Microwave-Induced Thermal Escape in Josephson Junctions

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We investigate, by experiments and numerical simulations, thermal activation processes of Josephson tunnel junctions in the presence of microwave radiation. When the applied signal resonates with the Josephson plasma frequency oscillations, the switching current may become multivalued in a temperature range far exceeding the classical to quantum crossover temperature. Plots of the switching currents traced as a function of the applied signal frequency show very good agreement with the functional forms expected from Josephson plasma frequency dependencies on the bias current. Throughout, numerical simulations of the corresponding thermally driven classical Josephson junction model show very good agreement with the experimental data.

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The Josephson tunnel junction is an intriguing solid state physics system due to the macroscopic quantum nature of the variables describing the governing equations [1]. Irradiation of junctions with microwave (ac) radiation has produced a number of significant nonlinear phenomena such as chaos and phase locking observed both in experiments and theoretical models. These phenomena have been investigated for almost four decades [2,3] and successful applications have significantly benefited from complete theoretical understanding of the relevant dynamics.

Measurements of the escape statistics from the zero-voltage state in Josephson junctions have been conducted extensively over the years, successfully confirming consistency with the classic Kramers model for thermally activated escape from a potential well [4,5]. Applying an ac field to a low-temperature system has been reported to produce anomalous switching distributions with signatures of two, or more, distinct dc bias currents for which switching is likely. These measurements have been interpreted as a signature of the ac field aiding the population of multiple quantum levels in a junction, thereby leading to enhancement of the switching probability for bias currents for which the corresponding quantum levels match the energy of the microwave photons; work performed in this direction appeared first in the literature two decades ago [6,7]. More recently another group showed that the application of microwaves is not the only condition under which level quantization can be observed in Josephson junctions [8].

The results mentioned in the previous paragraph have contributed to attract interest toward Josephson junction systems as possible basic elements in the field of quantum coherence and quantum computing[9–13]; within the framework of this research topic renovated attention has been devoted to phenomena generated by the application of ac signals to Josephson junctions during measurements of escape out of the washboard potential well. With the present Letter, we wish to contribute to these developments presenting a systematic study of the escape properties of a Josephson tunnel junction subject to external microwave radiation in a temperature range were data have not been previously reported, namely, from 0.36 K up to 1.6 K.

Figure 1 illustrates the process under investigation: in the classical 1 degree of freedom single-particle washboard potential of the Josephson junction [6], thermal excitations (shaded in the sketch) of energy $k_B T$ and the energy $E_{\rm ac}$ of forced oscillations due to microwave radiation, can cause the particle to escape from the potential well. This process can be traced by sweeping the current-voltage characteristics of the Josephson junction periodically. Escape from the potential well corresponds to an abrupt transition from the top of the Josephson-current zero-voltage state to a nonzero-voltage state. The statistics of the switching events, in the absence of time-varying perturbations, have been shown to be consistent with Kramers' model [4] for thermal escape from a one-dimensional potential. Since the thermal equilibrium Kramers model does not include the effect of nonequilibrium force terms, the results of the switching events generated by the presence of a microwave radiation on a Josephson junction can be investigated, in a thermal regime, only by a direct numerical simulations of the resistively and capacitively shunted junction (RCSJ) model [2,3],

$$\frac{\hbar C}{2e}\frac{d^2\varphi}{dt^2} + \frac{\hbar}{2eR}\frac{d\varphi}{dt} + I_c \sin\varphi = I_{\rm dc} + I_{\rm ac}\sin\omega_{\rm d} t + N(t). \tag{1}$$

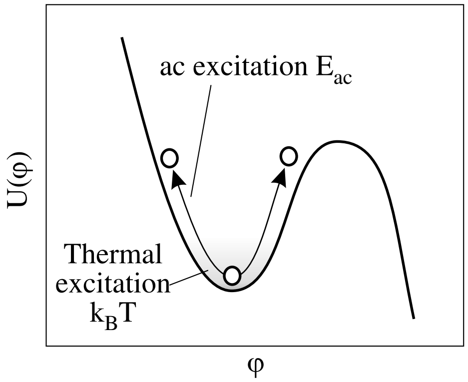

FIG. 1. Sketch of the physical phenomenon under investigation: a driven oscillation energy E_{ac} superimposed onto thermal excitations, may cause a particle to escape a washboard potential.

Here, φ is the phase difference of the quantum mechanical wave functions of the superconductors defining the Josephson junction, C is the magnitude of junction capacitance, R is the model shunting resistance, and I_c is the critical current, while I_{dc} and $I_{ac}\sin\omega_d t$ represent, respectively, the continuous and alternating bias current flowing through the junction. The microwave energy sketched in Fig. 1 will then be $\propto I_{ac}^2$. The term $N(t)$ represents the thermal noise current due to the resistor R given by the thermodynamic dissipation-fluctuation relationship [14]

$$\langle N(t)\rangle = 0 \tag{2}$$

$$\langle N(t)N(t')\rangle = 2\frac{k_B T}{R}\delta(t-t'), \tag{3}$$

with T being the temperature. The symbo, $\delta(t-t')$ is the Dirac delta function. Current and time are usually normalized, respectively, to the Josephson critical current I_c and to $(\omega_0)^{-1}$, where $\omega_0 = \sqrt{2eI_c/\hbar C}$ is the Josephson plasma frequency. With this normalization, the coefficient of the first-order phase derivative becomes the normalized dissipation $\alpha = \hbar\omega_0/2eRI_c$. It is also convenient to scale the energies to the Josephson energy $E_J = I_c\hbar/2e = I_c\Phi_0/2\pi$, where $\Phi_0 = h/2e = 2.07\times10^{-15}Wb$ is the flux quantum. Thus, the set of Eqs. (1)–(3) can be expressed in normalized form as

$$\ddot{\varphi} + \alpha\dot{\varphi} + \sin\varphi = \eta + \eta_d \sin\Omega_d\tau + n(\tau) \tag{4}$$

$$\langle n(\tau)\rangle = 0 \tag{5}$$

$$\langle n(\tau)n(\tau')\rangle = 2\alpha\theta\delta(\tau-\tau'), \tag{6}$$

where $\theta = \frac{k_B T}{E_J}$ is the normalized temperature $\eta = \frac{I_{dc}}{I_c}$ and $\eta_d = \frac{I_{ac}}{I_c}$

The plasma frequency is a key parameter of our work, as we shall see later, and this is why we use its inverse as a normalizing parameter. The conversion from our normalized units to others used in the Josephson effect has been reported in the literature [15](see also page 124 of Ref. [2]).

Figure 2 shows the results of numerical simulations of escape in a system with $\alpha = 0.00845$, $\theta = 4.76\times10^{-4}$, and continuous bias sweep rate $\frac{d\eta}{d\tau} = 2.1\times10^{-8}$. These parameters have been chosen in correspondence with the experimental measurements discussed below. However, wide parameters intervals were investigated in order to check the consistency of the results. Specifically, the effects of sweep rate were monitored over 5 orders of magnitude ($10^{-9} \le \frac{d\eta}{d\tau} \le 10^{-4}$) and the normalized temperature was varied in the range $1.6\times10^{-4} \le \theta \le 40\times10^{-4}$. Switching distributions (each corresponding to 10 000 events), obtained for different values of the normalized drive frequency, are shown as a function of the continuous bias, and we clearly observe that secondary peaks in the distribution may appear, along with the primary peak corresponding to the unperturbed ($\eta_d = 0$) switching distribution. The bias location of the secondary peak of each distribution is pointing by a dotted line to the intersection (marked with •) with the driving frequency on the top plot of Fig. 2. Also on the top plot, we have shown the linear plasma resonance curve $\Omega_p = (1-\eta^2)^{1/4}$ as a solid line, and we observe that the numerically obtained switching distribution peaks are

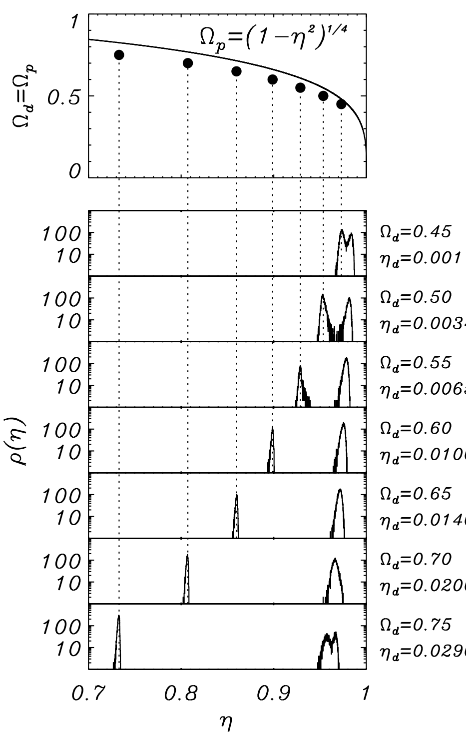

FIG. 2. Simulated switching distributions, $\rho(\eta)$, for the ac-driven junction model obtained for increasing values of the drive frequency. The frequency data points in the uppermost plot are relative to the position of the secondary peak in the plots. Normalized temperature is $\theta = 4.76\times10^{-4}$, bias sweep rate is $\dot{\eta} = 2.1\times10^{-8}$, and damping coefficient is $\alpha = 0.00845$. Solid curve in uppermost graph represents the linear plasma resonance Ω_p.

closely aligned with the linear plasma resonance near the driving frequency.

Notice that the values of the ac drive amplitude η_d were tuned (as indicated at the right margin of each plot) in order to achieve the resonant energy necessary to escape the potential well. Naturally, the lowest values of dc bias current need larger ac amplitudes to produce a significant number of switching events. The fact that the data points on the top graph are consistently below the predicted (linear) resonance curve can be directly attributed to the anharmonicity of the Josephson potential for large bias currents [16].

Thermal escape properties of the Josephson junction, as described by Eqs. (4)–(6), depend strongly on the value of the normalized frequency Ω_d. We have repeated the same numerical procedure as described in Fig. 2 for drive frequencies equal to subharmonics and superharmonics of the frequencies leading to the direct resonances observed above. Specifically, Fig. 3 reports results for driving frequencies $\Omega_d = q\Omega_p$ with $q = \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1, 2$ (notice that $q = 1$ is the case shown in Fig. 2). The numerical results for both super- and subharmonics are shown in Fig. 3 (circles $\circ$). In this figure we see that, even when driving at subharmonics or superharmonics, the positions of the secondary switching peaks follow closely what can be expected from the plasma resonance dependence on the bias current. It is worth noting that the escape histograms often revealed multipeaked distributions with more than two peaks. An example of this is shown in the lowest plot of Fig. 2 where we see two peaks close to each other and another well separated on the left; the variation in the bias current tunes the plasma frequency, which in turn may find some rational resonance with the applied frequency and increases the probability for escape.

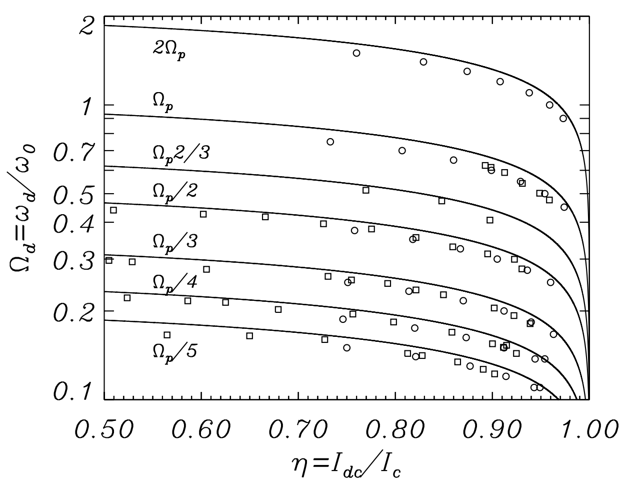

FIG. 3. The functional dependencies of the driving frequency upon the location of the secondary peak in $\rho(\eta)$ obtained for subharmonic and harmonic pumping. Circles ($\circ$) represent numerical results and squares ($\Box$) experimental data. Parameters are as given in Fig. 2. Solid curves represent $q\Omega_p$, for different q, as indicated.

Complementary experiments were performed on Josephson tunnel junctions fabricated according to classical Nb-NbAlOx-Nb procedures [17]. The samples had very good current-voltage characteristics and magnetic field diffraction patterns. The junctions were cooled in a ^{3}He refrigerator (Oxford Instruments Heliox system), providing temperatures down to 360 mK. The microwave-radiation, brought to the chip-holders by a coax cable, was coupled capacitively to the junctions. The junction had a maximum critical current of 143 μA and a total capacitance of 6pF from which we estimate a plasma frequency of 42.5 GHz. From this value of the (unbiased) plasma frequency the classical to quantum crossover temperature [18] $T^* = (\hbar\omega_0/2\pi k_B) =$ 320 mK between classical thermal and quantum mechanical behavior can be estimated. The sweep rate of the continuous current I_{dc} was $\dot{I}_{dc} = 800$ mA/s, and we verified that the experiment was being conducted in adiabatic conditions [8]. At the temperature $T = 1.6$ K the junction had a Josephson energy $E_J = 46.4 \times 10^{-21} J$, and effective resistance $R = 74\ \Omega$. We have gathered data in the temperature range of 360 mK − 1.6 K, but we will here show the results for $T = 1.6$ K. As mentioned above, the displayed simulation results were obtained on the basis of these real system parameters. The evaluation of the dissipation parameter was based on the hysteresis of the current-voltage characteristics of the junctions[2,3]; we anticipate that the dissipation parameter used in the simulations may not completely represent the experimental dissipation. Theory and simulations indicate that this dissipation parameter influences the switching distributions such that a larger dissipation parameter will result in a broader resonance, which in turn will require a larger ac amplitude to produce a multipeak switching distribution. For increasing dissipation, multipeaked distributions will eventually vanish due to a transition to overdamped, nonresonant dynamics.

Figure 4 displays the experimentally obtained complementary results to Fig. 2. It is clear that also the experiments exhibit close agreement between the bias location of the secondary switching current and the plasma frequency resonance when the value of drive frequency is near the plasma frequency resonance. We note that the level of the applied power is not a simple linear function (like in Fig. 2) because resonances in the coupling system caused a slightly irregular coupling from the microwaves to the junction. In Fig. 4 each plot is obtained by recording 2000 switching events.

Similarly to the simulations, we have also conducted escape measurements using subharmonic microwave radiation. These data are shown in Fig. 3 as squares ($\Box$) alongside the corresponding simulation results discussed above. In the experiments, as in the simulations, we see that the position of the secondary peak follows closely what would be expected from the plasma frequency dependence on the dc current bias. We emphasize that no fitting was used in Fig. 3 for neither numerical/

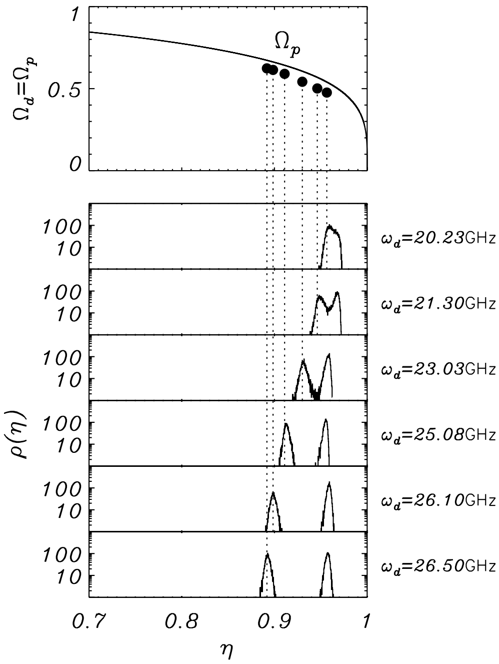

FIG. 4. The experimental complement to Fig. 2 for a thermodynamic temperature (1.6 K) equivalent to the numerical normalized value.

experimental data nor the plasma resonance curves. Data obtained at different temperatures in the range (360 mK − 1.6 K) showed features identical to those of Figs. 2–4 for both simulations and experiments. As was the case in the simulations, we also frequently observed switching distributions with three peaks in the experiments. These features, however, just like one would expect in classical-resonance regime (and just like what we observe in the simulations) would disappear when the quasi particle conductance, due to a raise of the temperature became too high. The straightforward agreement between theory and experiment exhibited in Fig. 3 shows that the RCSJ model is a very solid basis for the physical interpretation of our experiments. The adequacy of this model is also justified by the fact that frequency dependencies of junction'impedance [2,3] can be negligible when the system works essentially, as in our case, in stationary conditions.

In conclusion, our experiments on ac driven, thermal escape of a *classical* particle from a one-dimensional potential well have shown that resonant coupling (harmonic or subharmonic) between the applied microwaves and the plasma resonance frequency provides an enhanced opportunity for escape, and we have directly observed the signatures of such microwave-induced escape distributions in the form of anomalous multipeaked escape statistics which indicates a consistent interpretation of these switching distributions in the classical regime of Josephson junctions.

It is noted that previous experimental work on ac-induced escape distributions obtained at temperatures below T^* is consistent with the observations presented here. Those experiments have produced ac-induced peaks in the observed switching distributions, and the relevant peaks are located alongside the expected classical plasma resonance curve, as we have also found here. An important observation is that the microwave-radiation frequency necessary for populating an excited quantum level ($\hbar\omega_d$) in a quantum oscillator coincides with the classical-resonance frequency of the corresponding classical oscillator. It is evident then that multipeaked effects are not a unique signature of quantum behavior in the ac-driven Josephson junction. The experimental and numerical evidence herein reported, along with data existing below T^* [13], point again to the Josephson junction as an ideal vehicle for probing relevant issues in fundamental physics [19].

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[1] P.W. Anderson, in *Lectures on the Manybody Problem*, edited by E. R. Caianiello (Academic Press, NY, 1964), Vol. 2, p. 132.
[2] A. Barone and G. Paternó, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982).
[3] T. Van Duzer and C.W. Turner, *Principles of Superconductive Devices and Circuits* (Prentice-Hall, New York, 1998), 2nd ed..
[4] H. A. Kramers, Physica (Amsterdam) **7**, 284 (1940).
[5] T. A. Fulton and L. N. Dunkelberger, Phys. Rev. B **9**, 4760 (1974).
[6] J. M. Martinis, M. H. Devoret, and J. Clarke, Phys. Rev. Lett. **55**, 1543 (1985).
[7] M. H. Devoret, J. M. Martinis, and J. Clarke, Phys. Rev. Lett. **55**, 1908 (1985).
[8] P. Silvestrini, V. G. Palmieri, B. Ruggiero, and M. Russo, Phys. Rev. Lett. **79**, 3046 (1997).
[9] B. Ruggiero, M. G. Castellano, G. Torrioli, C. Cosmelli, F. Chiarello, V. G. Palmieri, C. Granata, and P. Silvestrini, Phys. Rev. B **59**, 177 (1999).
[10] J. M. Martinis, S. Nam, and J. Aumentado, Phys. Rev. Lett. **89**, 117901 (2002).
[11] Y. Lu, S. Han, Xi Chu, S. Chu, Z. Wang, Science **296**, 889 (2002).
[12] A. J. Berkley, H. Xu, M. A. Gubrud, R. C. Ramos, J. R. Anderson, C. Lobb, and F. C. Wellstood, Phys. Rev. B **68**, 060502 (2003).
[13] A. Wallraff, T. Duty, A. Lukashenko, and A.V. Ustinov, Phys. Rev. Lett. **90**, 037003 (2003).
[14] G. Parisi, *Statistical Field Theory* (Addison-Wesley, Reading, MA, 1988).
[15] M. Cirillo, J. Appl. Phys. **68**, 5729 (1990)
[16] N. Grønbech-Jensen and M. Cirillo, cond-mat/0404721.
[17] S. Morohashi and S. Hasuo, J. Appl. Phys. **61**, 4835 (1987).
[18] I. Affleck, Phys. Rev. Lett. **46**, 388 (1981).
[19] V. Averbukh and N. Moiseyev, Phys. Rev. A **57**, 1345 (1998).