

## Heavy Quark Energy Loss in a Nuclear Medium

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Multiple scattering, modified fragmentation functions, and radiative energy loss of a heavy-quark propagating in a nuclear medium are investigated in perturbative QCD. Because of the quark mass dependence of the gluon formation time, the medium size dependence of heavy-quark energy loss is found to change from a linear to a quadratic form when the initial energy and momentum scale are increased relative to the quark mass. The radiative energy loss is also significantly suppressed relative to a light quark due to the suppression of collinear gluon emission by a heavy quark.

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An energetic parton propagating in a dense medium suffers a large amount of energy loss due to multiple scattering and induced gluon radiation [1]. In a static medium, the total energy loss of a massless parton is found to have a quadratic dependence on the medium size [2–6] due to the non-Abelian Landau-Pomeranchuk-Migdal (LPM) interference effect. In an expanding medium, the total energy loss can be cast into a line integral weighted with local gluon density along the path of parton propagation [7,8]. Therefore, the measurement of parton energy loss can be used to study properties of the medium. Recent experimental measurements [9,10] of high- $p_T$  hadron suppression agree very well [11] with such a parton energy loss mechanism.

Because of the small velocity  $v \approx 1 - M^2/2E^2$  of a heavy quark, the formation time,  $\tau_f \sim 1/(\omega M^2/2E^2 + \ell_T^2/2\omega)$  of the radiated gluon with energy  $\omega$  and transverse momentum  $\ell_T$ , is reduced relative to that of a light quark. One should then expect the LPM effect to be significantly reduced for intermediate energy heavy quarks. In addition, the heavy-quark mass also suppresses gluon radiation amplitude at small angles [12] relative to that off a light quark. Both mass effects lead to a heavy-quark energy loss different from a light quark. This might explain why one has not observed significant heavy-quark energy loss from the recent measurement of the single electron spectrum from charm decay in Au + Au collisions at  $\sqrt{s} = 130$  GeV [13]. In this Letter, we report a study on medium induced energy loss and the modified fragmentation function of a heavy quark. In particular, we show how the mass effects reduce the total energy loss and how the dependence on medium size changes from a linear to a quadratic dependence when the energy of the heavy quark or the momentum scale is increased. Similar results have been reported in Refs. [14,15] during and after the completion of this work. The current study is within the framework of twist expansion and can be applied to heavy-quark propagation in both cold nuclear medium and hot partonic matter.

To separate the complication of heavy-quark production and propagation, we consider a simple process of charm quark production via the charge-current interaction in deep inelastic scattering (DIS) off a large nucleus. The results can be easily extended to heavy-quark propagation in a hot medium. The differential cross section for the semi-inclusive process  $\ell(L_1) + A(p) \rightarrow \nu_\ell(L_2) + H(\ell_H) + X$  can be expressed as

$$E_{L_2} E_{\ell_H} \frac{d\sigma_{\text{DIS}}}{d^3 L_2 d^3 \ell_H} = \frac{G_F^2}{(4\pi)^3 s} L_{\mu\nu}^{cc} E_{\ell_H} \frac{dW^{\mu\nu}}{d^3 \ell_H}. \quad (1)$$

Here  $L_1$  and  $L_2$  are the four momenta of the incoming lepton and the outgoing neutrino,  $\ell_H$  is the observed heavy meson momentum,  $p = [p^+, m_N^2/2p^+, \mathbf{0}_\perp]$  is the momentum per nucleon in the nucleus, and  $s = (p + L_1)^2$ .  $G_F$  is the four-fermion coupling constant and  $q = L_2 - L_1 = [-Q^2/2q^-, q^-, \mathbf{0}_\perp]$  is the momentum transfer via the exchange of a  $W$  boson. The charge-current leptonic tensor is given by  $L_{\mu\nu}^{cc} = 1/2 \text{Tr}[\not{L}_1 \gamma_\mu (1 - \gamma_5) \not{L}_2 (1 + \gamma_5) \gamma_\nu]$ . We assume  $Q^2 \ll M_W^2$ . The semi-inclusive hadronic tensor is defined as

$$E_{\ell_H} \frac{dW^{\mu\nu}}{d^3 \ell_H} = \frac{1}{2} \sum_X \langle A | J_\mu^+ | X, H \rangle \langle X, H | J_\nu^{+\dagger} | A \rangle 2\pi \times \delta^4(q + p - p_X - \ell_H), \quad (2)$$

where  $\sum_X$  runs over all possible final states and  $J_\mu^+ = \bar{c} \gamma_\mu (1 - \gamma_5) s_\theta$  is the hadronic charged current. Here,  $s_\theta = s \cos\theta_C - d \sin\theta_C$  and  $\theta_C$  is the Cabibbo angle. To the leading twist in collinear approximation, the semi-inclusive cross section factorizes into the product of quark distribution  $f_{s_\theta}^A(x_B + x_M)$ , the heavy-quark fragmentation function  $D_{Q \rightarrow H}(z_H)$  ( $z_H = \ell_H^-/\ell_Q^-$ ), and the hard partonic part  $H_{\mu\nu}^{(0)}(k, q, M)$  [16]. Here,  $x_B = Q^2/2p^+q^-$  is the Bjorken variable and  $x_M = M^2/2p^+q^-$ .

Similar to the case of light quark propagation in nuclear medium [6], the generalized factorization of multiple scattering processes [17] is employed. We con-

sider only the leading contributions from double parton scattering, in which a heavy quark is produced by the charge-current interaction in nucleon and then scatters again with a gluon from another nucleon in the nucleus. The cross section is proportional to two-parton correlation within a nucleus that is proportional to the size of the nucleus and hence is enhanced by a factor of  $A^{1/3}$ . By power counting, this high-twist term should also be proportional to  $\alpha_s/Q^2$ . Such a contribution is a leading higher-twist term in the generalized collinear expansion, assuming a small expansion parameter  $\alpha_s A^{1/3}/Q^2$ . The evaluation of 23 cut diagrams is similar to the case of a light quark [16]. The dominant contribution comes from central cut diagrams, giving the semi-inclusive tensor for heavy-quark fragmentation from double quark-gluon scattering,

$$\begin{aligned} \frac{W_{\mu\nu}^D}{dz_h} &= \sum \int dx H_{\mu\nu}^{(0)} \int_{z_h}^1 \frac{dz}{z} D_{Q \rightarrow H} \left( \frac{z_H}{z} \right) \frac{C_A \alpha_s}{2\pi} \frac{1+z}{1-z} \\ &\times \int \frac{d\ell_T^2}{[\ell_T^2 + (1-z)^2 M^2]^4} \ell_T^4 \frac{2\pi\alpha_s}{N_c} T_{qg}^{A,C}(x, x_L, M^2) \\ &+ (g - \text{frag.}) + (\text{virtual corrections}). \end{aligned} \quad (3)$$

$$T_{qg}^{A,C}(x, x_L, M^2) = \frac{1}{2} \int \frac{dy^-}{2\pi} dy_1^- dy_2^- \tilde{H}_C^D \langle A | \bar{\psi}_q(0) \gamma^+ F_\sigma^+(y_2^-) F^{+\sigma}(y_1^-) \psi_q(y^-) | A \rangle e^{i(x+x_L)p^+ y^-} \theta(-y_2^-) \theta(y^- - y_1^-). \quad (5)$$

This twist-4 parton matrix is essentially the quark-gluon correlation inside the nucleus and is probed only by double scattering. Here  $\tilde{H}_C^D$  contains all the phase factors arising from the LPM interference between different radiation amplitudes. It also contains coefficients that are polynomial functions of  $M^2/\ell_T^2$  as a result of the collinear expansion of quark-gluon scattering amplitude [16]. In the case of  $M = 0$ , one recovers the light quark result and in the soft bremsstrahlung limit ( $z \rightarrow 1$ ),

$$\tilde{H}_C^D(M=0) \approx (1 - e^{-i\tilde{x}_L p^+ y_2^-}) (1 - e^{-i\tilde{x}_L p^+ (y^- - y_1^-)}), \quad (6)$$

where  $\tilde{x}_L \equiv x_L + (1-z)x_M/z$  [ $x_L = \ell_T^2/2p^+ q^- z(1-z)$ ] is the additional momentum fraction carried by the initial quark or gluon in the rescattering, depending on whether the gluon radiation happens before or after the secondary scattering. These phase factors result from the LPM interference of different radiation amplitudes. The geometrical nuclear size within the nuclear wave function restricts the spatial integration over  $y_2^-$  or  $y^- - y_1^-$  in

$$\tilde{D}_{Q \rightarrow H}(z_H, \mu^2) \equiv D_{Q \rightarrow H}(z_H, \mu^2) + \int_0^{\mu^2} \frac{d\ell_T^2}{\ell_T^2 + (1-z)^2 M^2} \frac{\alpha_s}{2\pi} \int_{z_h}^1 \frac{dz}{z} \Delta \gamma_{q \rightarrow qg}(z, x, x_L, \ell_T^2, M^2) D_{Q \rightarrow H} \left( \frac{z_H}{z} \right) + (g - \text{frag.}), \quad (8)$$

where  $D_{Q \rightarrow H}(z_H, \mu^2)$  is the leading-twist fragmentation function of the heavy quark. The contribution from gluon fragmentation to heavy-quark meson is similar in form but should be negligible as compared to the first term. The modified splitting function is given by Eq. (3), as

$$\Delta \gamma_{q \rightarrow qg}(z) = \left[ \frac{1+z^2}{(1-z)} T_{qg}^{A,C}(x, x_L, M^2) + \text{v.c.} \right] \frac{2\pi C_A \alpha_s \ell_T^4}{[\ell_T^2 + (1-z)^2 M^2]^3 N_c f_q^A(x)}, \quad (9)$$

The above is very similar to the case of double scattering of a light quark [6] and resembles that of gluon radiation off a heavy quark in vacuum. The transverse momentum distribution  $\ell_T^4/[\ell_T^2 + (1-z)^2 M^2]^2$  is typical of bremsstrahlung from a heavy particle. It vanishes in the small angle  $\ell_T \rightarrow 0$ . One can rewrite such a suppression relative to the gluon radiation from a light quark as

$$f_{Q/q} = \left[ \frac{\ell_T^2}{\ell_T^2 + z^2 M^2} \right]^4 = \left[ 1 + \frac{\theta_0^2}{\theta^2} \right]^{-4}, \quad (4)$$

where  $\theta_0 = M/q^-$  and  $\theta = \ell_T/q^- z$ . This is often referred to as the ‘‘dead-cone’’ phenomenon that suppresses small angle gluon radiation and therefore reduces radiative energy loss of a heavy quark [12]. The contribution from gluon fragmentation is similar to that from quark fragmentation with  $z \rightarrow 1-z$ . The virtual correction can be obtained via unitarity constraint.

The power of  $\ell_T$  spectra in Eq. (3) is a result of the collinear expansion and is simple to understand in terms of power counting, since the contribution from double scattering is proportional to a twist-4 parton matrix,

Eq. (5). It is clear that the final radiative gluon spectrum that is proportional to the effective twist-4 parton matrix  $T_{qg}^{A,C}(x, x_L, M^2)$  depends on the formation time for gluon radiation from a heavy quark,

$$\tau_f \equiv \frac{1}{p^+ \tilde{x}_L} = \frac{2z(1-z)q^-}{\ell_T^2 + (1-z)^2 M^2}. \quad (7)$$

When the formation time is much larger than the nuclear size, the LPM interference suppresses gluon radiation induced by the secondary scattering. One can see that  $\tau_f$  for gluon radiation from a heavy quark is shorter than that from a light quark. This should have significant consequences for the effective modified quark fragmentation function and the heavy-quark energy loss.

Rewriting the sum of single and double scattering contributions in a factorized form for the semi-inclusive hadronic tensor, one can define a modified heavy-quark fragmentation function  $\tilde{D}_{Q \rightarrow H}(z_H, \mu^2)$  from Eq. (3) as

where other variables in  $\Delta\gamma$  are suppressed. The virtual correction (v.c.) can be obtained from the real one in the first term through unitarity constraint. Given the twist-4 quark-gluon correlation in a nucleus,  $T_{qg}^{A,C}(x, \ell_T^2, M^2)$ , one should be able to evaluate the modified heavy-quark fragmentation function. We focus instead on the heavy-quark energy loss in this Letter.

As discussed previously [18], one can assume a factorized form of the twist-4 parton matrix

$$T_{qg}^{A,C}(x, x_L, M^2) \approx \frac{\tilde{C}}{x_A} f_q^A(x) [(1 - e^{-\tilde{x}_L^2/x_A^2})a_1 + a_2] \quad (10)$$

in the limit  $x_L \ll x$ , where  $x_A \equiv 1/m_N R_A$ . The coefficients  $a_1$  and  $a_2$  are polynomial functions of  $M^2/\ell_T^2$  and become  $(1+z)/2$  and  $C_F(1-z)^2/2C_A$ , respectively, for  $M=0$  [16]. The coefficient  $\tilde{C}$  is proportional to the soft gluon distribution inside nucleon. The suppression factor  $1 - \exp(-\tilde{x}_L^2/x_A^2)$  due to the LPM interference results from the phase factor in Eq. (6) integrated with a Gaussian nuclear distribution that has a radius  $R_A$ .

With this simplified form of twist-4 matrix, one can then calculate the heavy-quark energy loss, defined as the fractional energy carried by the radiated gluon,

$$\begin{aligned} \langle \Delta z_g^Q \rangle(x_B, Q^2) &= \frac{\alpha_s}{2\pi} \int_0^{Q^2} d\ell_T^2 \int_0^1 dz \frac{\Delta\gamma_{q \rightarrow qq}(1-z)}{\ell_T^2 + z^2 M^2} z \\ &= \frac{\tilde{C} C_A \alpha_s^2 x_B}{N_c Q^2 x_A} \int_0^1 dz \frac{1+z^2}{z(1-z)} \\ &\quad \times \int_{\tilde{x}_M}^{\tilde{x}_\mu} d\tilde{x}_L \frac{(\tilde{x}_L - \tilde{x}_M)^2}{\tilde{x}_L^4} \\ &\quad \times [(1 - e^{-\tilde{x}_L^2/x_A^2})a_1 + a_2], \end{aligned} \quad (11)$$

where  $\tilde{x}_M = (1-z)x_M/z$  and  $\tilde{x}_\mu = \mu^2/2p^+q^-z(1-z) + \tilde{x}_M$ . Note that the virtual correction in  $\Delta\gamma_{q \rightarrow qq}$  does not contribute to the energy loss. Also,  $\tilde{x}_L/x_A = L_A^-/\tau_f$  with  $L_A^- = R_A m_N/p^+$  the nuclear size in the chosen frame. The second term proportional to  $a_2$  corresponds to a finite contribution in the factorization limit. This term survives in the limit of complete LPM cancellation when double scattering acts like a single scattering for induced gluon radiation. We have neglected such a term in the study of light quark propagation since it is proportional to  $R_A$ , as compared to the  $R_A^2$  dependence from the first term due to the LPM effect. In this study we have to keep the second term for heavy-quark propagation since the first term with the LPM interference effect has a similar nuclear dependence when the mass dependence of the gluon formation time is important.

Since  $\tilde{x}_L/x_A = L_A^-/\tau_f \sim x_B M^2/x_A Q^2$ , its value should control the LPM interference effect and the behavior of the total heavy-quark energy loss. There are two distinct limiting behaviors of the energy loss for different values of  $x_B/Q^2$  relative to  $x_A/M^2$ . When  $x_B/Q^2 \gg x_A/M^2$  for small quark energy (large  $x_B$ ) or small  $Q^2$ , the formation

time of gluon radiation off a heavy quark is always smaller than the nuclear size. In this case,  $1 - \exp(-\tilde{x}_L^2/x_A^2) \approx 1$ . There is no destructive LPM interference. The integral in Eq. (11) becomes independent of  $R_A$ , and the heavy-quark energy loss

$$\langle \Delta z_g^Q \rangle \sim C_A \frac{\tilde{C} \alpha_s^2}{N_c} \frac{x_B}{x_A Q^2} \quad (12)$$

is linear in nuclear size  $R_A$ . Such a behavior is also found in a recent study in the framework of opacity expansion [14]. In the opposite limit,  $x_B/Q^2 \ll x_A/M^2$ , for large quark energy (small  $x_B$ ) or large  $Q^2$ , the quark mass becomes negligible. The gluon formation time could still be much larger than the nuclear size. Similarly as in the case of a light quark [6,19], the LPM suppression factor  $1 - \exp(-\tilde{x}_L^2/x_A^2)$  limits the available phase space for gluon radiation. The integral in Eq. (11) will be proportional to  $\int d\tilde{x}_L [1 - \exp(-\tilde{x}_L^2/x_A^2)]/\tilde{x}_L^2 \sim 1/x_A$ . The heavy-quark energy loss

$$\langle \Delta z_g^Q \rangle \sim C_A \frac{\tilde{C} \alpha_s^2}{N_c} \frac{x_B}{x_A^2 Q^2} \quad (13)$$

then has a quadratic dependence on the nuclear size similar to the light quark energy loss. Shown in Fig. 1 are the numerical results of the  $R_A$  dependence of charm quark energy loss, rescaled by  $\tilde{C}(Q^2)C_A\alpha_s^2(Q^2)/N_c$ , for different values of  $x_B$  and  $Q^2$ . One can clearly see that the  $R_A$  dependence is quadratic for large values of  $Q^2$  or small  $x_B$ . The dependence becomes almost linear for small  $Q^2$  or large  $x_B$ . The charm quark mass is set at  $M = 1.5$  GeV in the numerical calculation.

Another mass effect on the induced gluon radiation is the dead-cone phenomenon [12] that suppresses the the small angle gluon radiation. Since the size of the dead cone  $\theta_0 = M/q^-$  [Eq. (4)], within which the gluon radiation is suppressed, is inversely proportional to the quark's energy, the reduction of energy loss is stronger for a slower quark. For a heavy quark with either a high energy  $q^-$  or virtuality  $Q^2$ , its radiative energy loss should approach that of a light quark. Setting  $M=0$  in Eq. (11), we

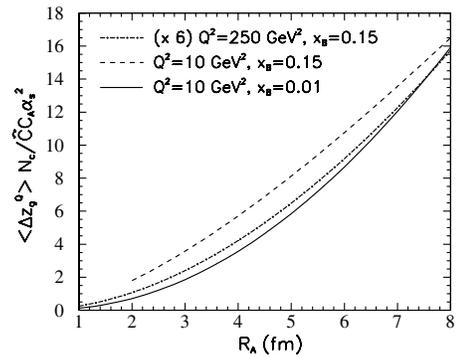


FIG. 1. The nuclear size,  $R_A$ , dependence of charm quark energy loss for different values of  $Q^2$  and  $x_B$ .

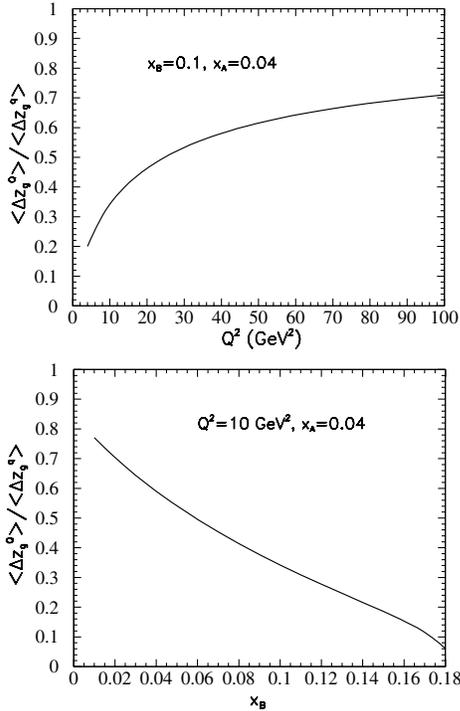


FIG. 2. The  $Q^2$  (upper panel) and the  $x_B$  (lower panel) dependence of the ratio between charm quark and light quark energy loss in a large nucleus.

recover the energy loss for light quarks as in our previous study [19]. To illustrate the mass suppression of radiative energy loss imposed by the dead-cone, we plot the ratio  $\langle \Delta z_g^c \rangle(x_B, Q^2) / \langle \Delta z_g^q \rangle(x_B, Q^2)$  of charm quark and light quark energy loss as functions of  $Q^2$  and  $x_B$  in Fig. 2. Apparently, the heavy-quark energy loss induced by gluon radiation is significantly suppressed as compared to a light quark when the momentum scale  $Q$  or the quark initial energy  $q^-$  is not too large as compared to the quark mass. Only in the limit  $M \ll Q, q^-$ , the energy loss approaches that of a light quark.

In summary, we have calculated medium modification of fragmentation and energy loss of heavy quarks in DIS in the twist expansion approach. We demonstrated that heavy-quark mass not only suppresses small angle gluon radiation due to the dead-cone effect but also reduces the gluon formation time. This leads to a reduced radiative energy loss as well as a different medium size dependence, as compared to a light quark. The result approaches that for a light quark when the quark mass is negligible as compared to the quark energy and the momentum scale  $Q$ . Similar to the case of light quark propagation [8], the result can be easily extended to a hot and dense medium, which will have practical consequences for heavy-quark production [20] and suppression in heavy-ion collisions. As the data on direct measurement of  $D$  meson in high-energy  $A + A$  collisions become available in the near future, one should be able to use the modified fragmenta-

tion function in a parton model to study the modification of the  $D$ -meson spectra and probe medium properties similarly as has been done with high  $p_T$  light hadrons [11]. The different pattern of energy loss for heavy quarks, such as energy and medium size dependence, will not only confirm the unique feature of non-Abelian energy loss but also give more confidence in using jet tomography to study properties of dense matter in heavy-ion collisions.

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- [1] M. Gyulassy and X.-N. Wang, Nucl. Phys. **B420**, 583 (1994); X.-N. Wang, M. Gyulassy, and M. Plümer, Phys. Rev. D **51**, 3436 (1995).
  - [2] R. Baier *et al.*, Nucl. Phys. **B483**, 291 (1997); **B484**, 265 (1997).
  - [3] B. G. Zakharov, JETP Lett. **63**, 952 (1996).
  - [4] M. Gyulassy, P. Lévai, and I. Vitev, Nucl. Phys. **B594**, 371 (2001); Phys. Rev. Lett. **85**, 5535 (2000).
  - [5] U. Wiedemann, Nucl. Phys. **B588**, 303 (2000).
  - [6] X. F. Guo and X.-N. Wang, Phys. Rev. Lett. **85**, 3591 (2000); X.-N. Wang and X. F. Guo, Nucl. Phys. **A696**, 788 (2001).
  - [7] M. Gyulassy, I. Vitev, and X. N. Wang, Phys. Rev. Lett. **86**, 2537 (2001); C. A. Salgado and U. A. Wiedemann, Phys. Rev. Lett. **89**, 092303 (2002).
  - [8] E. Wang and X. N. Wang, Phys. Rev. Lett. **89**, 162301 (2002).
  - [9] PHENIX Collaboration, K. Adcox *et al.*, Phys. Rev. Lett. **88**, 022301 (2002); PHENIX Collaboration, S. S. Adler *et al.*, Phys. Rev. Lett. **91**, 072301 (2003).
  - [10] C. Adler *et al.*, Phys. Rev. Lett. **89**, 202301 (2002); STAR Collaboration, J. Adams *et al.*, Phys. Rev. Lett. **91**, 172302 (2003).
  - [11] X. N. Wang, nucl-th/0305010.
  - [12] Y. L. Dokshitzer and D. E. Kharzeev, Phys. Lett. B **519**, 199 (2001).
  - [13] PHENIX Collaboration, K. Adcox *et al.*, Phys. Rev. Lett. **88**, 192303 (2002).
  - [14] M. Djordjevic and M. Gyulassy, Phys. Lett. B **560**, 37 (2003); Phys. Rev. C **68**, 034914 (2003); Nucl. Phys. **A733**, 265 (2004).
  - [15] N. Armesto, C. A. Salgado, and U. A. Wiedemann, Phys. Rev. D **69**, 114003 (2004).
  - [16] B. Zhang, E. Wang, and X. N. Wang (to be published).
  - [17] M. Luo, J. Qiu, and G. Sterman, Phys. Lett. B **279**, 377 (1992); Phys. Rev. D **50**, 1951 (1994); **49**, 4493 (1994).
  - [18] J. Osborne and X.-N. Wang, Nucl. Phys. **A710**, 281 (2002).
  - [19] B. W. Zhang and X. N. Wang, Nucl. Phys. **A720**, 429 (2003).
  - [20] D. Kharzeev and K. Tuchin, Nucl. Phys. **A735**, 248 (2004).