Critical Properties of the N-Color London Model

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The critical properties of the N-color London model are studied in $d = 2 + 1$ dimensions. The model is dualized to a theory of N vortex fields interacting through a Coulomb and a screened potential. The model with $N = 2$ shows two anomalies in the specific heat. From the critical exponents α and ν, the mass of the gauge field, and the vortex correlation functions, we conclude that one anomaly corresponds to an *inverted* 3Dxy fixed point, while the other corresponds to a 3Dxy fixed point. There are N fixed points, namely, one corresponding to an inverted 3Dxy fixed point, and $N - 1$ corresponding to neutral 3Dxy fixed points. This represents a novel type of quantum fluid, where superfluid modes arise out of charged condensates.

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Ginzburg-Landau (GL) theories with several complex scalar matter fields minimally coupled to one gauge field are of interest in a wide variety of systems, such as multiple component (color) superconductors, metallic phases of light atoms such as hydrogen [1,2], and as effective theories for easy plane quantum antiferromagnets [3–5]. The model also is highly relevant in particle physics where it is called two-Higgs doublet model [6]. In metallic hydrogen the scalar fields represent Cooper pairs of electrons and protons, which excludes the possibility of intercolor pair tunneling, i.e., there is no Josephson coupling between different components of the condensate. The same two-color action in (2 + 1) dimensions, where the matter fields originate in a bosonic representation of spin operators, is claimed to be the critical sector of a field theory separating a Néel state and a paramagnetic (valence bond ordered) state of a two-dimensional quantum antiferromagnet at zero temperature with an easy plane anisotropy present [3,5]. This happens because, although the effective description of the antiferromagnet involves an *a priori* compact gauge field, it must be supplemented by Berry phase terms in order to properly describe $S = 1/2$ spin systems [7,8]. Berry phases cancel the effects of monopoles at the critical point [3,5]. In this Letter, we point out novel physics of the quantum fluid that arises out of an N-color charged condensate when no intercolor Josephson coupling is present.

For a detailed analysis of the phase transitions in such a generalized GL model, we study an N component GL theory in (2 + 1) dimensions with no Josephson coupling term. The model is defined by N complex scalar fields $\{\Psi^{(\alpha)}(\mathbf{r})|\alpha = 1\dots N\}$ coupled through the charge e to a fluctuating gauge field $\mathbf{A}(\mathbf{r})$, with Hamiltonian

$$H = \sum_{\alpha=1}^{N} \frac{|(\nabla - \mathrm{i}e\mathrm{A})\Psi^{(\alpha)}|^2}{2M^{(\alpha)}} + V(\{\Psi^{(\alpha)}\}) + \frac{1}{2}(\nabla \times \mathbf{A})^2, \tag{1}$$

where $M^{(\alpha)}$ is the α-component condensate mass. The potential $V(\{\Psi^{(\alpha)}(\mathbf{r})\})$ is assumed to be only a function of $|\Psi^{(\alpha)}(\mathbf{r})|^2$. The model is studied in the phase only (London) approximation $\Psi^{(\alpha)}(\mathbf{r}) = |\Psi_0^{(\alpha)}| \exp[i\theta^{(\alpha)}(\mathbf{r})]$ and is discretized on a lattice with spacing $a = 1$ [9]. In the Villain approximation the partition function reads

$$Z = \int_{-\infty}^{\infty} \mathcal{D}\mathbf{A} \prod_{\gamma=1}^{N} \int_{-\pi}^{\pi} \mathcal{D}\theta^{(\gamma)} \prod_{\eta=1}^{N} \sum_{\mathbf{n}^{(\eta)}} \exp(-S),$$

$$S = \sum_{\mathbf{r}} \Big[\sum_{\alpha=1}^{N} \frac{\beta|\Psi_0^{(\alpha)}|^2}{2M^{(\alpha)}} (\Delta\theta^{(\alpha)} - e\mathbf{A} + 2\pi\mathbf{n}^{(\alpha)})^2 + \frac{\beta}{2}(\Delta \times \mathbf{A})^2 \Big], \tag{2}$$

where $\mathbf{n}^{(\alpha)}(\mathbf{r})$ are integer vector fields ensuring 2π periodicity, and the lattice position index vector $\mathbf{r}$ of the fields is suppressed. The symbol Δ denotes the lattice difference operator and $\beta = 1/T$ is the inverse temperature. *Here, we stress the importance of keeping track of the 2π periodicity of the individual phases.* The kinetic energy terms are linearized by introducing N auxiliary fields $\mathbf{v}^{(\alpha)}$. Integration over all $\theta^{(\alpha)}$ produces the local constraints $\Delta \cdot \mathbf{v}^{(\alpha)} = 0$, which are fulfilled by the replacement $\mathbf{v}^{(\alpha)} \to \Delta \times \mathbf{h}^{(\alpha)}$. We recognize $\mathbf{h}^{(\alpha)}$ as the dual gauge fields of the theory. By fixing the gauge $n_z^{(\alpha)} = 0$ and performing a partial integration we may introduce the vortex fields $\mathbf{m}^{(\alpha)} = \Delta \times \mathbf{n}^{(\alpha)}$. We integrate out the gauge field $\mathbf{A}$ and get a theory in the dual gauge fields $\mathbf{h}^{(\alpha)}$ and the vortex fields $\mathbf{m}^{(\alpha)}$ where $\Delta \cdot \mathbf{m}^{(\alpha)} = 0$

$$S = \sum_{\mathbf{r}} \Big[2\pi \mathrm{i} \sum_{\alpha=1}^{N} \mathbf{m}^{(\alpha)} \cdot \mathbf{h}^{(\alpha)} + \sum_{\alpha=1}^{N} \frac{(\Delta \times \mathbf{h}^{(\alpha)})^2}{2\beta|\psi^{(\alpha)}|^2} + \frac{e^2}{2\beta}\Big(\sum_{\alpha=1}^{N} \mathbf{h}^{(\alpha)}\Big)^2 \Big], \tag{3}$$

where $|\psi^{(\alpha)}|^2 = |\Psi_0^{(\alpha)}|^2/M^{(\alpha)}$. Note how the *algebraic sum* of the dual photon fields is massive. This differs from the case $N = 1$, where e produces one massive dual photon with bare mass $e^2/2$, and the model describes a vortex field $\mathbf{m}$ interacting through a *massive* dual gauge field $\mathbf{h}$. However, when $N \geq 2$, since $\Delta \cdot \mathbf{m}^{(\alpha)} = 0$, a gauge transformation $\mathbf{h}^{(\alpha)} \rightarrow \mathbf{h}^{(\alpha)} + \Delta g^{(\alpha)}$ for $\alpha = 1 \ldots N$ leaves the action invariant if one of the gauge fields, say $\mathbf{h}^{(\eta)}$ compensates the sum in the last term in (3) with $\Delta g^{(\eta)} = -\sum_{\alpha \neq \eta} \Delta g^{(\alpha)}$.

Integrating out the dual gauge fields we get a generalized theory of N interacting vortex fields

$$Z = \sum_{\mathbf{m}^{(1)}} \cdots \sum_{\mathbf{m}^{(N)}} \delta_{\Delta \cdot \mathbf{m}^{(1)},0} \cdots \delta_{\Delta \cdot \mathbf{m}^{(N)},0} \times e^{-S_V},$$
$$S_V = \sum_{\mathbf{r},\mathbf{r}'} \sum_{\alpha,\eta} \mathbf{m}^{(\alpha)}(\mathbf{r}) D^{(\alpha,\eta)}(\mathbf{r} - \mathbf{r}') \mathbf{m}^{(\eta)}(\mathbf{r}'), \tag{4}$$

where $\delta_{x,y}$ is the Kronecker delta, and the vortex interaction potential $D^{(\alpha,\eta)}(\mathbf{r})$ is the inverse discrete Fourier transform of $\tilde{D}^{(\alpha,\eta)}(\mathbf{q})$, where

$$\frac{\tilde{D}^{(\alpha,\eta)}(\mathbf{q})}{2\pi^2 \beta |\psi^{(\alpha)}|^2} = \frac{\lambda^{(\eta)}}{|\mathbf{Q_q}|^2 + m_0^2} + \frac{\delta_{\alpha,\eta} - \lambda^{(\eta)}}{|\mathbf{Q_q}|^2}, \tag{5}$$

$\lambda^{(\alpha)} = |\psi^{(\alpha)}|^2/\psi^2$ and $\psi^2 = \sum_{\alpha=1}^{N} |\psi^{(\alpha)}|^2$. Here, $m_0^2 = e^2\psi^2$ is the square of the bare inverse screening length in the intervortex interaction, and $|\mathbf{Q_q}|^2$ is the Fourier representation of the lattice Laplace operator. The first term of the vortex interaction potential (5) is a Yukawa screened potential, *while the second term mediates long range Coulomb interaction between vortex fields.* If $N = 1$ the latter cancels out exactly and we are left with the well studied vortex theory of the GL model, which has a charged fixed point for $e \neq 0$ [10,11]. For $N \geq 2$ we find a theory of vortex loops of N colors interacting through long range Coulomb interaction. If N goes to infinity, then $\psi^2 \rightarrow \infty$ and, therefore, the vortex fields interact via a diagonal $N \times N$ Coulomb matrix. This reflects the inability of one single gauge field $\mathbf{A}$ to screen a large number of vortex species. *The case $N \geq 2$ has features with no counterpart in the case $N = 1$* [9,11], namely, neutral superfluid modes arising out of charged condensates.

The above vortex system may be formulated as a field theory, introducing N complex matter fields $\phi^{(\alpha)}$ for each vortex species, minimally coupled to the dual gauge fields $\mathbf{h}^{(\alpha)}$. This generalizes the dual theory for $N = 1$ pioneered in [12]. The theory reads (see also [5])

$$S_{\text{dual}} = \sum_{\mathbf{r}} \Bigg\{ \sum_{\alpha=1}^{N} \Bigg[m_\alpha^2 |\phi^{(\alpha)}|^2 + |(\Delta - i\mathbf{h}^{(\alpha)})\phi^{(\alpha)}|^2 + \frac{(\Delta \times \mathbf{h}^{(\alpha)})^2}{2\beta |\psi^{(\alpha)}|^2} \Bigg] + \frac{e^2}{2\beta} \Bigg(\sum_{\alpha=1}^{N} \mathbf{h}^{(\alpha)} \Bigg)^2 + \sum_{\alpha,\eta} g^{(\alpha,\eta)} |\phi^{(\alpha)}|^2 |\phi^{(\eta)}|^2 \Bigg\}. \tag{6}$$

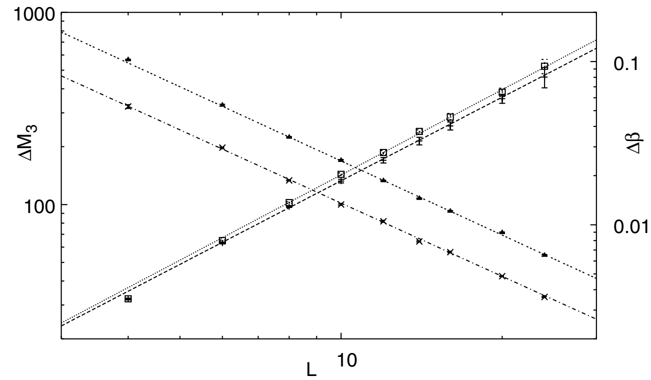

FIG. 1. The FSS of the peak-to-peak value of the third moment ΔM_3 labeled (□) and (+) for T_{c1} and T_{c2} respectively. The scaling of the width between the peaks $\Delta\beta$ labeled (▲) and (×) for T_{c1} and T_{c2}, respectively. The lines are power law fits to the data for $L > 6$ used to extract α and ν.

Here we have added chemical potential (core energy) terms for the vortices as well as steric short-range repulsion interactions between vortex elements. In the $N = 1$ case, a renormalization-group method treatment of the mass term of the dual gauge field yields $\partial e^2/\partial \ln l = e^2$, and hence this term scales up, suppressing the dual gauge field. Correspondingly, for $N \geq 2$, this suppresses $\sum_\alpha \mathbf{h}^{(\alpha)}$, but not each individual dual gauge field. For the particular case $N = 2$, assuming the same to hold, we end up with a gauge theory of two complex matter fields coupled minimally to one massless gauge field, which was also precisely the starting point. Thus the theory is self-dual for $N = 2$ [4,5]. For $N = 1$, it is known that a charged theory in $d = 2 + 1$ dualizes into a $|\phi|^4$ theory and vice versa [11]. The vortex tangle of the 3Dxy model is incompressible and the dual theory is a massless gauge theory such that $\langle \phi \rangle \neq 0$ is prohibited. For $e \neq 0$, the dual theory has global symmetry, and vortex condensation and $\langle \phi \rangle \neq 0$ is possible [11].

For $N = 2$, Monte Carlo (MC) simulations have been carried out for the action (4) with parameters $|\psi^{(1)}|^2 = 1/2$, $|\psi^{(2)}|^2 = 1$, $e^2 = 1/4$, and $m_0^2 = 3/8$. Here, $|\psi^{(1)}|^2$ and $|\psi^{(2)}|^2$ have been chosen to have well separated bare energy scales associated with the twist of the two types of phases, and m_0 has been chosen to be of the order of the inverse lattice spacing in the problem to avoid difficult finite-size effects. One MC update consists of inserting elementary vortex loops of random direction and species according to the Metropolis algorithm.

We observe two anomalies in the specific heat at T_{c1} and T_{c2} where $T_{c1} < T_{c2}$. We find T_{c1} and T_{c2} from scaling of the second moment of the action $\langle (S_V - \langle S_V \rangle)^2 \rangle$ to be $T_{c1} = 1.4(6)$ and $T_{c2} = 2.7(8)$. To check the criticality of these anomalies we have calculated the critical exponents α and ν by applying finite-size scaling (FSS) of $M_3 = \langle (S_V - \langle S_V \rangle)^3 \rangle$ [13]. The peak-to-peak value of this quantity scales with system size L as $L^{(1+\alpha)/\nu}$, the width between the peaks scales as $L^{-1/\nu}$. The advantage of this is that asymptotically correct behavior is reached for practical system sizes. The FSS plots for system sizes $L = 4, 6, 8, 10, 12, 14, 16, 20$, and 24 are shown in Fig. 1.

From the scaling we conclude that both anomalies are in fact critical points, and we obtain $\alpha = -0.02 \pm 0.02$ and $\nu = 0.67 \pm 0.01$ for T_{c1} and $\alpha = -0.03 \pm 0.02$ and $\nu = 0.67 \pm 0.01$ for T_{c2}. These values are consistent with those of the 3Dxy and the *inverted* 3Dxy universality classes found with high precision to be $\alpha = -0.0146(8)$ and $\nu = 0.67155(3)$ [14].

To characterize these phase transitions further, we consider $\mathcal{G}_{\mathbf{A}}(q) = \langle \mathbf{A}_q \cdot \mathbf{A}_{-q} \rangle$ and $\mathcal{G}_{\Sigma\mathbf{h}}(q) = \langle (\sum_\alpha \mathbf{h}_q^{(\alpha)}) \cdot (\sum_\alpha \mathbf{h}_{-q}^{(\alpha)}) \rangle$, expressed in terms of $G^{(+)}(q) = \langle |\sum_\alpha |\psi^{(\alpha)}|^2 \mathbf{m}_q^{(\alpha)}|^2 \rangle$ as

$$\begin{aligned} \mathcal{G}_{\mathbf{A}}(q) &= \frac{2/\beta}{|\mathbf{Q_q}|^2 + m_0^2}\left(1 + \frac{2\pi^2\beta m_0^2}{|\mathbf{Q_q}|^2}\frac{G^{(+)}(q)}{|\mathbf{Q_q}|^2 + m_0^2}\right), \\ \mathcal{G}_{\Sigma\mathbf{h}}(q) &= \frac{2\beta\psi^2}{|\mathbf{Q_q}|^2 + m_0^2}\left(1 - \frac{2\pi^2\beta}{\psi^2}\frac{G^{(+)}(q)}{|\mathbf{Q_q}|^2 + m_0^2}\right). \end{aligned} \tag{7}$$

The masses of $\mathbf{A}$ and $\sum_\alpha \mathbf{h}^{(\alpha)}$ are defined by $m_{\mathbf{A}}^2 = \lim_{q\to 0} 2\mathcal{G}_{\mathbf{A}}(q)^{-1}/\beta$ and $m_{\Sigma\mathbf{h}}^2 = \lim_{q\to 0} 2\beta\psi^2\mathcal{G}_{\Sigma\mathbf{h}}(q)^{-1}$.

We briefly review the case $N = 1$ [11]. The dual field theory of the neutral fixed point ($m_0^2 = 0$) is a charged theory describing an incompressible vortex tangle. The leading behavior of the vortex correlator is $\lim_{q\to 0} 2\pi^2\beta G^{(+)}(q) \sim [1 - C_2(T)]q^2$, $q^2 - C_3(T)q^{2+\eta_{\mathbf{h}}}$, and $q^2 + C_4(T)q^4$ for $T < T_c$, $T = T_c$, and $T > T_c$, respectively. For $T < T_c$ we have $m_{\Sigma\mathbf{h}}^2 = 0$ ($N = 1$), however for $T > T_c$ the $1/q^2$ terms in $\mathcal{G}_{\Sigma\mathbf{h}}(q)$ cancel out exactly and this mass attains an expectation value. At the charged fixed point ($m_0^2 \neq 0$) of the GL model, the effective field theory of the vortices is a neutral theory. The vortex tangle is compressible with a scaling ansatz for the vortex correlator $\lim_{q\to 0} G^{(+)}(q) \sim q^2$, $q^{2-\eta_{\mathbf{A}}}$, and $c(T)$ for $T < T_c$, $T = T_c$, and $T \geq T_c$, respectively. Consequently, from (7), the mass $m_{\mathbf{A}}$ drops to zero at T_c, and the mass of the dual gauge field $m_{\mathbf{h}}$ is finite for all temperatures and has a kink at T_c. Renormalization group arguments yield $\eta_{\mathbf{A}} = 4 - d$ where d is the dimensionality [10,15], which has recently been verified numerically [11,16].

The vortex correlator for $N = 2$ is sampled in real space and $G^{(+)}(q)$ is found by discrete Fourier transformation, it is shown in Fig. 2. At $T = T_{c1}$ the leading behavior is $G^{(+)}(q) \sim q^2$ on both sides of T_{c1}. Consequently, due to (7), $m_{\mathbf{A}}$ and $m_{\Sigma\mathbf{h}}$ are finite in this regime. This shows that the vortex tangle is incompressible and that the anomalous scaling dimension $\eta_{\mathbf{A}} = 0$ corresponds to a neutral fixed point. Below T_{c2} the dominant behavior is $G^{(+)}(q) \sim q^2$ whereas $G^{(+)}(q) \sim c(T)$ above T_{c2}. At $T = T_{c2}$, $G^{(+)}(q) \sim q$ indicating $\eta_{\mathbf{A}} = 1$. Accordingly, $m_{\mathbf{A}}$ is finite below T_{c2} and zero for $T \geq T_{c2}$.

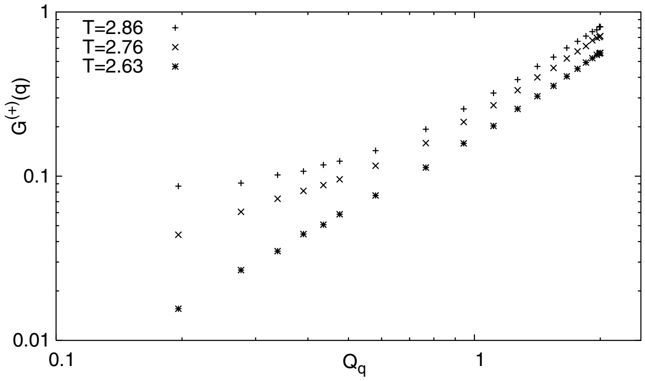

FIG. 2. $G^{(+)}(q)$ for $N = 2, L = 32$. For $T = 2.86 > T_{c2}$, $T = 2.76 \simeq T_{c2}$, and $T = 2.63 < T_{c2}$, $\lim_{\mathbf{q}\to 0} G^{(+)}(q) \sim c(T)$, $\sim q$, and $\sim q^2$, respectively.

For $T \lesssim T_{c2}$, $m_{\mathbf{A}}$ scales according to $\mathcal{G}_{\mathbf{A}}(q)^{-1} \times \frac{2}{\beta} = m_{\mathbf{A}}^2 + Cq^{2-\eta_{\mathbf{A}}} + \mathcal{O}(q^\delta)$ for small q where $\delta > 2 - \eta_{\mathbf{A}}$ [16], with a corresponding ansatz for $\mathcal{G}_{\Sigma\mathbf{h}}(q)$. For each coupling we fit $\mathcal{G}_{\mathbf{A}}(q)^{-1}$ data from system sizes $L = 8$, 12, 20, and 32 to the ansatz. The results for $m_{\mathbf{A}}$ (and $m_{\Sigma\mathbf{h}}$, found similarly), are given in Fig. 3. The system exhibits Higgs mechanism at $T = T_{c2}$ when $m_{\mathbf{A}}$ drops to zero. Furthermore $m_{\mathbf{A}}$ has a kink at T_{c1} due to ordering of $\theta^{(1)}$. The anomalies in $m_{\mathbf{A}}$ coincide precisely with T_{c1} and T_{c2} determined from scaling of $\langle (S_V - \langle S_V \rangle)^2 \rangle$. Note also how $m_{\Sigma\mathbf{h}}$ changes abruptly at T_{c2}. This is due to a sudden change in screening of $\sum_{\alpha=1}^N \mathbf{h}^{(\alpha)}$ by the vortex-loop proliferation at $T = T_{c2}$, giving an abrupt increase in $m_{\Sigma\mathbf{h}}$, analogously to what happens for $N = 1$, $e \neq 0$ [11].

Above T_{c2}, $\mathbf{A}$ is massless, giving a compressible vortex tangle which accesses configurational entropy better than an incompressible one. Below T_{c2}, $\mathbf{A}$ is massive and merely renormalizes $|\Psi|^4$ terms in Eq. (1). The theory is effectively a $|\Psi|^4$ theory in this regime. Thus, the remaining proliferated vortex species originating in the matter

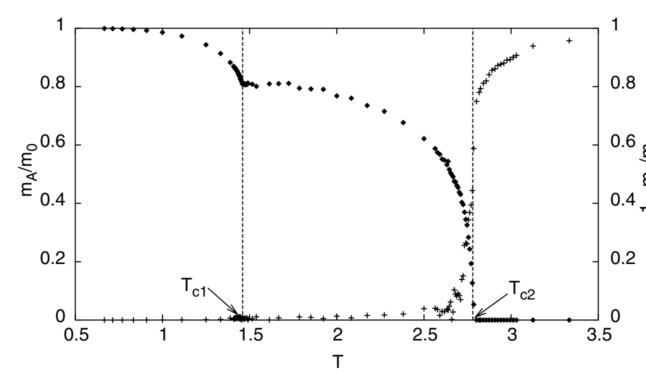

FIG. 3. The mass $m_{\mathbf{A}}$ (◆) and $1 - m_0/m_{\Sigma\mathbf{h}}$ (+) found from Eq. (7). Two nonanalyticities can be seen in $m_{\mathbf{A}}$ at T_{c1} and T_{c2}, corresponding to a neutral fixed point and a charged Higgs fixed point, respectively. An abrupt increase in $m_{\Sigma\mathbf{h}}$ due to vortex condensation is located at T_{c2}.

fields with lower bare stiffnesses form vortex tangles as if they originated in a neutral superfluid. For the general N case, a Higgs mass is generated at the highest critical temperature, after which $\mathbf{A}$ merely renormalizes the $|\Psi|^4$ term, such that the Higgs fixed point is followed by $N-1$ neutral fixed points as the temperature is lowered.

We now discuss the vortex mode $\mathbf{m}^{(1)} - \mathbf{m}^{(2)}$, demonstrating that it should be identified as a superfluid mode in the system. Its properties are controlled by $\mathcal{G}_{\Delta\mathbf{h}}(q) \equiv \langle|\mathbf{h}_q^{(1)} - \mathbf{h}_q^{(2)}|^2\rangle$. A dual Higgs phenomenon for $N=2$, $T=T_{c1}$ involving $\mathcal{G}_{\Delta\mathbf{h}}(q)$ may be demonstrated as follows. Introducing $G^{(-)}(q) = \langle|\mathbf{m}_q^{(1)} - \mathbf{m}_q^{(2)}|^2\rangle$ and $G^{(\mathrm{m})}(q) = \langle(\mathbf{m}_q^{(1)} - \mathbf{m}_q^{(2)}) \cdot (\sum_{\alpha=1}^2 |\psi^{(\alpha)}|^2 \mathbf{m}_{-q}^{(\alpha)})\rangle$ we find, in the notation used in Eqs. (5) and (7)

$$\mathcal{G}_{\Delta\mathbf{h}}(q) = \frac{8\beta\lambda^{(1)}\lambda^{(2)}\psi^2}{|\mathbf{Q_q}|^2}\left\{1 - \frac{2\pi^2\beta\lambda^{(1)}\lambda^{(2)}\psi^2 G^{(-)}(q)}{|\mathbf{Q_q}|^2} - \frac{2\pi^2\beta(\lambda^{(1)} - \lambda^{(2)})G^{(m)}(q)}{|\mathbf{Q_q}|^2 + m_0^2}\right\} + (\lambda^{(1)} - \lambda^{(2)})^2 \mathcal{G}_{\Sigma\mathbf{h}}(q). \quad (8)$$

The $G^{(-)}(q)$ correlation function is always $\sim q^2$, $q \to 0$, but has a nonanalytic coefficient of q^2, determined by the helicity modulus Υ of the neutral mode $\mathbf{m}^{(1)} - \mathbf{m}^{(2)}$. When Υ vanishes at T_{c1} through a disordering of $\theta^{(1)}$, thus destroying the superfluid neutral mode, the first and second term in the bracket cancel, which in turn cancels the $1/q^2$ term in $\mathcal{G}_{\Delta\mathbf{h}}(q)$. This produces a dual Higgs mass $m_{\Delta\mathbf{h}}$ defined by $\mathcal{G}_{\Delta\mathbf{h}}(q) \sim 1/(q^2 + m_{\Delta\mathbf{h}}^2)$ for $T > T_{c1}$. The remaining terms in Eq. (8) contribute to determining the actual value of $m_{\Delta\mathbf{h}}$. Thus, while $\mathbf{h}^{(1)} + \mathbf{h}^{(2)}$ is always massive, cf. Equation (3), $\mathbf{h}^{(1)} - \mathbf{h}^{(2)}$ is massless below T_{c1} and massive above T_{c1}. Therefore $\mathbf{h}^{(1)} - \mathbf{h}^{(2)}$ plays the role of a gauge degree of freedom, providing a dual counterpart to $\mathbf{A}$ in Eq. (1). This is evident when $|\psi^{(1)}|^2 = |\psi^{(2)}|^2$. Then Eq. (8) for $N=2$, $e \neq 0$ has the same form as the dual gauge field correlator for the case $N=1$, $e=0$, which exhibits a dual Higgs phenomenon [11]. Thus, for $N=2$, $e \neq 0$, $\mathbf{m}^{(1)} - \mathbf{m}^{(2)}$ behaves as vortices for $N=1$, $e=0$, *i.e., it is a superfluid mode arising out of superconducting condensates.* A nonzero $m_{\Delta\mathbf{h}}$ is *produced* by disordering $\theta^{(1)}$ at T_{c1} while a nonzero $m_\mathbf{A}$ is *destroyed* by disordering $\theta^{(2)}$ at T_{c2}.

We have analyzed the N-color London model Eq. (2) in vortex representation Eqs. (4) and (5). The dual theory is given by Eqs. (3) and (6). For $N=2$, we have performed large scale Monte Carlo simulations computing (i) critical exponents α and ν, (ii) gauge field and dual gauge field correlators, (iii) the corresponding masses, and (iv) critical couplings using FSS. For $\psi^{(1)} \neq \psi^{(2)}$ we find one *neutral* low-temperature critical point at T_{c1}, and one *charged* critical point at $T_{c2} > T_{c1}$. For general N, a Higgs mass $m_\mathbf{A}$ is generated at the highest critical temperature, followed by $N-1$ neutral fixed points as the temperature is lowered.

These results apply to electronic and protonic condensates in liquid metallic hydrogen under extreme pressure. Estimates exist for T_{c2} for such systems, $T_{c2} \approx 160$ K [2], and hence $T_{c1} \approx 0.1$ K. Hence, in addition to the emergence of the Meissner effect at T_{c2} and a corresponding divergence in the magnetic penetration length $\lambda \sim |1 - T/T_{c2}|^{-\nu/(2-\eta_A)}$ [17], there will also be a novel effect, namely, a low-temperature anomaly in the magnetic penetration length $\lambda \sim 1/m_\mathbf{A}$ at T_{c1}, cf. Figure 3, due to the appearance of superfluid modes arising from superconducting condensates.

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