# Scaling Ion Trap Quantum Computation through Fast Quantum Gates

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We propose a method to achieve scalable quantum computation based on fast quantum gates on an array of trapped ions, without the requirement of ion shuttling. Conditional quantum gates are obtained for any neighboring ions through spin-dependent acceleration of the ions from periodic photon kicks. The gates are shown to be robust to influence all the other ions in the array and insensitive to the ions' temperature.

Trapped ions constitute one of the most promising systems for implementation of quantum computation. Significant theoretical and experimental advances have been achieved for this system [1–9], and quantum gates have been demonstrated at the level of a few ions [7–9]. The current central problem is to find methods to scale up this system for larger-scale quantum computation [10–13]. A particularly promising approach to scalability is based on shuttling ions in complex traps to different regions for storage and for quantum gate operations [10,11]. Interesting initial experiments have been reported on separating, shuttling, and sympathetic cooling of the ions [14–16]. Nevertheless, these experiments also indicate that fast separation of the target ions is a challenging task [14], which limits the speed of any collective quantum gate in a scalable structure.

In this Letter, we propose a scaling method based on fast quantum gates on an array of trapped ions. Very recently, Garcia-Ripoll, Zoller, and Cirac proposed a remarkable scheme of two-bit quantum gates [17], which can operator faster than the ion trap frequency. The original fast gate in [17] was designed for two ions, while keeping in mind that one achieves scalability through the ion shuttling. Here, we propose an efficient scaling method for ion trap quantum computation based on the concept of fast quantum gates. This scaling method provides an attractive alternative, avoiding a series of challenges associated with the ion shuttling and the near-ground-state ion cooling.

The extension of fast quantum gates from two ions to a large array of ions is actually quite challenging: even if the laser pulses are shined only on two ions, all the ions in the array affect each other through the long-range Coulomb interactions, and all the phonon modes need to be taken into account as the motional sideband addressing is not possible with a fast gate. The mutual strong influence between the ions normally sets a significant obstacle to scalable quantum computation. However, we show that as long as the gate speed is faster than the local ion oscillation frequency (specified below), this unwanted influence can be arbitrarily reduced with a remarkable

method for noise cancellation. Besides the proposal of an efficient scaling method, we also give a different design of the fast quantum gates with the use of only periodic laser pulses. Such periodic pulses are significantly easier for experimental realization.

We consider an array of trapped ions, which could be in any convenient geometry. For instance, Fig. 1(a) shows a possible configuration of the array from multiconnected linear Paul traps. The qubit is represented by two stationary internal states of the ion, denoted as $|0\rangle$ and $|1\rangle$ in general. We assume that the distance $d$ between the neighboring ions is appreciably large ($d > 5 \, \mu$m) so that single-bit operations and measurements can be done with no difficulty. The critical issue is then to implement prototypical two-bit gates for any neighboring ions, which, combined with simple single-bit gates, realize universal quantum computation.

For the purpose of the two-bit gate on any neighboring ions, say, $i$ and $j$, we assume that one can mechanically accelerate each ion, with the acceleration direction depending on its internal state $|0\rangle$ or $|1\rangle$ [as illustrated in
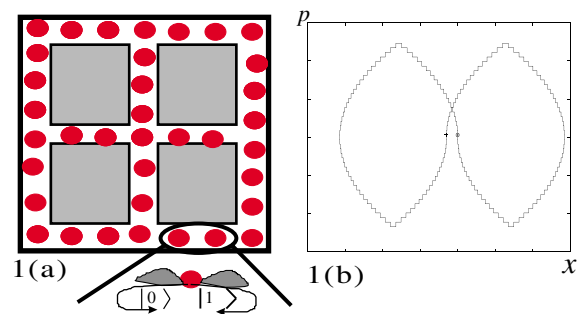


FIG. 1 (color online). A possible geometry of the ion array for scalable quantum computing from multiconnected linear Paul traps. Conditional quantum gates are achieved for any neighboring ions though application of a spin-dependent kicking force on these two ions. (b). The typical trajectory of the kicked ion in phase space, where "+" denotes the initial position, "○" and "×" denote the final position of the ion in $|0\rangle$ or $|1\rangle$ states, which in general is different from the initial position as the ion has an initial velocity.

Fig. 1(a)]. So the acceleration force is written as $F_{ij} = F\sum_{\alpha=i,j}\sigma_\alpha^z$, where $\sigma_\alpha^z = |1\rangle_\alpha\langle 1| - |0\rangle_\alpha\langle 0|$ is the Pauli operator. The state-dependent acceleration can be realized, for instance, through coherent photon kicks from fast laser pulses [17]. A pair of short Raman pulses with the wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ applied on the ions $i$ and $j$ will give a state-dependent momentum kick $\hbar(\mathbf{k}_1 - \mathbf{k}_2)\sigma_\alpha^z$ ($\alpha = i, j$) to the ions for each $\pi$ rotation of the states $|0\rangle_\alpha$ and $|1\rangle_\alpha$. For convenience, we assume $\mathbf{k}_1 \perp \mathbf{k}_2$ and $|\mathbf{k}_1| \approx |\mathbf{k}_2| \equiv k_c = 2\pi/\lambda_c$, where $\lambda_c$ is the carrier wave length. The $\mathbf{k}_1$ and $\mathbf{k}_2$ have a 45° angle to the two-ion axis and we alternatively apply the Raman pulses in the $(\mathbf{k}_1, \mathbf{k}_2)$ and $(-\mathbf{k}_1, -\mathbf{k}_2)$ directions. Each pulse sequence is periodic with a repetition frequency $f_r/2$ much larger than the ion trap frequency, so the net acceleration force on the ions is along the two-ion axis with the mean magnitude $F = \sqrt{2}\hbar k_c f_r$. The momentum kicks from the periodic laser pulses are actually discrete in time, but their effect is well approximated by a continuous kicking force $F$ if more than ten pulses are applied during each gate, as we have checked in our calculations.

To characterize this quantum gate, we need to take into account the Coulomb interactions between all the ions. The conventional approach is based on decomposition into the canonical phonon modes. However, for a large number of ions, this approach becomes very complicated. Here, we develop a different theoretical framework which is more appropriate for description of the fast gates on a large ion array. We note that with fast laser pulses, the acceleration force $F_{ij}$ can be significantly larger than the forces from the Coulomb interactions and the external trapping potentials. We therefore write the total Hamiltonian $H$ into two parts $H = H_0 + H_1$, where the dominant part

$$H_0 = -F(t)\sum_{\alpha=i,j}\sigma_\alpha^z x_\alpha + \sum_k \frac{p_k^2}{2m} \qquad (1)$$

accounts for the kicking force $F(t)$ on the target ions $i, j$ and the kinetic energy summarized over all the ions $k$ (including $i, j$); and the second part

$$H_1 = \sum_{k,k'} V_{kk'}(|x_k - x_{k'}|) + \sum_k V_T(x_k) \qquad (2)$$

accounts for the Coulomb interactions $V_{kk'}(|x_k - x_{k'}|)$ between every pair of ions $k, k'$ and the external trapping potentials $V_T(x_k)$ on each ion $k$. In Eqs. (1) and (2), $x_k$ denotes the displacement operator of the $k$th ion from its equilibrium position [so $\partial_{x_k} H_1(x_k) = 0$ by definition] and $p_k$ is the corresponding momentum operator [18].

To first entangle and then disentangle the ions' internal and motional states, we reverse the direction of the kicking force by 2 times as depicted in Fig. 2(a). So the force $F(t)$ in $H_0$ is time dependent, with $F(t) = F$ for $0 < t \leq t_1$ and $3t_1 < t \leq 4t_1$, and $F(t) = -F$ for $t_1 < t \leq 3t_1$ [see

Fig. 2(a), the value of $t_1$ will be specified later]. Under these laser kicks, Fig. 1(b) shows the typical state-dependent trajectory of the $i$ or $j$ ion in phase space under the "free" Hamiltonian $H_0$. Interestingly, independent of the initial position or velocity of the ion, the wave packets corresponding to the internal states $|0\rangle$ and $|1\rangle$ first split and then rejoin after a round trip. Under the Hamiltonian $H_0$, the position operator $x_k$ of the $k$th ion evolves as

$$x_k(t) = x_k + (t/m)p_k + s(t)\sum_{\alpha=i,j}\delta_{k\alpha}\sigma_\alpha^z, \qquad (3)$$

where $x_k$ and $p_k$ are the initial position and momentum operators, and $s(t) = \int_0^t \int_0^{\tau_1}[F(\tau_2)/m]d\tau_2 d\tau_1$.

To investigate the internal state evolution of the ions, we transfer to the interaction picture with respect to the free Hamiltonian $H_0$. The Hamiltonian $H_1$ in the interaction picture becomes $H_I(t) = U_0^\dagger(t)H_1(\{x_k\})U_0(t) = H_1(\{x_k(t)\})$, where $U_0(t) = \hat{T}\{\exp[-i\int_0^t H_0(\tau)/\hbar d\tau]\}$ ($\hat{T}\{\cdots\}$ represents the time-ordered integration) and $x_k(t) = U_0^\dagger(t)x_k U_0(t)$ is expressed as Eq. (3). The evolution operator in the interaction picture is given by $U_I(t) = \hat{T}\{\exp[-i\int_0^t H_I(\tau)/\hbar d\tau]\}$. Note that at the end of the gate ($t = 4t_1$), the $U_0(t)$ becomes an identity operators for the internal dynamics [as $s(4t_1) = 0$ in Eq. (3)], therefore the gate operation is wholly determined by the evolution operator $U_I(t)$ at time $t = 4t_1$.

To have an expression of $U_I(t)$, we note that there are three different length scales in our problem. They are the distance between the neighboring ions denoted as $d$, the length scale of the conditional displacement $s(t)$ denoted as $\bar{s}$, and the magnitudes of $x_k$ and $(t/m)p_k$ which are estimated by the ion oscillation length and denoted as $\xi$. For fast quantum gates, typically we have $d \gg \bar{s} \gg \xi$. Therefore, to the lowest (0th) order of the parameter $\xi/\bar{s}$, the coordinates $x_\alpha(t)$ ($\alpha = i, j$) are approximated by $s(t)\sigma_\alpha^z$. In this limit, at time $t = 4t_1$, the internal and external dynamics of $U_I(4t_1)$ become disentangled, and the internal state evolution is described by
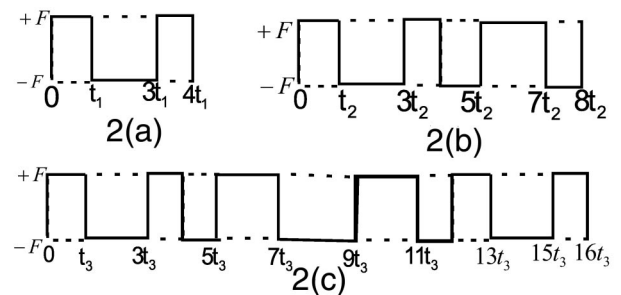


FIG. 2. The sequence of the spin-dependent kicking force for the one-cycle (a), two-cycle (b), and four-cycle (c) schemes, with $t_2 = t_1/2^{1/5}$ and $t_3 = t_1/2^{2/5}$. The solid (dashed) curves are, respectively, for the ions in $|0\rangle$ or $|1\rangle$ states.

$$U_{\text{in}} = \exp\left[-i(V_{ij}''/\hbar)\int_0^{4t_1} s^2(t)dt\,\sigma_i^z\sigma_j^z\right], \qquad (4)$$

where the derivative of the Coulomb potential $V_{ij}'' = \partial_{x_i}\partial_{x_j}V_{ij}(|x_i - x_j|) \simeq 2\hbar c/(137d^3)$ ($c$ is the light velocity). In writing Eq. (4), we have expanded the interaction Hamiltonian $H_I(t)$ to the second order of the small parameter $\bar{s}/d$ (the harmonic approximation). Equation (4) represents an ideal controlled phase flip gate on the neighboring ions $i, j$ if $(V_{ij}''/h)\int_0^{4t_1} s^2(t)dt = \pi/4$, which determines the gate time

$$T_g = 4t_1 \simeq 7.09[f_r^2 v_r^2 c/d^3]^{-1/5}. \qquad (5)$$

In deriving Eq. (5), we have substituted the expression of the kicking force $F$ into $s(t)$, and $v_r = \hbar k_c/m$ denotes the atom recoil velocity. Figure 3(a) shows the gate time $T_g$ as a function of the repetition frequency $f_r$ of the laser pulses for the $^{111}$Cd$^+$, $^{40}$Ca$^+$, and $^9$Be$^+$ ions. The gate speed ($1/T_g$) increases with the kicking frequency as $f_r^{2/5}$ and decreases with the neighboring-ion distance as $d^{-3/5}$. The total number of the laser pulses for each gate is given by $N = f_r T_g$, which increases with the gate speed as $(1/T_g)^{3/2}$.

We now examine the influence of the ions' oscillations by expanding the interaction Hamiltonian $H_I(t)$ to the next (1st) order of $\xi/\bar{s}$, which gives some intrinsic noise to the above conditional quantum gate after the ions' motional states are traced. This intrinsic noise comes from two respects: (i) The internal and motional states of the $i$ and $j$ ions do not become fully disentangled after the pulse sequence; (ii) Because of the mutual Coulomb interactions, the state-dependent trajectory of the $i, j$ ions
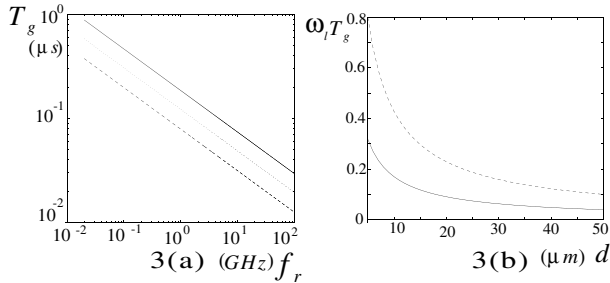


3(a) (GHz) $f_r$     3(b) ($\mu$m) $d$

FIG. 3. The gate time $T_g$ for the controlled phase flip operation is shown as a function of the repetition frequency of the kicking laser pulses for the $^{111}$Cd$^+$ (solid curve), $^{40}$Ca$^+$ (dotted curve), and $^9$Be$^+$ (dashed curve) ions, where the neighboring ions' distance $d$ is taken as $d = 10$ $\mu$m, and the carrier wave length for the $^{111}$Cd$^+$, $^{40}$Ca$^+$, and $^9$Be$^+$ ions are assumed to be 215 nm, 393 nm, and 313 nm, respectively. (b) The scaling parameter $\omega_l T_g$ in the gate fidelity is shown as a function of the neighboring ions' distance $d$ for $^{111}$Cd$^+$ with the pulse repetition frequency $f_r = 1$ GHz (dashed curve) and $f_r = 10$ GHz (solid curve), respectively. The curve is well fit by the scaling $\omega_l T_g \propto d^{-9/10}$.

yields a state-dependent force on all the other ions, which entangles the gate operation with the motional states of all the ions in the array. When the Hamiltonian $H_I(t)$ is expanded to the next order of $\xi/\bar{s}$, the evolution operator $U_I(4t_1)$ after the pulse sequence is decomposed as a product of the dominant part given by $U_{in}$ in Eq. (4), and the noise part $U_{ns}$ given by

$$U_{ns} = \exp\left[-i\sum_{\alpha=i,j;k}\sigma_\alpha^z(\beta_{\alpha k}x_k + \gamma_{\alpha k}p_k)\right]. \qquad (6)$$

The coefficients $\beta_{\alpha k} = (1 - \delta_{\alpha k}/2)H_{1\alpha k}''\int_0^{4t_1}s(t)dt$, $\gamma_{\alpha k} = (1 - \delta_{\alpha k}/2)H_{1\alpha k}''\int_0^{4t_1}(t/m)s(t)dt$, and the factor $H_{1\alpha k}'' \equiv \partial_{x_\alpha}\partial_{x_\kappa}H_1$ denotes the potential derivative at the ions' equilibrium position. The operator $U_{ns}$ entangles the internal state of the target ions with the motional states of all the ions in the array, and introduces a gate infidelity after we take trace over the motional states. To give a quantitative estimate of this intrinsic gate infidelity, we take the following simplifications: (i) We denote $\omega_L = \sqrt{H_{1kk}''/m}$ as the local oscillation frequency of the ion $k$ (with all the other ions fixed on their equilibrium positions) and assume that this frequency is roughly the same for all the ions ($\omega_L$ is $k$ independent); (ii) The states of the local oscillation modes ($x_k, p_k$) are assumed to be thermal with the mean phonon number estimated by $\bar{n} = k_BT_t/(\hbar\omega_l)$, where $T_t$ is the temperature; (iii) The gate fidelity $F_g$ is defined as $F_g = \langle\Psi_0|U_{in}^\dagger\rho_r U_{in}|\Psi_0\rangle$, where $|\Psi_0\rangle$ is the initial state of the $i, j$ ions, taken to be $(|0\rangle_i + |1\rangle_i)\otimes(|0\rangle_j + |1\rangle_j)/2$ as an example, and the final reduced internal state $\rho_r = Tr_m[U_{ns}U_{in}|\Psi_0\rangle\langle\Psi_0|U_{in}^\dagger U_{ns}^\dagger]$ is obtained by taking trace over the motional states; (iv) We define the parameters $\xi, \bar{s}$ quantitatively as $\xi = \sqrt{\hbar/(m\omega_L)}$ and $\bar{s} = [\int_0^{4t_1}s^2(t)dt]/[\int_0^{4t_1}s(t)dt]$. With these reasonable simplifications, we find that the gate fidelity $F_g$ is estimated by $F_g \approx \exp[-c_1(\bar{n} + 1/2)(\xi/\bar{s})^2]$, where the dimensionless constant $c_1$ is defined as $c_1 = \pi^2[(H_{1ii}''/2H_{1ij}'')^2 + \sum_{k\neq i}(H_{1ik}''/H_{1ij}'')^2]/4$. With the definitions of $\xi, \bar{s}$ and Eq. (5) for $T_g$, we can also express the gate fidelity as

$$F_g \approx \exp[-c_2(\bar{n} + 1/2)\omega_L T_g]. \qquad (7)$$

The dimensionless constant $c_2 \simeq 0.83c_1(H_{1ij}''/H_{1ii}'')$. To estimate the typical values of $c_1$ and $c_2$, in calculating $H_{1ii}''$ we take into account only the contributions from the intrinsic Coulomb interactions (neglecting the contribution from the external trapping potentials), and find that $c_1 \approx 8.6$ (12.0) and $c_2 \approx 3.0$ (3.0), respectively, for a 1 (2)-dimensional infinite ion array. Note that the values of $c_1$ and $c_2$ are well bounded from above even if we summarize over an infinite ion array, resulting form the fact that the noise comes from an effective mutual dipole interaction between the ions (described by the second

derivatives of $H_1$), which falls off very rapidly with the ions' distance.

The critical parameter $\omega_L T_g$ is shown in Fig. 3(b) as a function of the distance $d$ between the neighboring ions with the laser repetition frequency $f_r = 1$ (10) GHz. Although this parameter scales down with increase of the ions' distance, its value is not tiny in the typical parameter region, and it is ineffective to reduce this parameter by further increase of the laser repetition frequency due to the slow scaling $\omega_L T_g \sim f_r^{-2/5}$. A significant intrinsic infidelity for the quantum gate seems to be unavoidable if we substitute the typical value of $\omega_l T_g$ into Eq. (7). Fortunately, this is not the case as there is an elegant way to greatly reduce this noise. In the above analyses, we applied one cycle of the kicking force [shown in Fig. 2(a)] which pushes the ions to the left (right) side if they are in the $|0\rangle$ ($|1\rangle$) state (see Fig. 1). We can improve the scheme by using a two-cycle force as shown in Fig. 2(b). Each cycle of the evolution contributes the same amount of conditional phase which accumulated for a conditional phase flip gate (so in Fig. 2 we should choose $t_2 = t_1/2^{1/5} = T_g/4/2^{0.2}$). However, for these two cycles, the ions are pushed to the reverse directions, and the coefficients $\beta_{\alpha k}$ and $\gamma_{\alpha k}$ in the noise operator $U_{ns}$ (6) thus have a reverse sign. Because of this sign flip, the noise effects from these two cycles exactly cancel with each other. This noise cancellation is perfect up to the 1st order of the parameter $\xi/\bar{s}$. To estimate the residue noise with a two-cycle force, we take into account all the higher order contributions which basically rotate the operators $x_k$ and $p_k$ in Eq. (6) with a rotational angle $\theta \approx 4\omega_L t_2 \approx 0.87\omega_L T_g$ for two different cycles, so after a partial cancellation of the noise, the accumulated coefficients $\beta_{\alpha k}$ and $\gamma_{\alpha k}$ in the noise operator $U_{ns}$ are reduced effectively by a factor of $1 - e^{i\theta} \approx i\theta$, and the final gate fidelity $F_{g2}$ for the two-cycle scheme thus has the form $F_{g2} \approx \exp[-0.87^2 c_2(\bar{n} + 1/2)(\omega_L T_g)^3]$. We can further reduce the noise by using more cycles of the kicking force to get noise cancellation up to a higher order of the rotation angle $\theta$. For instance, Fig. 2(c) shows a four-cycle kicking force, which reduces the accumulated noise coefficients $\beta_{\alpha k}$ and $\gamma_{\alpha k}$ by a factor of $2(\omega_L T_g)^2 \times 4^{-2/5}$, and the corresponding gate infidelity $F_{g4}$ is given by

$$F_{g4} \approx \exp[-1.32 c_2(\bar{n} + 1/2)(\omega_L T_g)^5]. \qquad (8)$$

With such a improvement, the gate infidelity $\delta F = 1 - F_{g4}$ has been tiny in the typical parameter region. For instance, $\delta F \approx 0.006\%$ with the laser repetition frequency $f_r \approx 1$ GHz, the neighboring ions' distance $d \approx 50 \ \mu m$, and thermal phonon number $\bar{n} \sim 1$.

Summing up, we have proposed a scheme to achieve scalable quantum computation with trapped ions based on control from fast laser pulses. This scheme has the following features: (i) As the ion shuttling is not required, one can use an ion array in any convenient geometry from any types of ion traps, which greatly simplifies the trap design; (ii) The scheme is insensitive to the temperature of the ions, and requires no challenging cooling of the ion crystal to attain the Lamb-Dicke limit. (iii) The conditional gates are very fast even if we have a pretty large distance between the neighboring ions which allows easy separate addressing; (iv) The whole computation is also fast as the slow step of separating the ions is not required for achieving scalability in this scheme. The basic experimental challenge is to well control the fast laser pulses to induce a series of coherent photon kicks. Periodic laser pulses with a repetition frequency varying from hundreds of MHz to few THz have been reported in many experiments [19], which indicates realistic prospects for building scalable ion trap quantum computers based on this approach.

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