Quasiadiabatic Decay of Capillary Turbulence on the Charged Surface of Liquid Hydrogen

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We study the free decay of capillary turbulence on the charged surface of liquid hydrogen. We find that decay begins from the *high frequency* end of the spectral range, while most of the energy remains localized at low frequencies. The apparent discrepancy with the self-similar theory of nonstationary wave turbulent processes is accounted for in terms of a quasiadiabatic decay wherein fast nonlinear wave interactions redistribute energy between frequency scales in the presence of finite damping at all frequencies. Numerical calculations based on this idea agree well with experimental data.

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Introduction.—Turbulence on the surface of fluids has been the focus of numerous experimental and theoretical investigations during the last few years [1–9]. Interest in such phenomena, usually referred to as wave turbulence (WT), arises both from their great importance in terms of basic nonlinear physics and from numerous applications in engineering and the life sciences. One of the best known applications is in weather prediction, using information received from the measurements of the spectrum of waves on the sea surface. Turbulence in a system of capillary waves is of interest because its dynamics on this wavelength scale plays a significant role in the transfer and dissipation of energy on the liquid surface. Studies of nonstationary phenomena in systems of capillary waves are of particular importance because they could provide direct information about nonlinear wave interactions. In spite of the large number of experimental studies devoted to the nonlinear dynamics of surface waves, however, there are only a few recent reports [2–5] of experimental studies of capillary turbulence that are directly comparable with theoretical predictions.

Recently, the use of liquid hydrogen as a model medium has brought significant progress in the understanding of capillary turbulence. The remarkable properties of liquid hydrogen (its low viscosity and the high nonlinearity of its capillary waves, and the possibility of driving the electrically charged surface of the liquid directly) have allowed us to observe the following for the first time: the formation of the Kolmogorov power-law turbulent spectrum over a wide range of frequencies (10^2–10^4 Hz) [4], the modification of the scaling index of the turbulent spectrum in dependence on the spectral characteristics of the driving force [10], and the cutoff of the power spectrum of capillary turbulence at high frequencies due to the change of the energy transfer mechanism from nonlinear waves transformation to viscous damping [11]. Many important properties of waves on the surface of liquid hydrogen are similar to those of conventional liquids like water (cf. the capillary lengths for liquid hydrogen $\lambda = 0.19$ cm at $T = 15$ K, and for water $\lambda = 0.28$ cm at $T = 293$ K), providing an additional argument for using liquid hydrogen as a perfect test fluid for accurate tests of WT theory. The latter are of considerable importance, because WT theory is used to describe turbulent processes in a wide variety of media, including, e.g., plasmas [12], astrophysics [13], ocean surfaces [14], acoustic turbulence in superfluid He II [15], and nonlinear optics [16].

In this Letter we report the first observations of decay of the turbulent state in a system of capillary waves on the surface of liquid hydrogen. We have observed that the decay of the stationary turbulent spectrum starts in the *high frequency* domain, with the energy remaining localized in the low frequency range of the turbulent spectrum, i.e., near the driving frequency. At low frequencies the turbulent spectrum remains close to its unperturbed shape for a relatively long period of time after the driving force is switched off. This observation differs significantly from what might be expected from the self-similar theory of nonstationary WT processes [1], where the evolution of the spectrum is considered in the range of frequencies where viscous damping can be neglected.

We show that where there is finite damping at all frequencies, viscous losses cause qualitative changes in the evolution of the turbulent spectrum after removal of the driving force: rather than a propagation of perturbations from low to high frequencies, caused by the cascade transfer of energy (the scenario considered in [1]), there is a relatively fast decay of the high frequency domain of the spectrum. Although our observations are new, and came as a surprise, we show that they can still be understood qualitatively within the framework of the general WT theory [1].

Experimental observations.—The experimental arrangements were similar to those used in our earlier studies of steady-state turbulence on the charged surface of liquid hydrogen [17]. The measurements were made using an optical cell inside a helium cryostat. Hydrogen was condensed into a cup formed by a bottom capacitor plate and a guard ring 60 mm in diameter and 6 mm high. The layer of liquid was 6 mm thick. The top capacitor plate (a collector 60 mm in diameter) was located at a distance of 4 mm above the surface of the liquid. A two-dimensional positive charge layer was created just below the surface of the liquid with the aid of a radioactive plate placed at the bottom of the cup. The temperature of the liquid was 15–16 K. The waves on the charged surface were excited by a periodic driving voltage applied between the guard ring and the upper electrode. They were detected from the variation of the total power $P(t)$ of a laser beam reflected from the oscillating surface, which was measured with a photodetector, sampled with an analog-to-digital converter, and stored in a computer. Given the size of the light spot, the correlation function $I_\omega = \langle |\eta_\omega|^2 \rangle$ of the surface elevation $\eta(\mathbf{r}, t)$ in frequency representation is directly proportional to the squared modulus of the Fourier transform of the detected signal, $I_\omega = \mathrm{const} \times P_\omega^2$ at frequencies above 50 Hz.

In systems of finite size, the power spectrum of capillary turbulence is discrete; but, at frequencies much higher than the lowest resonant frequency (which in our experiments was $\sim$3 Hz), the intrinsic spectrum of resonant frequencies becomes quasicontinuous due to viscous broadening of the resonances and/or nonlinear broadening at pumping rates exceeding a critical value (see theory [8,18] and observations [2–5]). To establish the steady turbulent state at the surface of the liquid, a 95 Hz ac driving voltage was applied for $\sim$10 s. It was then switched off, and we observed the relaxation oscillations of the surface. The instantaneous power spectrum P_ω^2 of nonstationary surface oscillations was calculated by using a short-time Fourier transform of the measured signal $P(t)$ [19].

It is clearly evident from Fig. 1 that, during decay of the turbulence, it is the *high frequency* components of the power spectrum that are damped first. The amplitude of the main peak at the driving frequency remains larger than the amplitudes of peaks at the harmonics, all the time, both before and after the driving force is switching off, i.e., the surface of the liquid continues to oscillate mainly at the driving frequency. The wave amplitude in the turbulent distribution decreases homogeneously during the decay, and the power-law dependence of the spectrum persists, even for low frequencies down to 100 Hz.

Numerical calculations.—To try to understand the peculiar form of decay, we modeled numerically the time evolution of the initial steady-state turbulent power spectrum of the capillary waves. According to WT theory [1,6] the evolution of a turbulent spectrum can be described by the integro-differential kinetic equation (KE) for the classical occupation numbers n_k of surface waves, where k is a wave vector. In our calculations we used the so-called "local approximation" for the KE, where the collision integral was approximated by a differential operator. This approach is similar to the well-known Fokker-Plank equation approach in the kinetic theory of gases [20] and is based on nonlinear interactions between waves of comparable frequencies. Such a differential ana-

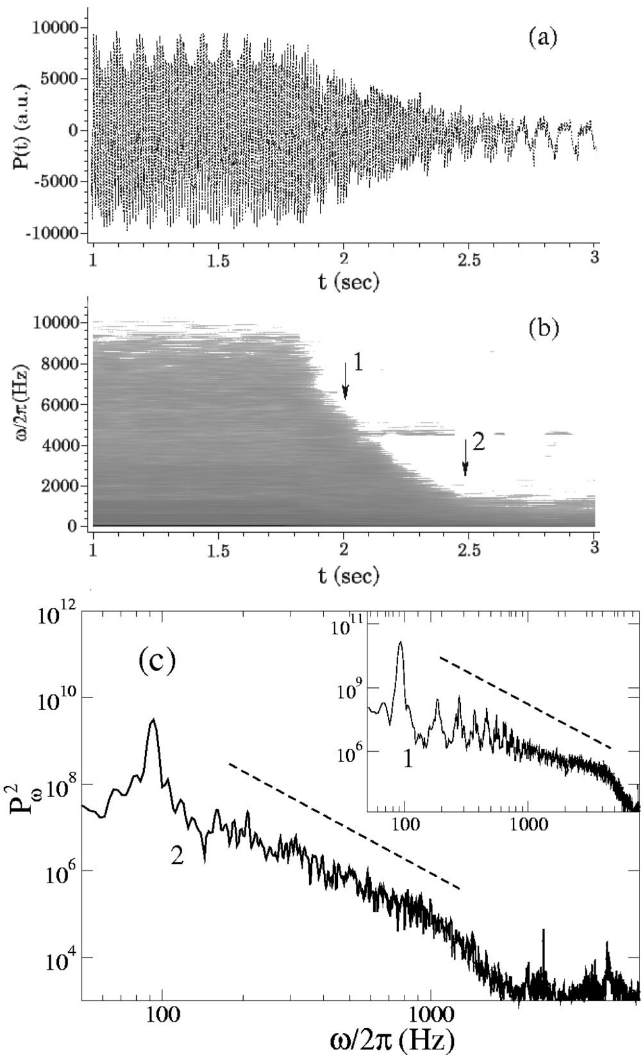

FIG. 1. (a) The measured signal $P(t)$. The periodic driving force was switched off at time $t = 1.8$ s. (b) Evolution of the turbulent power spectrum during the decay, calculated over $P(t)$. Grey shading indicates frequency components in the power spectrum whose P_ω^2 exceeds the threshold 10^4 (a.u.) in the graph below. (c) Instantaneous power spectra calculated at times indicated by the arrows in (b): curve 1 in the inset corresponds to time $t = 2$ s; curve 2 corresponds to $t = 2.5$ s. The spectra shown represent an ensemble average over ten identical measurements. The dashed line in (c) corresponds to the powerlike dependence $P_\omega^2 \sim \omega^{-7/2}$ predicted by theory [6,7].

log of the KE can be written in a unique way based on the known symmetry and scaling properties of the turbulent state. A similar phenomenological model of WT has been used successfully before, e.g., in studies of gravitational waves on the surface of liquid [14] and for optical turbulence [16], yielding results that are in a good agreement with the "exact" WT theory. To accord more accurately with the real conditions of the experiment, we introduce into Eq. (1) (below) a viscous damping term for waves at all frequencies, in addition to the nonlinear term. Capillary turbulence has not hitherto been studied using this approach.

Following the general recommendations of Refs. [14,16] we write the corresponding equation for the envelope curve of the turbulent spectrum as

$$\frac{\partial n_\omega}{\partial t} = \frac{C}{\omega^{4/3}} \frac{\partial}{\partial \omega}\left[\omega^7 n_\omega \frac{\partial}{\partial \omega}(\omega n_\omega)\right] - 2\gamma_\omega n_\omega + f_\omega(t). \quad (1)$$

For convenience in comparing numerical results with our experimental data, we use the occupation number in ω representation, $n_\omega(t) = n_k(t)|_{k=k(\omega)}$, as the main characteristic of the turbulent distribution, where $k = k(\omega)$ is the function inverse to the dispersion law for the capillary waves $\omega = \omega_k$. The first term on the right-hand side of (1) plays the role of the collision integral in the KE, $\gamma_\omega \propto \omega^{4/3}$ is the damping coefficient of capillary waves, the term $f_\omega(t)$ models the direct action of the driving force, and C is a dimensional constant. For capillary waves the correlation function $I_\omega = \text{const} n_\omega$ has the same scaling properties as n_ω. In our simulations we renormalized time by a dimensional constant, $t \to t/C$ and renormalized frequency, $\omega \to C\omega$. We have assumed a renormalized driving frequency $\omega_d/2\pi = 100$ to facilitate comparison of our numerical modeling results for the free decay shown in Fig. 2 with the experimental data of Fig. 1. The full curve shows the envelope of the initial steady-state turbulent distribution n_ω. For the frequency range $200 < \omega/2\pi < 800$ it can evidently be approximated by a powerlike dependence $n_\omega \sim \omega^{-7/2}$, in good agreement both with the existing theory of capillary turbulence [6,7] and with our current and previous experimental observations [10]. The cutoff in the power spectrum at the frequency $\omega_b/2\pi \approx 800$ is caused by viscosity. The dashed and dotted curves show the evolution of the envelope after the driving force was removed. The destruction of the powerlike dependence clearly begins from the high frequency domain of the turbulent spectrum. The energy of the oscillations is concentrated mainly in the low frequency spectral domain. That there is no destructive front propagating from low to high frequencies is entirely consistent with our experimental results.

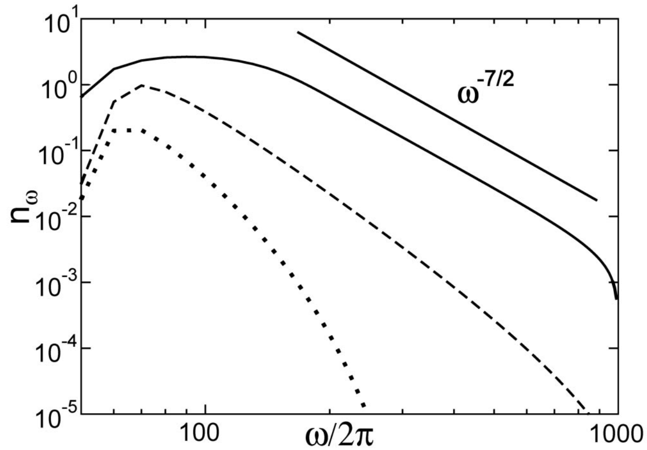

FIG. 2. Decay of the turbulent spectrum. The occupation number n_ω as a function of renormalized frequency calculated using (1) at three renormalized times: $t = 0$, the initial steady-state spectrum (full curve); $t = 3$ (dashes); $t = 10$ (dots). The straight line corresponds to $n_\omega \sim \omega^{-7/2}$ as predicted by the theory of capillary turbulence [6,7].

These results may be explained qualitatively in terms of the general theory of WT if we assume a fast redistribution of energy between different frequency scales inside the inertial range, leading to suppression of relaxation processes. Such a redistribution will stabilize the powerlike dependence of the turbulent spectrum at low frequencies. A similar evolution of the turbulent spectrum was observed in [11] with a smooth decrease of driving amplitude, where the turbulent system remained continuously in its steady state. Based on these observations we can claim that the evolution of the freely decaying turbulent spectrum has a quasiadiabatic character. The kinetic time of nonlinear wave interactions $\tau_k(\omega)$ plays the role of a fast time; the time $\tau_v(\omega)$, characterizing viscous damping of the waves, plays the role of a slow time within the inertial frequency range. From (1) one can estimate the frequency dependences of $\tau_k(\omega) \propto \omega^{-7/6}$ and $\tau_v(\omega) \propto \omega^{-4/3}$, and of their ratio as $r(\omega) = \tau_k(\omega)/\tau_v(\omega) \propto (\omega/\omega_b)^{1/6}$. It was shown in [11] that the inertial range is limited at high frequencies by the condition $r(\omega_b) \sim 1$, and that inside the inertial range the parameter r is small, i.e., nonlinear wave interactions occur faster than the viscous damping processes. At these frequencies the ratio $r(\omega)$ can be considered as a small adiabatic parameter. At frequencies of the order of ω_b, $r(\omega_b) \sim 1$, so that the nonlinear and viscous processes have comparable rates, and the adiabatic condition is violated. During the decay process the amplitudes of the waves decrease, so the kinetic time decreases as well [10,11], hence the shift of the boundary frequency ω_b towards the low frequency domain. At large times after the beginning of the decay, ω_b becomes comparable with the driving frequency ω_d. Dissipation is then starting to play an important role even at low frequencies. Thus we can claim that, for the understanding of nonstationary turbulent processes, viscous losses in a turbulent system are of central importance at all frequencies above the driving frequency: we have seen, both in experiments and in computations, that finite damping of the waves changes qualitatively the character of the turbulent decay compared to the scenario envisaged earlier. Note that these considerations do *not* apply to stationary turbulent phenomena, where dissipation can be neglected completely over a wide range of frequencies [8,9].

Conclusion.—Our experimental observations and numerical calculations have shown that the decay of capillary turbulence on the surface of a normal liquid begins with the damping of high frequency waves, so that the high frequency part of the powerlike turbulent spectrum is destroyed first. The energy-containing range of frequencies does not shift toward high frequencies in practice, but remains at frequencies of the order of the driving frequency. This remains true at all times during the decay, even at large times when the turbulent regime is replaced by the viscous damping of the waves. Transition processes in the turbulent system are damped by the fast nonlinear redistribution of energy between waves whose frequencies are inside the inertial range. These general conclusions may be useful for future studies of nonstationary turbulent processes in a wide class of problems, e.g., in hydrodynamics, nonlinear sound in solids and liquids, astrophysics, and plasma physics. In particular, our results support the phenomenological propositions made in [21] to account for the observed long-time properties of vorticity in the decay of quantum turbulence in superfluid helium. They could be also useful in relation to recent studies [18] of damping of the monochromatic capillary wave on a liquid surface.

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