Soliton Trains in Bose-Fermi Mixtures

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We theoretically consider the formation of bright solitons in a mixture of Bose and Fermi degenerate gases. While we assume the forces between atoms in a pure Bose component to be effectively repulsive, their character can be changed from repulsive to attractive in the presence of fermions provided the Bose and Fermi gases attract each other strongly enough. In such a regime the Bose component becomes a gas of effectively attractive atoms. Hence, generating bright solitons in the bosonic gas is possible. Indeed, after a sudden increase of the strength of attraction between bosons and fermions (realized by using a Feshbach resonance technique or by firm radial squeezing of both samples) soliton trains appear in the Bose-Fermi mixture.

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Solitonic solutions are a very general feature of nonlinear wave equations. They differ from ordinary wave packets as they retain their shape while propagating instead of spreading due to dispersion. This intriguing feature is based on the existence of a nonlinear interaction which compensates for dispersion and produces a self-focusing effect on the propagating wave packet.

Dilute atomic quantum gases offer a unique environment to study fundamental solitonic excitations in a pure quantum system with intrinsic nonlinearity. Since the interparticle interaction causing this nonlinearity can be both attractive and repulsive, the Gross-Pitaevskii equation describing the evolution of the condensate wave function exhibits both dark and bright solitonic solutions [1]. Dark solitons as a fundamental excitation in stable Bose-Einstein condensates with repulsive interparticle interaction have already been studied in [2–4].

Bright solitons have been observed in Bose-Einstein condensates of ^{7}Li in quasi-one-dimensional geometry [5,6]. However, in three-dimensional geometry usually used to prepare the sample the necessary large and negative scattering length leads to density-limited particle numbers (dynamical instability—collapse). The observation of bright solitons was therefore possible only due to the magnetic tuning of the interactions from repulsive (used to form a stable Bose-Einstein condensate) to attractive during the experiments.

Another experimental approach to bright matter wave solitons was realized in the recently reported observation of gap solitons [7] in a condensate with repulsive interactions by engineering of the matter wave dispersion relation via sophisticated manipulation in a periodic potential (concept of negative effective mass [8]).

In this Letter we propose a novel scheme to realize bright solitons in one-dimensional atomic quantum gases. In particular, we study the formation of bright solitons in a Bose-Einstein condensate embedded in a quantum degenerate Fermi gas. One important feature is that this mixture allows tuning of the one-dimensional interactions not only by Feshbach resonances but also by simply changing the trap geometry.

We consider the bare interaction between bosonic atoms to be repulsive ($g_B > 0$), whereas the particle interaction between bosonic and fermionic atoms is assumed to be strongly attractive ($g_{BF} < 0$). Bright solitons in Bose-Fermi gas mixtures are then produced as a result of a competition between two interparticle interactions: boson-boson repulsion versus boson-fermion attraction. Experimentally, this situation is, for example, accessible in Bose-Fermi mixtures of bosonic ^{87}Rb atoms and fermionic ^{40}K atoms.

We determine the critical strength of attraction between bosons and fermions necessary for the formation of bright solitons and show that these parameter regimes might be achievable in present experiments. We study the formation of bright solitons by different excitation mechanisms: either increasing the attractive boson-fermion interaction by Feshbach resonance techniques or by radial squeezing of the mixture which corresponds to increasing the effective one-dimensional scattering length. We contrast the response of the system following adiabatic and fast increase of the boson-fermion interaction strength.

We consider a Bose-Fermi mixture confined in a trap at zero temperature and describe this system in terms of the many-body wave function $\Psi(\mathbf{x}_1, \ldots, \mathbf{x}_{N_B}; \mathbf{y}_1, \ldots, \mathbf{y}_{N_F})$, where N_B and N_F are the numbers of bosons and fermions, respectively. We use the approach based on the Lagrangian density. Since the fermionic sample is spin polarized only boson-boson and boson-fermion interactions are included. At zero temperature we assume that the wave function of the Bose-Fermi mixture is a product of the Hartree ansatz for bosons and the Slater determinant (antisymmetric wave function) for fermions.

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Introduced in this way the single-particle wave functions $(\varphi^{(B)}, \varphi_1^{(F)}, \ldots, \varphi_{N_F}^{(F)})$ have to now be determined. Therefore, the assumed many-body wave function is inserted into the Lagrangian density and integrated over the $N_B - 1$ bosonic and $N_F - 1$ fermionic spatial coordinates. This leads to the mean-field single-particle Lagrangian (for details, see Ref. [9]) and the corresponding Euler-Lagrange equations are the basic equations of the presented approach (here, $j = 1, 2, ..., N_F$)

$$\begin{aligned} i\hbar \frac{\partial \varphi^{(B)}}{\partial t} &= -\frac{\hbar^2}{2m_B}\nabla^2\varphi^{(B)} + V_{\mathrm{trap}}^{(B)}\varphi^{(B)} \\ &\quad + g_B N_B |\varphi^{(B)}|^2\varphi^{(B)} + g_{BF}\sum_{i=1}^{N_F}|\varphi_i^{(F)}|^2\varphi^{(B)}, \\ i\hbar \frac{\partial \varphi_j^{(F)}}{\partial t} &= -\frac{\hbar^2}{2m_F}\nabla^2\varphi_j^{(F)} + V_{\mathrm{trap}}^{(F)}\varphi_j^{(F)} \\ &\quad + g_{BF} N_B |\varphi^{(B)}|^2\varphi_j^{(F)}. \end{aligned} \tag{1}$$

All of the above equations have a simple interpretation. Removing the last terms in these equations (i.e., neglecting the mean-field interaction energy between Bose and Fermi components in comparison with other energies) one recovers the Gross-Pitaevskii equation for a degenerate Bose gas and the set of Schrödinger equations describing a noninteracting Fermi system. It is easy to notice that when the Bose and Fermi gases attract each other strongly enough the mean-field energy connected with this attraction can overcome the repulsive mean-field energy for bosons. It means that the presence of a degenerate Fermi gas changes the character of the interaction between the bosonic atoms from repulsive to attractive.

Based on the above considerations one can write the condition for the value of the critical strength of attraction between bosons and fermions. It is given by $g_B n_B = |g_{BF}^{\mathrm{cr}}| n_F$, where $n_B = N_B|\varphi^{(B)}|^2$ and $n_F = \sum_{i=1}^{N_F}|\varphi_i^{(F)}|^2$ are the densities of both fractions, normalized to the number of particles, taken at the center of the trap. A rough estimation of g_{BF}^{cr} can be found assuming that the densities of both components are calculated within the Thomas-Fermi approximation and that the components do not interact. Then we have

$$|g_{BF}^{\mathrm{cr}}| = C\frac{N_B^{2/5}}{N_F^{1/2}} g_B, \tag{2}$$

where $C = C_1(a_\perp^B/a_B)^{3/5}(a_\perp^F/a_\perp^B)^3\lambda_B^{2/5}/\lambda_F^{1/2}$, $C_1 = 3^{9/10}5^{2/5}\pi/16 \approx 1.0$, $a_\perp$ is the radial harmonic oscillator length, $\lambda = \omega_z/\omega_\perp$ defines the aspect ratio of the axially symmetric trap, and a_B is the s-wave scattering length for the pure Bose gas related to the interaction strength through $g_B = 4\pi\hbar^2 a_B/m_B$. The condition (2) has several implications. Squeezing radially both the Bose and Fermi components decreases the value of critical g_{BF}. The same happens when the number of fermions is getting bigger in comparison with the number of bosons. For a particular trap the numbers of atoms are limited by the occurrence of a collapse [10]. In the case of the experiment of Ref. [10] it was found that the system was stable if the number of atoms in both species (^{87}Rb and ^{40}K in $|2, 2\rangle$ and $|9/2, 9/2\rangle$ hyperfine states, respectively) were smaller than 2×10^4. Taking the parameters of that experiment ($\omega_\perp^B = 2\pi \times 215$ Hz and $\omega_z^B = 2\pi \times 16.3$ Hz) and assuming the following numbers of atoms, $N_B = 10^3$ and $N_F = 10^4$, one gets the critical coupling $|g_{BF}^{\mathrm{cr}}| = 6.7 g_B$ which equals the natural value of g_{BF} for a ^{87}Rb-^{40}K mixture in the double-polarized state mentioned above. It is understood that relation (2) is only a necessary condition for creation of bright solitons. Another important factor is the geometry of the system.

In the following we concentrate on a one-dimensional geometry, which is most favorable for the appearance of solitons. In Fig. 1 we plot the density profiles (in oscillatory units defined as $1/a_{ho}$, where $a_{ho} = \sqrt{\hbar/m_F\omega_F}$) of the Fermi (dashed lines) and Bose (solid lines) components of the one-dimensional mixture in its ground state. The gases are confined in a trap with frequencies $2\pi \times 30$ Hz (for fermions) and $2\pi \times 20$ Hz (for bosons). To get these curves we solve numerically the set of Eqs. (1) by evolving adiabatically the coupling constant g_{BF} from zero to the given value. Even though the number of bosons is 10 times larger than the number of fermions, the size of the fermionic cloud is much bigger due to the Pauli exclusion principle. We see that after turning on the attractive forces between the components some number of fermions is drawn inside the bosonic cloud. When the attraction is increasing further the fermionic cloud is clearly divided into two distinguishable parts. One of

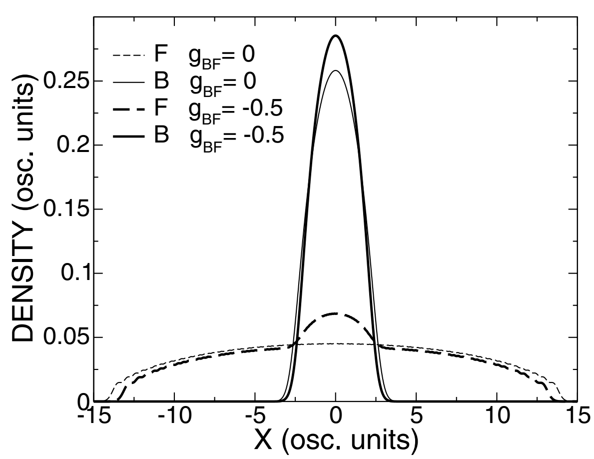

FIG. 1. Density profiles (normalized to one) of a one-dimensional Bose-Fermi mixture for $N_B = 1000$ bosons and $N_F = 100$ fermions for various strengths of interspecies attraction. The effective repulsion for bosons is $g_B = 0.0163$ oscillatory (osc.) units. Solid lines correspond to the Bose fraction, whereas the dashed ones indicate fermions.

them is a broad background gas, whereas the second is the narrow density peak hidden within the bosonic peak. Both peaks get higher and narrower when the attraction is getting stronger. This is a sign of effective attraction.

It turns out that for strong enough attraction between the Bose and Fermi components the central peaks (bosonic and fermionic ones) in Fig. 1 form a structure which after switching off the trapping potential persists without changing its shape. However, the broad fermionic background is lost. This is illustrated in Fig. 2, where the fermionic and bosonic densities are plotted some time after opening the trap. Upper and lower frames differ by the value of g_{BF}. In the case of the upper frame the strength of attraction is weaker than the critical one and no solitonic behavior is observed—the bosonic and fermionic clouds spread out. For the lower frame the attraction is strong enough and the formation of a pronounced peak in both Bose and Fermi components is observed. Such a structure can be forced to move by imposing a momentum on it (realized experimentally, e.g., by applying a Bragg scattering technique) and its shape again does not change. The critical value of the coupling constant is in our one-dimensional model approximately equal to $g_{BF}^{\rm cr} \approx -1.0$ osc. units ($\hbar\omega_F a_{ho}$) and can be compared to the value obtained based on the one-dimensional counterpart of condition (2). Taking the ratio $n_B(0)/n_F(0) = 57.5$ from Fig. 1 one gets $g_{BF}^{\rm cr} = -0.94$ osc. units what remains in agreement with the numerical estimation.

In Figs. 3 and 4 we show the densities of the Bose and Fermi components after the strength of attraction between bosons and fermions has been increased by using a Feshbach resonance technique. The other way of changing the interaction strength could be the firm radial squeezing of both samples. The basic observation is that some time after switching the mutual interaction the bosonic cloud explodes into several peaks that oscillate under the influence of the trapping potential. This happens provided that the final coupling (attraction) between both components is strong enough. Stronger attraction results in a bigger number of peaks. Each bosonic peak (the solid line) contains the fermionic density (the dashed line).

Each single peak of this two-component structure oscillates in the trap almost without changing its shape with slightly different initial velocity. The initial velocity is determined by the amount of energy injected into the system due to the change of the coupling constant (i.e., by its rate and magnitude). All peaks meet at the center of the trap, where they collide, every half of the oscillation period. Usually, the transfer of some number of atoms happens during the collision. The analysis of the collision of two slowly moving peaks under no axial confinement shows that they are repulsive. The shape of the wave packet is not a secans-hyperbolicus one as it is for the solutions of the Gross-Pitaevskii equation for an attractive uniform condensate. This is because of a different kind of nonlinearity present in the system due to mediation of the bosonic interaction by the fermions. After switching off the trap each peak travels with constant

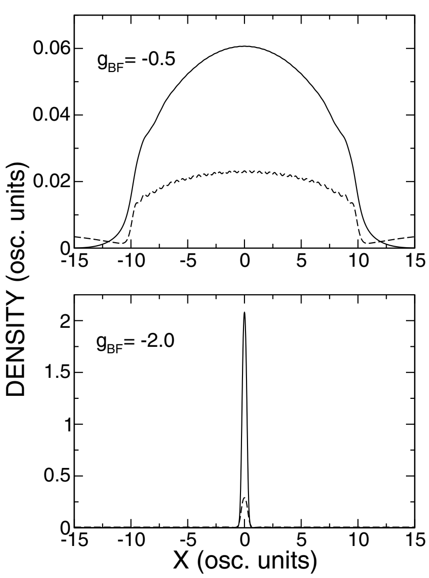

FIG. 2. Illustration of the solitonic character of the ground state of the Bose-Fermi mixture. Both frames show the densities 34 ms after switching off the trapping potential. Only when the attraction between the species is stronger than the critical one (the lower frame) the system forms a single-peak two-component structure which does not spread in time.

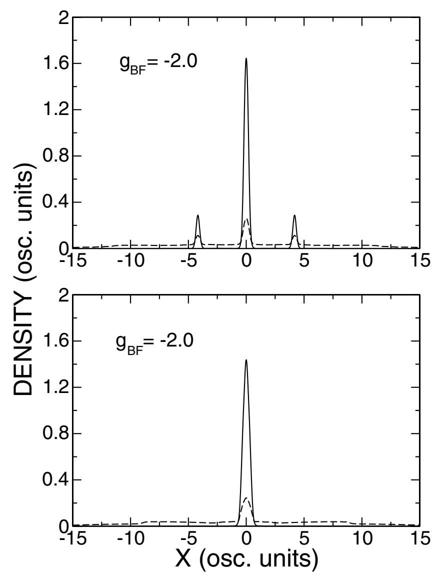

FIG. 3. Density profiles of a one-dimensional Bose-Fermi mixture after switching the attraction between the Bose and Fermi components from $g_{BF} = -0.5$ to $g_{BF} = -2.0$ osc. units. The interaction strength is changed linearly during 8.5 ms (upper frame) and 17 ms (lower frame). The snapshots are taken at 20 ms. A more adiabatic change of the strength of attraction results in a lower number of solitons.

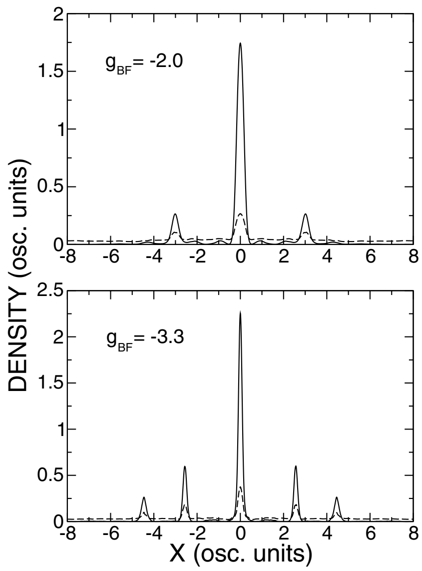

FIG. 4. Density profiles of a one-dimensional Bose-Fermi mixture 10.5 ms after changing the strength of attraction between two species from the initial value $g_{BF} = -0.8$ osc. units to the final one indicated by the label. Here, the strength of the interaction is increased instantaneously. The bosonic cloud breaks into several bright solitons; each soliton contains a piece of fermionic cloud. The bigger jump in the coupling g_{BF} results in a bigger number of solitons.

velocity without spreading. All the properties just discussed indicate that the observed structures can be called solitons.

In the case of Fig. 4 the strength of the coupling between the Bose and Fermi gases is changed instantaneously. The relative heights of the peaks depend on the initial width of the bosonic cloud and they get higher for a bigger number of bosons or smaller bosonic trap frequency. The latter can be realized only in the optical trap, since in the magnetic trap this ratio is fixed by the magnetic moments of the two atomic species. On the other hand, in Fig. 3 we show the response of the system when the mutual attraction is increased within a finite time which is on the order of the trap period. Slower switching off the interaction leads to a smaller number of solitons.

Based on the numerical results, we propose two schemes for generating bright solitons in degenerate Bose-Fermi mixtures. First of all, the system has to be pushed in the range of strong enough attraction between fermions and bosons. This can be achieved by producing a mixture in an appropriate doubly polarized state (for details, see Ref. [11]) and by using the Feshbach resonance technique. Another way is to follow the idea reported in Ref. [12] according to which squeezing radially both samples firmly enough imposes a one-dimensional geometry on the system with one-dimensional interaction strength given by $g_{3D}/\pi a_\perp^2$ ($a_\perp$ is the radial harmonic oscillator length). Increasing the radial confinement leads then to higher effective one-dimensional interaction strength. Therefore, one way to generate solitons in a Bose-Fermi mixture would follow the following scenario: (i) the ground state of the system is formed in an elongated trap at the natural value of g_{BF} and (ii) the strength of attraction g_{BF} is then increased. Another way could be performing the evaporation already under favorable conditions, i.e., at the presence of an appropriate magnetic field or a strong enough one-dimensional geometry. However, in this case only one single-peak two-component soliton is formed, placed at the center of the trap.

In conclusion, we have shown that bright solitons can be generated in a Bose-Fermi mixture as a result of a competition between two interparticle interactions: boson-boson collisions which are effectively repulsive and boson-fermion collisions which are attractive. Assuming that the strength of attraction is large enough both kinds of atoms start to mediate in the other species interaction introducing the system into a new regime where locally Bose and Fermi gases become gases of effectively attractive atoms. Therefore it becomes possible to generate bright solitons in the system under such conditions. Depending on how fast the change of the attraction strength is performed the system responds by forming a train of solitons (fast change) or a single soliton (adiabatic change). Each soliton is, in fact, the single-peak two-component structure with the fermionic cloud hidden within the bosonic one.

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[1] V. E. Zakharov and A. B. Shabat, Zh. Eksp. Teor. Fiz. **61**, 118 (1971) [Sov. Phys. JETP **34**, 62 (1972)]; **64**, 1627 (1973) [Sov. Phys. JETP **37**, 823 (1973)].
[2] S. Burger *et al.*, Phys. Rev. Lett. **83**, 5198 (1999).
[3] J. Denschlag *et al.*, Science **287**, 97 (2000).
[4] B. P. Anderson *et al.*, Phys. Rev. Lett. **86**, 2926 (2001).
[5] L. Khaykovich *et al.*, Science **296**, 1290 (2002).
[6] K. E. Strecker *et al.*, Nature (London) **417**, 150 (2002).
[7] B. Eiermann *et al.*, cond-mat/0402178.
[8] G. Lenz, P. Meystre, and E.M. Wright, Phys. Rev. A **50**, 1681 (1994).
[9] T. Karpiuk, M. Brewczyk, and K. Rzążewski, Phys. Rev. A **69**, 043603 (2004).
[10] G. Modugno *et al.*, Science **287**, 2240 (2002).
[11] A. Simoni *et al.*, Phys. Rev. Lett. **90**, 163202 (2003).
[12] M. Olshanii, Phys. Rev. Lett. **81**, 938 (1998).