Temporal Correlation Vortices and Topological Dispersion

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Interference measurements of a polychromatic partially coherent light source verify the existence of a temporal correlation vortex. Topological dispersion is found to destabilize this singularity.

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Vortices are found at many scales throughout physics. In systems benefiting from a wave description, vortex states may be described by azimuthally harmonic (i.e., quantized) wave functions. The realization that such waves are robust and carry angular momentum (which was first postulated by Bohr, advanced by de Broglie, and formalized by Schrödinger) has recently received considerable attention in optics [1]. The importance of vortex waves is extensive. The paramount feature of quantized vortex waves is the phase, which varies from zero to an integer multiple (the "topological charge") of 2π along a closed path enclosing a singular point [2]. The wave amplitude is understood to vanish at the singularity owing to total destructive interference. A problem with the wave description is that neither classical nor quantum systems are perfectly coherent; thus, total destructive interference cannot be strictly observed. A nonzero value of the time-averaged intensity at an expected singularity indicates a partially coherent field [3]. Here we show that the complex degree of coherence (CDC) may exist as a "temporal correlation vortex" (TCV). A TCV may allow anomalous statistical properties when the magnitude of the CDC vanishes at the vortex core. An understanding of zero coherence points may be useful beyond the field of optics. For example, the transition from coherent to incoherent behavior may link the correspondence between the wave (e.g., quantum) and classical realms [4]. Vortices may provide a means of exploring the transition between these realms. Owing to an artifact in our experiment, we discovered that frequency-dependent topological charge (i.e., "topological dispersion") may destabilize a TCV.

Statistical properties within a beam containing many vortices have been explored [5], and the spatial coherence properties of specially prepared beams have been investigated [6]. Applications taking advantage of the coherence filtering potential of optical vortices were proposed for astronomy and demonstrated for low-angle light scattering [3]. Investigations of the influence of temporal coherence on vortices within a spectrally distributed source have been few [7].

A measure of the temporal coherence between two fields may be found by recording the correlation between the fields at different time points, t and $t - \tau$, where τ is a relative delay between the two fields owing to different optical path lengths [8]. The mutual coherence function (MCF) in the measurement plane, $z = \text{const}$, having polar coordinates (r, θ), may be represented as a complex function given by an ensemble average over a representative set of statistically stationary random scalar fields, $\{E_1\}$ and $\{E_2\}$: $\Gamma_{12}(r, \theta; \tau) = \langle E_1(r, \theta; t) E_2{}^*(r, \theta; t - \tau)\rangle$. The CDC is given by the normalized MCF: $\gamma_{12}(r, \theta; \tau) = \Gamma_{12}(r, \theta; \tau)/(I_1 I_2)^{1/2}$, where $I_j = \Gamma_{jj}(r, \theta; 0)$ is the time-averaged intensity distribution of the jth field. We now explore the case where $\gamma_{12}(r, \theta; \tau)$ satisfies a vortex ansatz:

$$\gamma_{12}(r, \theta; \tau) = \langle h(r; \tau)\rangle \exp(iM\theta), \tag{1}$$

where $\langle h(r; \tau)\rangle$ is a radially symmetric complex amplitude function, and M, the TCV topological charge, is a nonzero signed integer. At $r = 0$, the phase is singular; hence the real and imaginary values of γ_{12} vanish identically: $\langle h(0; \tau)\rangle = 0$. Equation (1) may be called a correlation vortex, owing to the circulation of phase along a line $\vec{l}$ that encloses the singularity: $\oint \vec{\nabla}(M\theta) \cdot d\vec{l} = 2\pi M$, where θ ranges from 0 to 2π.

The correlation vortex described by Eq. (1) may occur, for example, when the electric fields are given by

$$E_j(r, \theta; t) = A_j(t) \exp(i\beta_j) g_j(r/w) \exp(im_j\theta) \exp[i(\omega_0 t)], \quad j = 1, 2, \tag{2}$$

where $A_j(t)$ is a random variable having stationary statistical properties, β_j is an arbitrary phase parameter, ω_0 is the mean angular frequency of the detected light, $g_j(r/w)$ is a radially symmetric function characterizing the beam envelope, and w characterizes the width of the electric field profile. Constant strengths for each field, $\langle |A_1|^2\rangle$ and $\langle |A_2|^2\rangle$, will be assumed. Although integer values of $M = m_1 - m_2$ are required, E_1 and E_2 need not be vortex fields; i.e., m_1 and m_2 need not be integers. In practice, a beam having a noninteger topological charge may form when a spiral phase plate produces a nonharmonic azimuthal phase variation [9]. Diffraction at the edge phase discontinuity is evident in such beams. We note that an autocorrelation function ($E_1 = E_2$) cannot produce a TCV since $M = 0$. Combining Eqs. (1) and (2), we find

h is independent of the amplitude functions, g_j:

$$h(r, \tau) = [\langle A_1(t)A_2^*(t-\tau)\rangle/(\langle|A_1|^2\rangle\langle|A_2|^2\rangle)^{1/2}] \times \exp[i(\omega_0\tau + \beta_1 - \beta_2)]. \quad (3)$$

The condition $h(0, \tau) = 0$ therefore requires $\langle A_1(t)A_2^*(t-\tau)\rangle = 0$ at $r = 0$. This finding suggests that a TCV is a "vortex filament" reminiscent of a vortex in an inviscid fluid. From an experimental point of view, we expect the fringe visibility, $\mathrm{Re}[\gamma_{12}(r, \theta, \tau)] = |\gamma_{12}(\tau)| \cos(\omega_0\tau + \beta_1 - \beta_2 + M\theta)$, to vanish at the vortex core.

A simple way to create a TCV is to correlate a nonvortex beam ($m_2 = 0$) with a vortex beam ($m_1 = M$). A single light source having a full-width at half-maximum (FWHM) spectral width, $\Delta\lambda$, and a Mirau interferometer (Veeco Optical Profiler model NT3300) were used to create these fields and to combine them at different delay times. As shown in Fig. 1, the vortex beam was produced by reflecting light from a surface resembling a spiral staircase having $N_s = 8$ steps. The commercial interferometer allowed us to reconstruct the surface profile [see Fig. 1(b)] and to determine the nominal height of seven steps: $\delta h = 0.22$ μm. Each step produced an additional round trip optical phase delay of $\delta\phi = 4\pi\delta h/\lambda$, resulting in a pitch $\Delta h = 7\delta h = 1.52$ μm. Owing to a different optical delay at each step, the magnitude of the degree of coherence is shifted. Thus we replace $|\gamma_{12}(r, \theta; \tau)|$ with $|\gamma_{12}(r, \theta; \tau_j)|$ where $\tau_j = \tau + 2j\delta h/c$, for $j = 0, 1, 2, \ldots, N_s - 1$. Furthermore, different wavelengths accumulate different round trip phase changes since $\delta\phi \propto 1/\lambda$. The mask therefore has a topologically dispersive affect on the beam, resulting in a spectral charge distribution $M(\lambda)$ that may be characterized by the deviation,

$$\Delta M = M_c \Delta\lambda/\lambda_0 = M_c/(T_c \nu_0), \quad (4)$$

where $M_c = 2\Delta h/\lambda_0$ is the average topological charge of the reflected beam for a mask having a continuously varying height, and λ_0 and $\nu_0 = c/\lambda_0$ are the mean detected wavelength and frequency, respectively. Equation (4) states that the topological charge varies by an amount, ΔM, across the FWHM spectral width of the light source. Consequently, topologically dispersive effects may become evident over optical delays of the order $T_c/\Delta M$.

When the mask topology varies discretely, rather than continuously, the finite number of steps may, in effect, subsample the phase. That is, there may be an insufficient number of phase steps to construct the desired phase topology. The effective charge generated by a discrete mask may be expressed as

$$M = (N_s/2\pi) \arccos[\cos(\Phi_c)], \quad (5)$$

where $\Phi_c = \mathrm{mod}(2\pi M_c/N_s, 2\pi)$. For example, if $\lambda_0 = 583$ nm and $\Delta\lambda = 20$ nm, then $M_c = 5.21$, $\Delta M = 0.18$, and $M = 2.79$. By displacing the reference mirror by roughly 25 μm and the object, respectively, we found that the beam developed into a well-characterized vortex near the core region by radiating nonvortex phase structures.

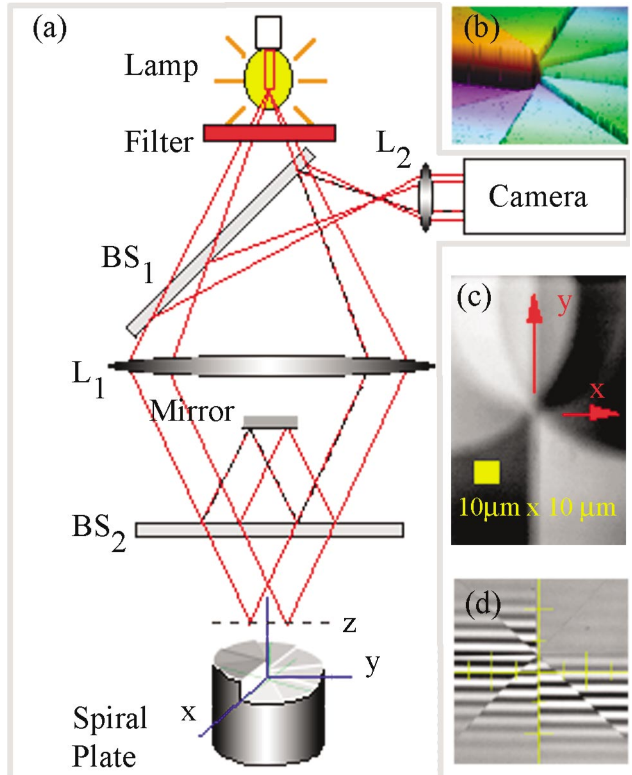

FIG. 1 (color online). (a) Simplified schematic of Mirau interferometer and eight-level spiral glass plate. The lamp and filter are characterized by a center wavelength, λ_0, and a FWHM bandwidth, $\Delta\lambda$. A $51.8\times$ microscope objective, L_1, having a numerical aperture of 0.55 was used along with a camera lens, L_2, to image the surface of the plate near the camera detector array. The optical path length difference was scanned by translating the mirror relative to the beam splitter, BS_2. (b) Reconstructed surface profile of the plate with a measured pitch of $\Delta h = 1.52$ μm. (c) Typical interference profile. (d) Interference profile obtained when the mirror is tilted, making the object and reference beams nonparallel.

Three-dimensional (x, y, τ) sets of interference fringes were recorded in the vicinity of the vortex core and near the condition of zero optical path delay. A single frame at $\tau \approx 0$ is shown in Fig. 1(c) when the beams are collinear. The spatial origin $(x, y) = (0, 0)$ has been placed at the point where the vertices of each of the eight phase regions meet. This point is the expected position of the vortex core. Tilting one of the beams results in fringe patterns as shown in Fig. 1(d). Note the shifted fringe visibility at each step. To find the expected zero in the temporal correlation function, we used a computer to render image slices in the x-τ and y-τ planes. Slices through the expected vortex core are shown in Fig. 2. Both the x-τ (at

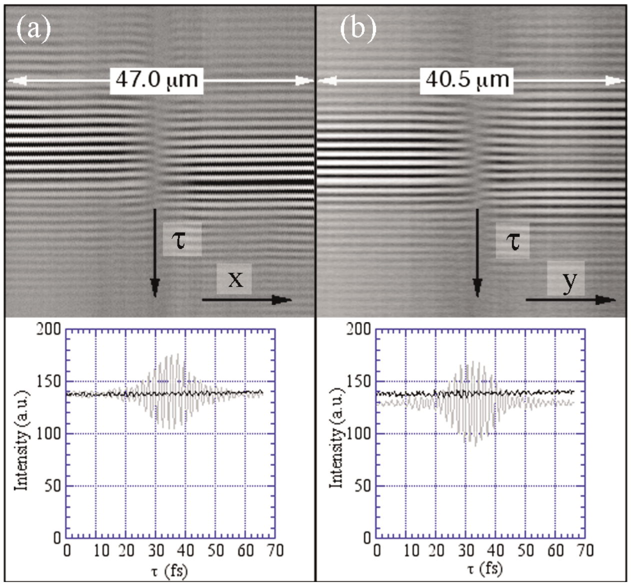

FIG. 2 (color online). Correlograms at $\lambda_0 = 583$ nm through the correlation vortex core for a $\Delta\lambda = 20$ nm bandwidth light source showing a stable correlation vortex having negligible fringe visibility in both the (a) x-τ ($y = 0$) and (b) y-τ ($x = 0$) planes. The graphs depict the different fringe visibility inside (black) and outside (gray) the vortex core.

$y = 0$) slice in Fig. 2(a) and the y-τ (at $x = 0$) slice in Fig. 2(b) exhibit distinct narrow lines of vanishing fringe visibility for all values of τ. We attribute the small nonvanishing fringe visibility in the plots to the resolution of the detection system—the vortex area is on the order of the pixel size of our camera. The fringe registration is out of phase across the τ axis, indicating that an odd integer-valued vortex was generated in the object beam. The light source in this experiment had a center frequency of $\lambda_0 = 583$ nm, and thus the beam was expected to develop into a vortex of charge $M \approx 3$. A careful inspection of the fringes reveals a subtle shift of the fringe registration, which may be attributed to the small topological dispersion ($\Delta M = 0.2$). These figures verify that a stable correlation vortex exists across the coherence time of the light source. Indeed the plots in Fig. 2 indicate that the interference fringes nearly vanish along the line ($x = 0, y = 0, \tau$) (black curves), whereas the visibility reaches significantly greater values outside the vortex core region (gray curves).

An experiment using a larger bandwidth source ($\Delta\lambda = 300$ nm) demonstrates the instability of a correlation vortex to strong topological dispersion. The topological dispersion and charge for this case is $\Delta M = 2.34$ and $M = 3.13$, respectively, where$\lambda_0 = 624$ nm and $M_c = 4.87$. The correlograms in Fig. 3 indicate the correlation vortex is unstable over optical delay times smaller than the coherence time. In the x-τ plane, the path of zero fringe visibility is a sloping line, and the y-τ plane exhibits multiple phase dislocations. The position and

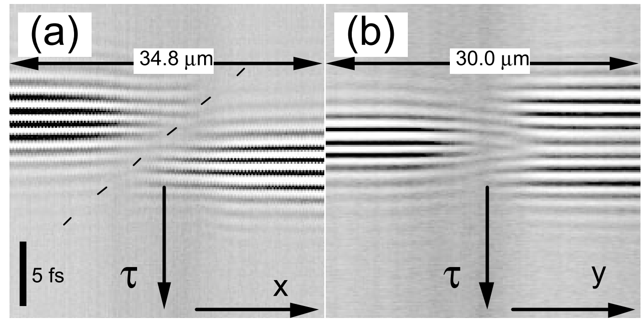

FIG. 3. Correlograms of an unstable correlation vortex having a bandwidth of $\Delta\lambda = 300$ nm.

charge of the correlation vortex is therefore unstable to large values of topological dispersion, ΔM. Fourier analysis of the correlograms may provide additional information about correlation vortices and topological dispersion.

In summary, we have predicted and observed a temporal correlation vortex. When the effective topological charge varied across the spectrum, we discovered that the TCV became unstable. Although observed in the optical domain, TCV's may be expected in any partially coherent wave system. From a historical point of view, these experiments are high-frequency (time-averaged) analogs of the original sonar and radio wave experiments [2] that helped launch the field of singular optics.

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[9] Formally, a vortex of fractional charge m' may be expressed as an infinite series of vortices having integer charge m: $f(\theta) = \exp(im'\theta) = \sum_{m=-\infty}^{+\infty} C_m \exp(im\theta)$, where $C_m = (-1)^m \mathrm{sinc}[\pi(m' - m)] \exp(i\pi m')$, assuming $f(\theta) = f(\theta + 2\pi l)$ for $l = \pm 1, \pm 2, \ldots$. [See, for example, H. Stark, J. Opt. Soc. Am. **69**, 1519 (1979).]