

## Low Reheating Temperature and the Visible Sterile Neutrino

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We present here a scenario, based on a low reheating temperature  $T_R \ll 100$  MeV at the end of (the last episode of) inflation, in which the coupling of sterile neutrinos to active neutrinos can be as large as experimental bounds permit (thus making this neutrino “visible” in future experiments). In previous models this coupling was forced to be very small to prevent a cosmological overabundance of sterile neutrinos. Here the abundance depends on how low the reheating temperature is. For example, the sterile neutrino required by the Liquid Scintillator Neutrino Detector result may not have any cosmological problem within our scenario.

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In inflationary models, the beginning of the radiation dominated era of the Universe results from the decay of coherent oscillations of a scalar field and the subsequent thermalization of the decay products into a thermal bath with the so-called “reheating temperature”  $T_R$ . This temperature may have been as low as 0.7 MeV [1] (a very recent analysis strengthens this bound to  $\sim 4$  MeV [2]). It is well known that a low reheating temperature inhibits the production of particles which are nonrelativistic or decoupled at  $T \lesssim T_R$  [3]. The final number density of active neutrinos starts departing from the standard number for  $T_R \lesssim 8$  MeV but stays within 10% of it for  $T_R \lesssim 5$  MeV. For  $T_R = 1$  MeV the number of tau- and muon-neutrinos is about 2.7% of the standard number. This would have allowed one of the active neutrinos to be a warm dark matter (WDM) candidate [4]. Experimental bounds force now all three mostly active neutrino masses to be in the range of hot dark matter (HDM), but we can use the same idea on sterile neutrinos.

Sterile neutrinos without extra-standard model interactions are produced in the early Universe through their mixing with active neutrinos [5]. Dodelson and Widrow [6] (see also Ref. [7]) provided the first analytical calculation of the production of sterile neutrinos in the early Universe, under the assumption (which we maintain here) of a negligible primordial lepton asymmetry. Figure 2 of Ref. [8] shows that mostly sterile neutrinos produced in this manner have an acceptable abundance only if their mixing with active neutrinos is very small, for example,  $\sin^2 2\theta < 10^{-7}$  for masses  $m_s \gtrsim 1$  keV. In the presence of a large lepton asymmetry, sterile neutrinos are produced resonantly [8,9] with a nonthermal spectrum which favors low energies (“cool” dark matter candidate).

The main idea of this Letter is that the primordial abundance of sterile neutrinos does not necessarily impose their mixing to active neutrinos to be as small as usually believed. We can thus consider sterile neutrinos of any mass and coupling, as long as other experimental and cosmological bounds are satisfied. These neutrinos could

therefore be revealed in future experiments. Only for simplicity we do not deal here with neutrinos heavier than 1 MeV or with a large chemical potential, or with reheating temperatures lower than 5 MeV, but our idea clearly applies to all of these cases too [10].

In Ref. [6] it is shown that most of the sterile neutrinos in the early Universe are produced at a temperature  $T_{\max} \simeq 133(m_s/\text{keV})^{1/3}$  MeV. Thus, if  $T_R < T_{\max}$ , the production of sterile neutrinos is suppressed. We follow the calculations of Ref. [6] but consider that the production of sterile neutrinos, through the conversion of active neutrinos produced in collisions, starts when the temperature of the Universe is  $T_R < T_{\max}$ .

In the calculation, the active neutrinos are assumed to have the usual thermal equilibrium distribution  $f_A = [\exp(E/T) + 1]^{-1}$ ; thus following Ref. [4], we restrict ourselves to reheating temperatures  $T_R \gtrsim 5$  MeV. We also restrict ourselves to the case of sterile neutrinos with mass  $m_s < 1$  MeV, so that we do not need to consider their decays into electron pairs. These sterile neutrinos are therefore relativistic at production.

In the approximation of two-neutrino mixing,  $\sin\theta$  is the amplitude of the heavy mass eigenstate in the composition of the active neutrino flavor eigenstate  $\nu_\alpha$ ,  $\alpha = e, \mu, \tau$ .

For some range of masses which depend on  $T_R$ , one can neglect all matter effects, so the oscillations are as in vacuum. For example, for  $T_R = 5$  MeV, the specific value of the reheating temperature we use in this Letter, this happens for  $m_s \gtrsim 0.2$  eV (0.1 eV) for  $\nu_e \leftrightarrow \nu_s$  ( $\nu_{\mu,\tau} \leftrightarrow \nu_s$ ). In this case, the  $\nu_s$  distribution function turns out to be

$$f_s(E, T) \simeq 3.2d_\alpha \left( \frac{T_R}{5 \text{ MeV}} \right)^3 \sin^2 2\theta \left( \frac{E}{T} \right) f_\alpha(E, T), \quad (1)$$

where  $d_\alpha = 1.13$  for  $\nu_\alpha = \nu_e$ , and  $d_\alpha = 0.79$  for  $\nu_\alpha = \nu_{\mu,\tau}$  [11]. This distribution results in a number fraction of sterile over active neutrinos plus antineutrinos

$$f \equiv \frac{n_{\nu_s}}{n_{\nu_\alpha}} \approx 10 d_\alpha \sin^2 2\theta \left( \frac{T_R}{5 \text{ MeV}} \right)^3. \quad (2)$$

Notice that the number density of sterile neutrinos depends only on the active-sterile mixing angle and the reheating temperature. A low reheating temperature ensures a small sterile number density, even for very large active-sterile mixing angles, as large as other experimental bounds permit. This makes sterile neutrinos in our scenario potentially detectable in future experiments. The  $\nu_s$ -number density is independent of the mass of the sterile neutrinos, contrary to the result of Ref. [6]. Thus the mass density of nonrelativistic sterile neutrinos,  $\Omega_s h^2 = (m_s n_{\nu_s} / \rho_c) h^2$ , depends linearly on the mass and on  $\sin^2 2\theta$ ,

$$\Omega_s h^2 \approx 0.1 d_\alpha \left( \frac{\sin^2 2\theta}{10^{-3}} \right) \left( \frac{m_s}{1 \text{ keV}} \right) \left( \frac{T_R}{5 \text{ MeV}} \right)^3. \quad (3)$$

The condition  $\Omega_s h^2 \lesssim \Omega_{\text{DM}} h^2 = 0.135$  [12] rejects the triangular, gray region of masses and mixings shown in Figs. 1 and 2. The values of masses and mixings for which sterile neutrinos constitute 10% of the dark matter are also shown with a dotted line.

Figures 1 and 2 show bounds for  $\nu_\alpha = \nu_e$  and  $\nu_\alpha = \nu_{\mu,\tau}$ , respectively, and for  $T_R = 5$  MeV. They show that  $\nu_s$  in our scenario are viable HDM candidates, while neutrinos with  $m_s > 1$  keV are disfavored, if not rejected, as WDM or cold dark mass (CDM), by bounds coming from supernovae cooling and astrophysical bounds due to radiative decays, explained below.

Through  $\nu_\alpha \leftrightarrow \nu_s$  oscillations, sterile neutrinos can be produced in supernova (SN) cores and escape, carrying away a large amount of the released energy. The observations of  $\nu_e, \bar{\nu}_e$  from SN1987A constrain the energy loss in  $\nu_s$  and yield a bound on the mixing angle. For

$m_s \lesssim 45$  keV,  $\nu_\alpha \leftrightarrow \nu_s$  oscillations are matter suppressed and the forbidden range is [13]

$$0.22 \text{ keV} \lesssim |m_s (\sin^2 2\theta)^{1/4}| \lesssim 17 \text{ keV}. \quad (4)$$

For  $m_s \gtrsim 45$  keV, the matter effects are negligible and neutrinos oscillate as in vacuum. In this case, the forbidden range is [13]

$$7 \times 10^{-10} \lesssim \sin^2 2\theta \lesssim 2 \times 10^{-2}. \quad (5)$$

These bounds exclude the diagonally hatched region with thin lines in Figs. 1 and 2. The effective matter potential in the SN core for  $\nu_e \rightarrow \nu_s$  conversions,  $V_m$ , might be driven to its zero equilibrium fixed point,  $V_m \approx 0$ , during the explosion [8,14]. In this case, the  $\nu_\alpha \rightarrow \nu_s$  conversion happens as in vacuum. However, using the SN parameters of Ref. [13], this does not change the bound in Eqs. (4) and (5). The SN1987A bound on neutrino radiative decays [15] excludes the region above the line labeled SMM (Solar Maximum Mission satellite) in both figures.

If the sterile neutrinos produced in nonresonant  $\nu_e \rightarrow \nu_s$  conversion in fact carry away a sizable fraction of the energy emitted in a SN explosion, asymmetric emission of  $\nu_s$  due to the presence of a strong magnetic field could explain the very large velocities of pulsars [14] (see diagonally hatched region with thick lines in Fig. 1).

Having restricted ourselves to  $m_s < 1$  MeV, the dominant decay mode of the mostly sterile  $\nu$  is into three neutrinos. Assuming neutrinos are Majorana particles, a lifetime  $\tau$  equal to the lifetime of the Universe  $t_U = 4.32 \times 10^{17}$  s is indicated in the figures by the solid thick line (for Dirac neutrinos, the lifetime is larger by a factor of  $\sim 2$ ). Equivalent lines corresponding to shorter or longer lifetimes can be easily obtained knowing that the lifetime is proportional to  $(\sin^2 2\theta m_s^5)^{-1}$ . The decay mode into a neutrino and a photon happens with a branch-

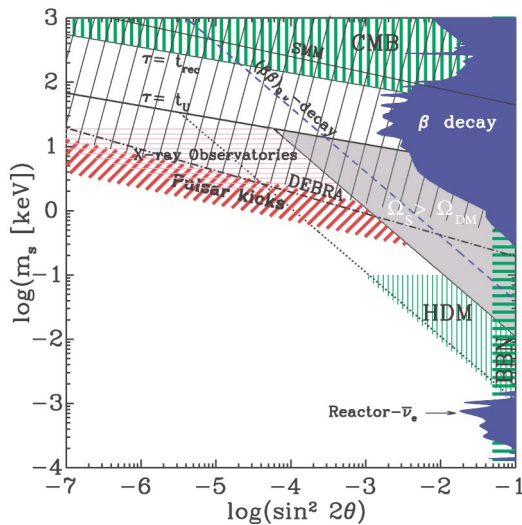


FIG. 1 (color online). Bounds and sensitivity regions for  $\nu_e \leftrightarrow \nu_s$  oscillations. See text.

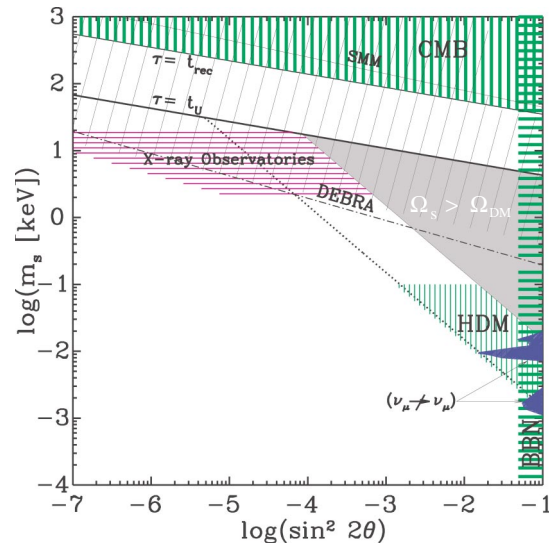


FIG. 2 (color online). Same as Fig. 1 for  $\nu_{\mu,\tau} \leftrightarrow \nu_s$ . For  $\nu_\tau \leftrightarrow \nu_s$ , the darkest gray excluded region does not apply. See text.

ing ratio  $0.8 \times 10^{-2}$  [this decay is not Glashow-Iliopoulos-Maian (GIM) suppressed, contrary to the decay of an active neutrino into another active neutrino and a photon [16]]. The diffuse extragalactic background radiation (DEBRA) imposes a bound [17] that, in the relevant range of masses, can be well approximated by

$$I_\gamma \lesssim (E/0.05 \text{ MeV})^{-1} (\text{cm}^2 \text{ sr s})^{-1}, \quad (6)$$

where  $I_\gamma$  is the differential photon flux.

For decays with  $\tau > t_{\text{rec}}$  ( $t_{\text{rec}}$  is the time of recombination in the early Universe; the line  $\tau = t_{\text{rec}}$  is also shown in the figures) the bound obtained for unclustered neutrinos would reject the region above the dot-dashed line labeled DEBRA in the figures. In particular, for  $\tau > t_U$ , for unclustered sterile neutrinos, the bound would be

$$\left(\frac{m_s}{1 \text{ keV}}\right) \lesssim 0.10 d_\alpha^{-1/6} \left(\frac{5 \text{ MeV}}{T_R}\right)^{1/2} \frac{1}{(\sin^2 2\theta)^{1/3}}. \quad (7)$$

However, this bound affects neutrinos with mass  $m_s \gtrsim 100 \text{ eV}$  which are gravitationally clustered. Therefore the bound is not entirely correct. But the actual estimate of how much of the diffuse photon background would be due to neutrinos which decay after structures in the universe form is missing in the literature.

Abazajian *et al.* [18] proposed to observe clusters of galaxies with the Chandra and XMM-Newton observatories, in their high sensitivity range for x-ray photon detection of 1–10 keV. They proposed to reach a detection energy flux of  $10^{-13} \text{ erg}/(\text{cm}^2 \text{ s})$  with the Chandra observatory, which would allow one to observe a monochromatic signal from the Virgo cluster, if

$$\left(\frac{m_s}{2 \text{ keV}}\right) \gtrsim 2.1 d_\alpha^{-1/6} \left(\frac{5 \text{ MeV}}{T_R}\right)^{1/2} \left(\frac{10^{-6}}{\sin^2 2\theta}\right)^{1/3}, \quad (8)$$

for  $m_s = 2\text{--}20 \text{ keV}$  (horizontally hatched region with thin lines in the figures). Here the density fraction of sterile neutrinos within clusters is assumed to coincide with the cosmological energy fraction ( $\Omega_s/\Omega_{\text{DM}}$ ).

The lack of distortions in the cosmic microwave background (CMB) spectrum due to neutrino radiative decays excludes all the vertically hatched region with thick lines in the figures [19].

Structure formation arguments impose sterile neutrinos if they constitute the whole of the dark matter, as WDM, to have  $m_s > 2.9 \text{ keV}$  [20]. Note we use 2.9 keV instead of the 2.6 keV in Ref. [20] because of our choice of cosmological parameters. Besides, our sterile neutrinos are hotter than those of Ref. [20], so the lower bound in our scenario should be even somewhat larger than 2.9 keV.

In the mass range in which sterile neutrinos can be part of the HDM, we can apply the bounds on the sum of the contributions of active and sterile neutrinos to the HDM density. The  $3\sigma$  bound on the sum of the neutrino masses (see Fig. 2 of Ref. [21]) is

$$\sum m_i + f m_s \leq 1.1 \text{ eV}, \quad i = 1, 2, 3, \quad (9)$$

where  $m_i$  are the light neutrino masses. Combining Eq. (9) with an estimate of the light neutrino masses, we obtain an upper limit on  $m_s$ . If the neutrino mass spectrum is normal hierarchical, oscillation data impose the sum of the active neutrino masses to be about 0.05 eV. This provides the most conservative bound on  $m_s$ :

$$m_s \sin^2 2\theta \lesssim 0.1 d_\alpha^{-1} (T_R/5 \text{ MeV})^{-3} \text{ eV}, \quad (10)$$

which we plot for  $m_s \lesssim 100 \text{ eV}$  (vertically hatched region with thin lines in Figs. 1 and 2).

The  $3\sigma$  upper bound imposed by big bang nucleosynthesis (BBN) on any extra contribution to the energy density, parametrized as extra neutrino species,  $\Delta N_\nu$ , is  $\Delta N_\nu \leq 0.73$  (see Fig. 7 of Ref. [22]), which translates into (horizontally hatched region with thick lines in Figs. 1 and 2)

$$\sin^2 2\theta \lesssim 5.6 d_\alpha^{-1} 10^{-2} (T_R/5 \text{ MeV})^{-3}. \quad (11)$$

Let us turn now to experimental bounds. So far, disappearance experiments have reported negative results [23–25]. Reactor- $\bar{\nu}_e$  experiments CHOOZ [23] and Bugey [24] constrain the mixing angle relevant in  $\bar{\nu}_e \rightarrow \bar{\nu}_s$  conversion [23] to be  $\sin^2 2\theta < 0.1$  for  $m_s > 1.7 \text{ eV}$ . For smaller masses (down to  $m_s \sim \text{few } 10^{-1} \text{ eV}$ ) the bound is stronger by a factor of 2–5 [24] (darkest region in Fig. 1). Accelerator- $\nu_\mu$  disappearance experiments [25] impose the mixing angle which controls  $\nu_\mu \rightarrow \nu_s$  oscillations to be  $\sin^2 2\theta < 0.02$  for  $13.8 \text{ eV} < m_s < 17.9 \text{ eV}$ . Less stringent bounds apply for other values of  $m_s$  (darkest region in Fig. 2). Future experiments looking for  $\bar{\nu}_e$  [26] and  $\nu_\mu$ ,  $\bar{\nu}_\mu$  [27,28] disappearance might explore part of the now allowed parameter space; e.g.,  $\nu$  factories may reach  $\sin^2 2\theta \sim 10^{-3}$ .

Appearance experiments searching for  $\nu_\alpha \rightarrow \nu_\beta$  oscillations ( $\alpha, \beta = e, \mu, \tau$ ) are sensitive to the product of the mixing angles between  $\nu_\alpha, \nu_\beta$  and the mostly sterile mass eigenstate. These experiments have reported no positive signal, except for the Liquid Scintillator Neutrino Detector (LSND) experiment [29], which found evidence of  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  conversion. MiniBooNE [27] will test this result. The analysis of the data requires the mixing of at least four neutrinos. However, in the so-called 3 + 1 scheme, the usual parameter  $\theta_{\text{LSND}} \simeq \theta_{es} \times \theta_{\mu s}$ , and the less restrictive bound on both two-neutrino mixing angles is obtained when  $\theta_{es} = \theta_{\mu s} = \theta$ . Since  $\sin^2 2\theta_{\text{LSND}}$  needs to be about 0.001, then  $\sin^2 2\theta \simeq 0.06$  for  $m_s = O(1) \text{ eV}$ , a value that in our model is cosmologically and astrophysically allowed for  $T_R \simeq 5$  to 4 MeV.

In the mass range of interest,  $\beta$ -decay experiments searching for kinks in the energy spectra of the emitted electron constrain the mixing angle between  $\nu_e$  and the mostly sterile neutrino mass eigenstate. Different nuclei

have been used and negative results have been found so far (for a complete review, see Ref. [30]). The limits are strongly mass dependent (darkest region in Fig. 1).

If neutrinos are Majorana particles, the neutrinoless double beta  $[(\beta\beta)_{0\nu}]$  decay is allowed. The half-life time depends on the effective Majorana mass  $\langle m \rangle$  (see, e.g., Ref. [31]). The contribution of the mostly sterile neutrino is of the form  $\langle m \rangle_s = m_s \sin^2 \theta e^{i\beta_s}$ , where  $\sin^2 2\theta$  is the mixing parameter in  $\nu_e \rightarrow \nu_s$  conversions and  $\beta_s$  is a Majorana  $CP$ -violating phase. At present, the most stringent bound on  $|\langle m \rangle|$  is  $|\langle m \rangle| < 0.35\text{--}1.05$  eV [32], which conservatively translates into (dashed line in Fig. 1):

$$m_s \sin^2 2\theta < 4 \text{ eV}. \quad (12)$$

Possible cancellations in  $|\langle m \rangle|$  among contributions due to different mass eigenstates would weaken this bound. Barring this possibility, the present and future  $(\beta\beta)_{0\nu}$ -decay experiments (see, e.g., Ref. [33]) will probe part of the cosmologically relevant region, possibly up to  $m_s \sin^2 2\theta \sim 0.05$  eV, in which  $\nu_s$  could be an important part of the dark matter or produce pulsar kicks.

The sterile neutrinos with  $\sin^2 2\theta \approx 0.1\text{--}0.01$  would have the cross section required in Ref. [34] to separate atmospheric showers produced by these neutrinos from showers generated by active neutrinos, in future experiments such as EUSO and OWL [35] by using the Earth as a filter. The required flux of ultrahigh energy neutrinos would be very large.

We presented here a scenario, based on a low reheating temperature at the end of inflation, in which the coupling of sterile neutrinos to active neutrinos can be as large as experimental bounds permit. The experimental discovery of a sterile neutrino in the region of  $m_s - \sin^2 2\theta$  opened up in this Letter would require an unusual cosmology, such as one with a low reheating temperature as presented here. In this case, baryon asymmetry might be generated through the Affleck-Dine mechanism [36], and the bulk of the dark matter (if not made of sterile neutrinos) should consist of other nonthermally produced particles.

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