Ghost Imaging with Thermal Light: Comparing Entanglement and Classical Correlation

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We consider a scheme for coherent imaging that exploits the classical correlation of two beams obtained by splitting incoherent thermal radiation. This case is analyzed in parallel with the configuration based on two entangled beams produced by parametric down-conversion, and a precise formal analogy is pointed out. This analogy opens the possibility of using classical beams from thermal radiation for ghost imaging schemes in the same way as entangled beams.

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The ghost imaging technique [1–7] exploits the quantum entanglement of the state generated by parametric down-conversion (PDC) in order to retrieve information about an unknown object. In the photon-counting regime of PDC, the photons of a pair are spatially separated and propagate through two distinct imaging systems. In the path of one of the photons an object is located. Information about the spatial distribution of this object is obtained by registering the coincidence counts as a function of the other photon position [1–3]. In the regime of a large number of photon pairs, this procedure is generalized to the measurement of the signal-idler spatial correlation function of intensity fluctuations [4]. Such a two-arm configuration opens the possibility of performing coherent imaging by using, in a sense, spatially incoherent light, since each of the two down-converted beams taken separately is described by a thermal-like mixture and only the two-beam state is pure [3,4].

In this Letter, we show that such a scheme can be implemented with classical incoherent light, as the radiation produced by a thermal (or quasithermal) source. A comparison between thermal and biphoton emission is performed in [7], where an underlying duality accompanies the mathematical similarity between the two cases. Here we consider a different scheme (Fig. 1), appropriate for correlated imaging, in which a thermal beam is divided by a beam splitter, and the two outgoing beams are handled in the same way as the PDC beams in entangled imaging. A precise formal analogy between the PDC and the thermal case will emerge from our analysis.

Currently there is a debate whether quantum entanglement is necessary to perform ghost imaging [3–6]. A recent experiment [5] reproduced the results of a ghost image experiment by using classically correlated beams. However, the theoretical discussion in [5] suggested that, although any single experiment in ghost imaging could be reproduced by classically correlated beams, a non-entangled source could not emulate the behavior of an entangled source for all possible imaging schemes. Similar conclusions were reached in [4], where, in particular, it was suggested that entanglement played a crucial role in providing the ability to reconstruct both the image and the diffraction pattern of an object. In contrast to this, we will show here that the spatial correlation of the two beams produced by splitting thermal light, although being completely classical, is enough to mimic qualitatively *all* the features of entangled imaging, even in ways which were not believed possible before.

For the sake of comparison, we will treat in parallel the cases of entangled beams and of thermal light. For simplicity, we ignore the time argument, which corresponds to using a narrow frequency filter. We will come back to this point in the final part of the Letter. In addition, we assume translational invariance in the transverse plane.

In the entangled case, the signal and idler fields are generated in a type II $\chi^{(2)}$ crystal by a PDC process. Our starting point is the input-output relations of the crystal, which in the plane-wave pump approximation read [4,8,9]

$$b_i(\vec{q}) = U_i(\vec{q})a_i(\vec{q}) + V_i(\vec{q})a_j^\dagger(-\vec{q}), \qquad i \neq j = 1, 2. \tag{1}$$

Here $b_i(\vec{q}) = \int \frac{d\vec{x}}{2\pi} e^{-i\vec{q}\cdot\vec{x}} b_i(\vec{x})$, where $b_i(\vec{x})$ are the signal ($i = 1$) and idler ($i = 2$) field envelope operators at the output face of the crystal (distinguished by their orthogonal polarizations), $\vec{x}$ being the position in the transverse plane. a_i are the corresponding fields at the input face of

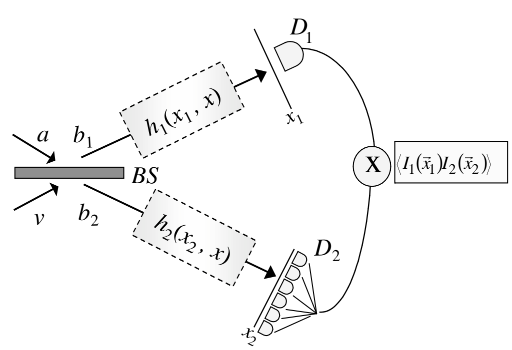

FIG. 1. Correlated imaging with incoherent thermal light. The thermal beam a is split into two beams which travel through two distinct imaging systems. Arm 1 includes an object. BS is a beam splitter, D_1 is a pointlike detector, D_2 is an array of pixel detectors, and v is a vacuum field.

the crystal and are in the vacuum state. The gain functions U_i, V_i are, for example, given in [8].

In the thermal case, we start from the input-output relations of a beam splitter:

$$b_1(\vec{x}) = ra(\vec{x}) + tv(\vec{x}), \qquad b_2(\vec{x}) = ta(\vec{x}) + rv(\vec{x}), \quad (2)$$

where t and r are the transmission and reflection coefficients of the mirror, a is a thermal field, and v is a vacuum field uncorrelated from a. We assume that the thermal state of a is characterized by a Gaussian field statistics, in which any correlation function of arbitrary order is expressed via the second order correlation function [10]:

$$\langle a^\dagger(\vec{x})a(\vec{x}')\rangle = \int \frac{d\vec{q}}{(2\pi)^2} e^{-i\vec{q}\cdot(\vec{x}-\vec{x}')}\langle n(\vec{q})\rangle_{\text{th}}, \quad (3)$$

where $\langle n(\vec{q})\rangle_{\text{th}}$ is the mean value of the photon number in mode $\vec{q}$ in the thermal state, and we assumed translational invariance of the source. In particular, the following factorization property holds [10]:

$$\langle : a^\dagger(\vec{x})a(\vec{x}')a^\dagger(\vec{x}'')a(\vec{x}''') :\rangle = \langle a^\dagger(\vec{x})a(\vec{x}')\rangle\langle a^\dagger(\vec{x}'')a(\vec{x}''')\rangle + \langle a^\dagger(\vec{x})a(\vec{x}''')\rangle\langle a^\dagger(\vec{x}'')a(\vec{x}')\rangle, \quad (4)$$

where :: indicates normal ordering. In both the PDC and the thermal case, the two outgoing beams travel through two distinct imaging systems, described by their impulse response functions h_1, h_2 (see Fig. 1). Arm 1 includes an object. Beam 1 is detected either by a pointlike detector D_1 or by a "bucket" detector [3], in any case giving no information on the object spatial distribution. On the other side, detector D_2 spatially resolves the light fluctuations, as, for example, an array of pixel detectors. The fields at the detection planes are given by

$$c_i(\vec{x}_i) = \int d\vec{x}_i' h_i(\vec{x}_i, \vec{x}_i')b_i(\vec{x}_i') + L_i(\vec{x}_i), \qquad i = 1, 2, \quad (5)$$

where L_1, L_2 account for possible losses in the imaging systems, and depend on vacuum field operators uncorrelated from b_1, b_2. Information about the object is extracted by measuring the spatial correlation function of the intensities detected by D_1 and D_2, as a function of the position $\vec{x}_2$ of the pixel of D_2:

$$\langle I_1(\vec{x}_1)I_2(\vec{x}_2)\rangle = \langle c_1^\dagger(\vec{x}_1)c_1(\vec{x}_1)c_2^\dagger(\vec{x}_2)c_2(\vec{x}_2)\rangle. \quad (6)$$

All the object information is concentrated in the correlation function of intensity fluctuations:

$$G(\vec{x}_1, \vec{x}_2) = \langle I_1(\vec{x}_1)I_2(\vec{x}_2)\rangle - \langle I_1(\vec{x}_1)\rangle\langle I_2(\vec{x}_2)\rangle, \quad (7)$$

where $\langle I_i(\vec{x}_i)\rangle = \langle c_i^\dagger(\vec{x}_i)c_i(\vec{x}_i)\rangle$ is the mean intensity of the ith beam. Since c_1 and $c_2^\dagger$ commute, all the terms in Eqs. (6) and (7) are normally ordered and L_1, L_2 can be neglected, thus obtaining

$$G(\vec{x}_1, \vec{x}_2) = \int d\vec{x}_1' \int d\vec{x}_1'' \int d\vec{x}_2' \int d\vec{x}_2'' h_1^*(\vec{x}_1, \vec{x}_1'')h_1(\vec{x}_1, \vec{x}_1')h_2^*(\vec{x}_2, \vec{x}_2'')h_2(\vec{x}_2, \vec{x}_2')[\langle b_1^\dagger(\vec{x}_1'')b_1(\vec{x}_1')b_2^\dagger(\vec{x}_2'')b_2(\vec{x}_2')\rangle - \langle b_1^\dagger(\vec{x}_1'')b_1(\vec{x}_1')\rangle\langle b_2^\dagger(\vec{x}_2'')b_2(\vec{x}_2')\rangle]. \quad (8)$$

In the thermal case, by taking into account the transformation (2) and that v is in the vacuum state, b_1 and b_2 in Eq. (8) can be simply replaced by ra and ta. Next, by using Eq. (4), we arrive at the final result

$$G(\vec{x}_1, \vec{x}_2) = |rt|^2 \left| \int d\vec{x}_1' \int d\vec{x}_2' h_1^*(\vec{x}_1, \vec{x}_1')h_2(\vec{x}_2, \vec{x}_2')\langle a^\dagger(\vec{x}_1')a(\vec{x}_2')\rangle \right|^2. \quad (9)$$

On the other hand, also in the PDC case the four-point correlation function in Eq. (8) has special factorization properties. As it can be obtained from Eq. (1) [8],

$$\langle b_1^\dagger(\vec{x}_1'')b_1(\vec{x}_1')b_2^\dagger(\vec{x}_2'')b_2(\vec{x}_2')\rangle = \langle b_1^\dagger(\vec{x}_1'')b_1(\vec{x}_1')\rangle\langle b_2^\dagger(\vec{x}_2'')b_2(\vec{x}_2')\rangle + \langle b_1^\dagger(\vec{x}_1'')b_2^\dagger(\vec{x}_2'')\rangle\langle b_1(\vec{x}_1')b_2(\vec{x}_2')\rangle. \quad (10)$$

By inserting this result into Eq. (8), one obtains

$$G(\vec{x}_1, \vec{x}_2) = \left| \int d\vec{x}_1' \int d\vec{x}_2' h_1(\vec{x}_1, \vec{x}_1')h_2(\vec{x}_2, \vec{x}_2')\langle b_1(\vec{x}_1')b_2(\vec{x}_2')\rangle \right|^2, \quad (11)$$

where, by using relations (1),

$$\langle b_1(\vec{x}_1')b_2(\vec{x}_2')\rangle = \int \frac{d\vec{q}}{(2\pi)^2} e^{i\vec{q}\cdot(\vec{x}_1'-\vec{x}_2')}U_1(\vec{q})V_2(-\vec{q}). \quad (12)$$

There is a clear analogy between the results in the two cases. Apart from the numerical factor $|rt|^2$ and the presence of h_1^* instead of h_1, the second order correlation $\langle a^\dagger(\vec{x})a(\vec{x}')\rangle$ and the function $\langle n(\vec{q})\rangle_{\text{th}}$ play the same role in Eq. (9) as the correlation functions $\langle b_1(\vec{x})b_2(\vec{x}')\rangle$ and $U_1(\vec{q})V_2(-\vec{q})$ in Eq. (11). Most importantly, in both Eqs. (9) and (11) the modulus is outside the integral, a feature that ensures the possibility of coherent imaging via correlation function (see, e.g., [3]). The correlation function $\langle a^\dagger(\vec{x})a(\vec{x}')\rangle$ governs the properties of spatial coherence of the thermal source [10]. The correlation length, or transverse coherence length l_{coh}, is determined by the inverse of the bandwidth Δq of the function $\langle n(\vec{q})\rangle_{\text{th}}$. The same holds for the correlation $\langle b_1(\vec{x})b_2(\vec{x}')\rangle$, and the function $U_1(\vec{q})V_2(-\vec{q})$ in the entangled case.

Let us now analyze two paradigmatic examples of imaging systems [4]. In both examples the setup of arm 1 is fixed and consists of an object, described by a complex transmission function $T(\vec{x})$, and a lens located at a focal distance f from the object and from the detection plane. Hence, $h_1(\vec{x}_1, \vec{x}_1') \propto \exp(-i\vec{x}_1 \cdot \vec{x}_1' k/f) T(\vec{x}')$, with k being the field wave number. In arm 2, a single lens (identical to the other one) is placed at a distance z from the source and from the detection plane.

In the first example, we assume $z = f$, so that $h_2(\vec{x}_2, \vec{x}_2') \propto \exp(-i\vec{x}_2 \cdot \vec{x}_2' k/f)$. By using Eqs. (9) and (3), we obtain, for the thermal correlation function,

$$G(\vec{x}_1, \vec{x}_2) \propto |\langle n(-\vec{x}_2 k/f)\rangle_{\rm th} \tilde{T}[(\vec{x}_2 - \vec{x}_1)k/f]|^2, \qquad (13)$$

where $\tilde{T}(\vec{q}) = \int \frac{d\vec{x}}{2\pi} e^{-i\vec{q}\cdot\vec{x}} T(\vec{x})$. This has to be compared with the result of the entangled case [see Eq. (7) of [4]], where the combination $\vec{x}_2 + \vec{x}_1$ appears instead of $\vec{x}_2 - \vec{x}_1$, and $U_1 V_2$ instead of $\langle n\rangle_{\rm th}$. The object diffraction pattern $|\tilde{T}(\vec{q})|^2$ can be reconstructed in both cases, provided $l_{\rm coh}$ is small compared to the characteristic length scale of the object l_o. Best performances of the scheme are achieved for spatially incoherent light, $l_{\rm coh} \to 0$. The condition $l_{\rm coh} < l_o$ implies that no information about the diffraction pattern can be retrieved by direct detection of the intensity distribution in arm 1.

In the second example, we set $z = 2f$, so that $h_2(\vec{x}_2, \vec{x}_2') = \delta(\vec{x}_2 + \vec{x}_2') \exp(-ik|\vec{x}_2|^2/2f)$. Inserting this into Eq. (9),

$$G(\vec{x}_1, \vec{x}_2) \propto \left| \int \frac{d\vec{q}}{2\pi} \langle n(\vec{q})\rangle_{\rm th} \tilde{T}(\vec{x}_1 k/f - \vec{q}) e^{i\vec{q}\cdot\vec{x}_2} \right|^2 \qquad (14)$$

$$\approx |\langle n(\vec{x}_1 k/f)\rangle_{\rm th}|^2 |T(-\vec{x}_2)|^2, \qquad (15)$$

where in the second line $l_{\rm coh} < l_o$ was assumed, so that $\langle n(\vec{q})\rangle_{\rm th}$ is roughly constant in the region of the $\vec{q}$ plane where the diffraction pattern does not vanish, and it can be taken out from the integral in (14). In this example the correlation function provides information about the image of the object. A similar result holds for the case of entangled beams [see Eq. (8) of [4]].

Our results appear surprising, if one has in mind the case of a coherent beam impinging on a beam splitter, where the two outgoing fields are uncorrelated, i.e., $G(\vec{x}_1, \vec{x}_2) = 0$. However, when the input field is an intense thermal beam, i.e., the photon number per mode is not too small, the two outgoing field are well correlated in space. To prove this point, let us consider the number of photons detected in two small identical portions R ("pixels") of the beams in the near field immediately after the beam splitter, $N_i = \int_R d\vec{x} b_i^\dagger(\vec{x}) b_i(\vec{x})$, $i = 1, 2$, and the difference $N_- = N_1 - N_2$. Making use of the transformation (2), it can be proven that, for $|r|^2 = |t|^2 = 1/2$, the variance $\langle \delta N_-^2\rangle = \langle N_-^2\rangle - \langle N_-\rangle^2$ is given by

$$\langle \delta N_-^2\rangle = \langle N_1\rangle + \langle N_2\rangle, \qquad (16)$$

which corresponds exactly to the shot noise level. On the other side, by using the identity $\langle \delta N_-^2\rangle = \langle \delta N_1^2\rangle + \langle \delta N_2^2\rangle - 2\langle \delta N_1 \delta N_2\rangle$ and taking into account that $\langle \delta N_1^2\rangle = \langle \delta N_2^2\rangle$ for $|r|^2 = |t|^2$, the normalized correlation is given by

$$C \stackrel{\rm def}{=} \frac{\langle \delta N_1 \delta N_2\rangle}{\sqrt{\langle \delta N_1^2\rangle}\sqrt{\langle \delta N_2^2\rangle}} = 1 - \frac{\langle N_1\rangle}{\langle \delta N_1^2\rangle}. \qquad (17)$$

For any state, $0 \leq |C| \leq 1$, where the lower bound corresponds to the coherent state level, and the upper bound is imposed by Cauchy-Schwarz inequality. For the thermal state, provided that the pixel size is on the order of $l_{\rm coh}$, $\langle \delta N_1^2\rangle \approx \langle N_1\rangle + \langle N_1\rangle^2$, so that the correlation (17) never vanishes. For thermal systems with a large number of photons, $\langle N_1\rangle/\langle \delta N_1^2\rangle \ll 1$, and C can be made close to its maximum value (see [11] for more details). Even more important, it is not difficult to show that Eqs. (16) and (17) hold in any plane linked to the near field plane by a Fresnel transformation, in the absence of losses [11]. In particular, a high level of pixel-by-pixel correlation can be observed in the far-field plane. Notice that, although C can be made close to 1 by increasing the mean number of photons, the correlation never reaches the quantum level, as shown by Eq. (16).

For the entangled PDC beams, spatial correlation is present both in the near-field and in the far-field, with the ideal result $\langle \delta N_-^2\rangle = 0$, $C = 1$ in both planes [8]. In [4] we analyzed the effect of replacing the pure PDC entangled state with two nonentangled mixtures that exactly preserve the spatial signal-idler quantum correlations, either in the far field or in the near field. It turned out that with each mixture either the $z = f$ result or the $z = 2f$ result could be exactly reproduced, but the whole set of the results could not. The two beams generated by splitting thermal light are instead imperfectly correlated both in the near field and in the far field. However, by using intense thermal light, the classical intensity correlation is strong enough to reproduce qualitatively the results of both the $z = f$ and the $z = 2f$ configuration.

When comparing the performances of the classical and quantum regimes, a key role is played by the issue of the visibility of the information. This is retrieved by subtracting the background term $\langle I_1(\vec{x}_1)\rangle\langle I_2(\vec{x}_2)\rangle$ from the measured correlation function (6) [see Eq. (7)]. A measure of the visibility is given by $\mathcal{V} = G(\vec{x}_1, \vec{x}_2)/\langle I_1(\vec{x}_1) I_2(\vec{x}_2)\rangle$. A first remark concerns the presence of $\langle n(\vec{q})\rangle_{\rm th}$ in Eq. (9) in place of $U_1(\vec{q})V_2(-q)$ in Eq. (11). As a consequence, in the thermal case $G(\vec{x}_1, \vec{x}_2)$ scales as $\langle n(\vec{q})\rangle_{\rm th}^2$, while in the entangled case, it scales as $|U_1(\vec{q})V_2(-q)|^2 = \langle n(\vec{q})\rangle + \langle n(\vec{q})\rangle^2$, where $\langle n(\vec{q})\rangle = |V_2(-\vec{q})|^2 = |V_1(\vec{q})|^2$ is the mean number of photons per mode in the PDC beams, and $|U_1(\vec{q})|^2 = 1 + |V_1(\vec{q})|^2$ (see, e.g., [8]). This difference is immaterial when $\langle n(\vec{q})\rangle \gg 1$, while it becomes relevant for a small photon number, because the background $\langle I_1(\vec{x}_1)\rangle\langle I_2(\vec{x}_2)\rangle \propto \langle n(\vec{q})\rangle^2$ is negligible with respect to $G(\vec{x}_1, \vec{x}_2) \propto \langle n(\vec{q})\rangle$. Hence, in the regime of single photon-pair detection, the entangled case presents a

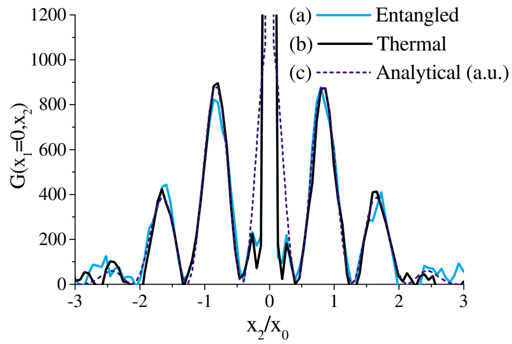

FIG. 2 (color online). Numerical simulation of the reconstruction of the diffraction pattern of a phase double slit. $G(\vec{x}_1 = 0, \vec{x}_2)$ after 80 000 shots is shown for (a) entangled signal or idler beams from PDC and (b) classically correlated beams by splitting the idler beam. (c) is $\tilde{T}(\vec{x}_2)$. Parameters are those of a 4 mm β-barium-borate crystal ($l_{\mathrm{coh}} = 16.6\ \mu$m, $\tau_{\mathrm{coh}} = 0.97$ ps). The pump waist and duration are 664 μm and 1.5 ps. $x_0 = \Delta q \lambda f / 2\pi$.

much better visibility of the information with respect to classically correlated thermal beams (see also [12]). A second remark concerns the role of the temporal argument. Standard calculations [10] show that the visibility scales as the ratio between the coherence time of the source τ_{coh} and the detection time (see also [7]). Hence a suitable source should present a relatively long coherence time, as the chaotic light produced by scattering a laser beam through a random medium [13]. As a special example of a thermal source, one can consider one of the two beams generated by PDC. Figure 2 is a numerical simulation of the reconstruction of the diffraction pattern of a phase double slit, $T(x) = -1$ inside the slits, $T(x) = +1$ elsewhere, in the scheme $z = f$ [14]. It compares the use of signal or idler entangled beams and two classically correlated beams obtained by symmetrically splitting the idler beam. The parametric gain is such that the mean photon number of beams b_1 and b_2 is the same in the two simulations. This shows that the diffraction pattern of a pure phase object can be reconstructed with incoherent thermal light and that in the regime of high photon number the quantum and classical correlation offer similar performances. More extended simulations not shown here [11] confirm that, by operating only on arm 2, both the diffraction pattern and the image of an object can be reconstructed via the classical correlation of thermal beams.

To conclude, the main result of this Letter is expressed by Eq. (9). When compared to Eq. (11), it shows that the results of correlation measurements performed on the thermal beams can emulate those of the PDC beams, provided that the thermal source coherence properties are properly engineered. As it was already recognized in other contexts (see, e.g., [15]), in the small photon number regime, a definite advantage of the quantum configuration is represented by a better visibility.

Our results imply that it is possible to perform coherent imaging without spatial coherence, which is reminiscent of the Hanbury-Brown and Twiss interferometric method for determining the stellar diameter [16]. However, here we define a technique to achieve a full coherent imaging, which, e.g., permits one to reconstruct the diffraction pattern of a pure phase object (Fig. 2).

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Note added.—After our Letter was submitted [17], the central result Eq. (9) found applications in the context of x-ray diffraction [18].

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