$$

for the positive-energy, spacelike roots of the dispersion relation (8). Noting that $d\dot{P}_\mu = p_\mu d\Gamma$, our above considerations lead to

$$\dot{\vec{P}} \sim -\frac{q^2}{8\pi}k^0\vec{k} \quad (20)$$

as a rough estimate for the net radiated momentum per time in the charge's rest frame. Comparison with Eq. (15) supports the validity of our phase-space result (20). *We conclude that in the context of vacuum Čerenkov radiation phase-space considerations can provide useful estimates for momentum-emission rates.*

Experimental studies employing the Čerenkov effect in the Maxwell-Chern-Simons model are unlikely to improve the existing tight bound of $O(k^\mu) \lesssim 10^{-42}$ GeV

[16]. However, Eq. (16) identifies an average alignment of charged-matter velocities in the plane perpendicular to $\vec{k}$ as a potential signature in a cosmological context. This effect might have been enhanced before electroweak symmetry breaking: First, radiation is not yet decoupled from the matter resulting in a large number of free charges that can be affected. Second, lightlike 4-momenta of massless charged matter imply that all wave frequencies can contribute to vacuum Čerenkov radiation [19]. We also note that our general philosophy and methods are applicable in other Lorentz-violating situations. For instance, some components of the dimensionless $(k_F)^{\mu\nu\rho\sigma}$ parameter in the SME are currently bounded only at the 10^{-9} level [21]. Moreover, the rate might be less suppressed in this case offering the possibility of improved constraints via vacuum Čerenkov radiation.

In conclusion, a generic conceptual picture of the Čerenkov effect, which complements the conventional one, has been developed and illustrated explicitly in the Maxwell-Chern-Simons model. This physical picture offers an interesting avenue for further insight into various aspects of Čerenkov radiation in general Lorentz-breaking vacua and macroscopic media. It paves the way for additional studies in a quantum context and provides a solid foundation for phenomenological explorations of Lorentz violation via the vacuum Čerenkov effect.

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