Achronal Cosmic Future

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The spherically symmetric accretion of dark and phantom energy onto Morris-Thorne wormholes is
considered. It is obtained that the accretion of phantom energy leads to a gradual increase of the
wormhole throat radius which eventually overtakes the superaccelerated expansion of the Universe and
becomes infinite at a time in the future before the occurrence of the big rip singularity. After that time,
as it continues accreting phantom energy, the wormhole becomes an Einstein-Rosen bridge whose
corresponding mass decreases rapidly and vanishes at the big rip.

The Universe which we live in is subject to a current evolution which is certainly causal. That happens no matter whether or not it is expanding in an accelerating fashion. But one cannot be so sure about causality in the primordial universe. A rather natural scenario was in fact proposed by Gott [1] in which the Universe created itself in a classically and quantum-mechanically stable way [2], starting with a noncausal patch filled with closed time-like curves which might be followed by an immediate ekpyrotic quantum evolution [3]. However, nobody keeps any doubts about causality in the future. There is a rather tacit consensus that, whatever the fate of the Universe, it will be governed by causal, chronal principles which probably respect the second law of thermodynamics [4]. It is currently thought, following present scientific logic, that, after all, the present Universe is big enough as for consistently admitting a classical description where no time traveling or similar oddities that disrupt causal evolution appear to be allowed [5]. We are going nevertheless to consider a possible cosmological framework where really there could be a disruption of the causal evolution of the accelerating Universe in the future. Actually such a prediction appears to be unavoidable if the universal equation of state for dark energy is characterized by a constant parameter $w = p/\rho < -1$ (where p and ρ respectively denote pressure and energy density), a case which is dubbed phantom energy [6] and which is not at all excluded by present cosmological constraints [7]. That phantom energy has a fair collection of weird properties which are all based on the requirement that the phantom stuff violates the dominant energy condition, that is, $p + \rho < 0$ [8]. But violating that energy condition would also make possible the existence of natural traversible wormholes [9], even at scales larger than the Planck scale, if were it not for the fact that such classical wormholes appear to be unstable to quantum vacuum fluctuations [5]. The evolution of wormholes and ringholes [10] induced by the accelerated expansion of the Universe has been already considered [11] for the case of dark energy with both $w > -1$ and $w < -1$. It was seen that in all

cases the scale of the tunneling increased as the scale factor became larger and blew up at the big rip [11] when the Universe was filled with phantom energy. Once we have learned what kind of kinematic effects can be expected from dark energy on wormholes, we shall consider in this Letter the effect of dark energy accretion onto submicroscopic and macroscopic wormholes. Our main result is that, as a consequence of dark energy accretion, whereas the wormhole throat gradually decreases down to a minimum size if $w > -1$, whenever $w < -1$ the wormhole throat grows rapidly up to reaching an infinite size before the Universe gets into the big rip singularity.

The mass μ of the spherical thin shell of exotic matter in a Morris-Thorne wormhole can be given by [12,13]

$$\mu = -\pi b_0/2, \tag{1}$$

where we have used units such that $G = \hbar = c = 1$, and b_0 is the radius of the spherical wormhole throat. Now, since, very approximately, the mass in Eq. (1) is just the negative of the amount of mass required to produce a normal Schwarzschild wormhole (i.e., the wormhole connecting a black hole with mass $M \simeq -\mu$ to its corresponding white hole, making an Einstein-Rosen bridge [14]), the rate at which the exotic mass of a wormhole changes by accreation of dark energy can be approximately equalized to the negative of the rate of the mass change of the black hole making the Schwarzschild wormhole, due to accreation of dark energy. Quite recently, Babichev, Dokuchaev, and Eroshenko [15] have obtained that, as a consequence of fluid accretion, the mass of a black hole changes at a rate $\dot{M} = 4\pi A M^2 (p + \rho)$, where A is a positive dimensionless constant. For an equation of state of the fluid $p = w\rho$, which will be assumed to be constant throughout the present Letter, we can therefore write for the rate of change of the throat radius of a Morris-Thorne wormhole due to dark energy accretion the expression

$$\dot{b}_0 = -2\pi^2 D b_0^2 (1 + w)\rho, \tag{2}$$

with D another positive dimensionless constant, $D \simeq A$, and ρ the energy density of the dark energy fluid.

It is known that the dark energy density is given by

$$\rho = \rho_0 a^{-3(1+w)}, \tag{3}$$

where ρ_0 is an arbitrary constant playing the role of the initial value of the energy density at the onset of dark energy domination, and $a = a(t)$ is the scale factor. A general solution for that scale factor in the case of a universe dominated by dark energy can be written as [11]

$$a(t) = \left[a_0^{3(1+w)/2} + \frac{3}{2}(1+w)\sqrt{\frac{8\pi\rho_0}{3}}(t-t_0) \right]^{2/[3(1+w)]}$$
$$\equiv T^{2/[3(1+w)]}, \tag{4}$$

in which a_0 and t_0 respectively are the initial values for the scale factor and time at the onset of dark energy domination. From Eqs. (1), (2), and (3) we have then

$$\dot{b}_0 = -2\pi^2 D(1+w)\rho_0 b_0^2 T^{-2}. \tag{5}$$

Trivial integration of this equation finally produces

$$b_0 = \frac{b_{0i}}{1 + \frac{b_{0i}(t-t_0)}{\dot{b}_{0i}a_0^{3(1+w)/2}T}}, \tag{6}$$

$$\dot{b}_{0i} = \frac{1}{2\pi^2 D\rho_0(1+w)}. \tag{7}$$

Thus, for all quintessential models with $w > -1$, the radius of the wormhole throat, and hence the mass of exotic matter contained in it, will gradually decrease with time and tends to a constant minimum value

$$b_{0\min} = \frac{b_{0i}}{1 + \frac{4\pi^2 D\rho_0 b_{0i}}{3a_0^{3(1+w)/2}\sqrt{8\pi\rho_0/3}}}, \tag{8}$$

as $t \to \infty$. The case of a universe filled with phantom energy for which $w < -1$ is actually quite more interesting. If $w < -1$ we have in fact

$$b_0 = \frac{b_{0i}}{1 - \frac{(t-t_0)b_{0i}}{\dot{b}'_{0i}(t_* - t_0)T'}}, \tag{9}$$

$$\dot{b}'_{0i} = \frac{3\sqrt{8\pi\rho_0}}{4\pi^2 D\rho_0}, \tag{10}$$

$$T' = a_0^{-3(|w|-1)/2} - \frac{3}{2}(|w|-1)\sqrt{8\pi\rho_0/3}(t-t_0), \tag{11}$$

$$t_* = t_0 + \frac{2}{3(|w|-1)a_0^{3(|w|-1)/2}\sqrt{8\pi\rho_0/3}}, \tag{12}$$

t_* being the time at which the big rip singularity takes place. While the remarkable, but not fully unexpected, prediction of Eq. (9) is that when the wormhole accretes

phantom energy, the radius of its throat will gradually increase (notice that after all, the wormhole contains the same kind of phantom energy as the stuff that it is accreting); that equation also shows the rather surprising result that, as a consequence from such an exotic matter increase, the size of the spherical wormhole throat will turn out to blow up at a time

$$\tilde{t} = t_0 + \frac{t_* - t_0}{1 + b_{0i}a_0^{3(|w|-1)/2}/\dot{b}'_{0i}} \tag{13}$$

which necessarily occurs before the big rip singularity at t_* (see Fig. 1). We note, however, that at time $t = \tilde{t}$, the exotic energy density becomes zero and that there is not any curvature singularity [9]; therefore, one need not cut the evolution of the Universe at that point. Notice as well that after $\tilde{t}$, the mass of the wormhole μ becomes positive, and this means that what initially was a Morris-Thorne wormhole is converted after time $\tilde{t}$ into an Einstein-Rosen bridge, which will immediately pinch off to leave a black-white hole pair that will rapidly lose its mass by the Babichev-Dokuchaev-Eroshenko mechanism [15] to vanish at the big rip.

The above results can be expected to happen in all dark energy models that predict a big rip singularity in the future once they are enforced to violate the dominant energy condition. Thus, in the essentially distinct case where the dark energy which is accreted by the Morris-Thorne wormhole corresponds to what is denoted as a k-
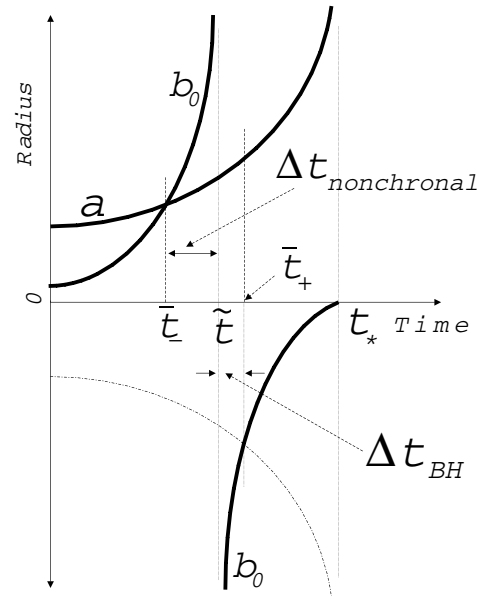

FIG. 1. Evolution of the radius of the wormhole throat b_0 induced by accretion of phantom energy. At time $t = \tilde{t}$, the negative exotic mass becomes infinite and then changes sign, so converting the wormhole into an Einstein-Rosen bridge whose associated mass decreases down to zero at the big rip at $t = t_*$. During the time interval $\Delta t_{nonchronal}$ there will be a disruption of the causal evolution of the whole Universe.

essence field [16] equipped with noncanonical kinetic energy, we have for the phantom regime [17]

$$p + \rho = -3(1 - \xi)H^2/\xi, \tag{14}$$

with $0 < \xi < 1$ and $H = \dot{a}/a$, where

$$a \propto (t - t_b)^{-2\xi/[3(1-\xi)]}, \tag{15}$$

in which t_b is an arbitrary time at which the big rip takes place; we obtain

$$b_0 = \frac{b_{0i}}{1 - \frac{t b_{0i}}{\dot{b}_{0i} t_b (t_b - t)}}, \tag{16}$$

$$\dot{b}_{0i} = \frac{3(1 - \xi)}{8\pi^2 D \xi}. \tag{17}$$

It can be readily seen that the properties of Eq. (16) qualitatively match those implied by Eq. (9), with the time at which the radius of the wormhole throat becomes infinite being given in this case by

$$\tilde{t} = \frac{t_b}{1 + \frac{b_{0i}}{\dot{b}_{0i} t_b}}; \tag{18}$$

that is, again $\tilde{t} < t_b$. Exactly the same expressions as those given in Eqs. (16) and (18) (but for a slightly different $\dot{b}_{0i}$) are also obtained when one uses the simple solution employed by Babichev, Dokuchaev, and Eroshenko [15].

The most bizarre implication stemming from the above results is that for a given cosmic time interval in the future starting at $t = \bar{t}_-$ and ending at $t = \bar{t}_+$ (for which times $a = b_0$), first the size of the wormhole throat will exceed the size of the whole Universe, and then, after time $\tilde{t}$, the Universe is contained for a cosmic while inside a giant black hole (see Figs. 1 and 2). Since, starting at time $\tilde{t}$, whereas the size of the Universe continues steadily increasing toward infinity, the size of the black hole will gradually decrease toward zero, the Universe will tend to first equalize and then exceed the black hole size before reaching the big rip. In the above simpler k-essence model we have

$$\bar{t}_{\pm} = t_b \pm \frac{a_0(t_b \dot{b}_{0i} + b_{0i})}{2 t_b \dot{b}_{0i} b_{0i}} - \frac{\sqrt{a_0^2(t_b \dot{b}_{0i} + b_{0i})^2 \pm 4 t_b^2 a_0 \dot{b}_{0i} b_{0i}^2}}{2 t_b t_{0i} \dot{b}_{0i}}, \tag{19}$$

with similar, albeit more complicated, expressions for the cosmic model that corresponds to the scale factor (4).

It follows that, during the time interval where the Universe is inside the Morris-Thorne wormhole throat, one must necessarily consider that the wormhole ought to either (A) be first disconnected from the regions of the original phantom universe at which it was previously connected, to be instead connected during that interval
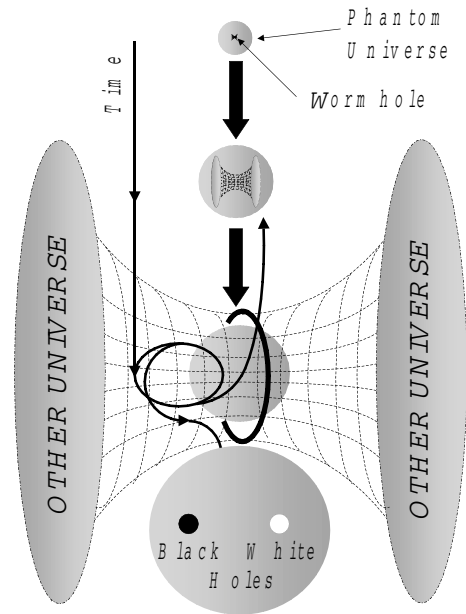

FIG. 2. Pictorial representation of the evolution of a universe filled with phantom energy which is being accreted onto an initially submicroscopic wormhole for the case (A) (see the text) when the wormhole is reconnected to other two extra larger universes, during the time period on which the wormhole throat is larger than the Universe and time follows closed curves.

to very large regions of two extra larger universes (such as it is shown on Fig. 2), or (B) keep its connections to the considered phantom universe according to the topology depicted on Fig. 3. Since the two wormhole mouths should be moving relative to one another, due to both the accelerating expansion of the phantom universe and the wormhole in case (B), and just to the gradual growth of the wormhole in case (A), in both cases, the phantom universe could be regarded as just being a "time traveler" through a gigantic Morris-Thorne wormhole during that time interval. Thus, the phantom universe could eventually either travel to its past to repeat the previous evolution process again or travel to its future where it will find itself either inside a decreasing black hole or containing a smaller black hole when nearer the big rip or even evolving, without containing any kind of holes, on the contracting phase after the big rip [11]. Some of these possible universal time traveling processes are also depicted in Figs. 2 and 3.

A key question however remains. Even though Morris-Thorne wormholes and other topological extensions from Misner space [10] appear to be stable at scales of the order the Planck length [18], macroscopic holes have been claimed to be unstable to quantum vacuum fluctuations [5]. We shall argue nevertheless that wormholes grown up by accreting phantom energy would keep their essential quantum nature, and hence their submicroscopic stability,
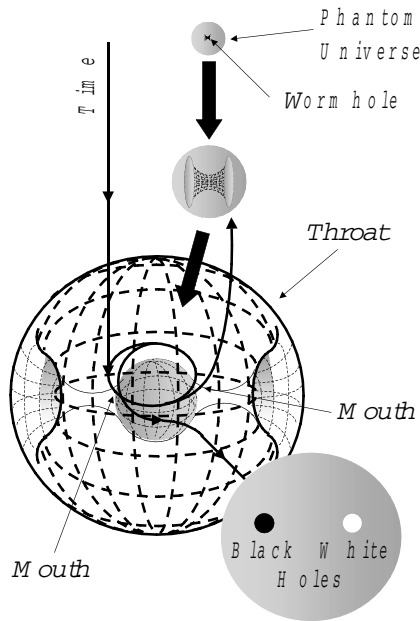
FIG. 3. Pictorial representation of the evolution of a universe filled with phantom energy which is being accreted onto an initially submicroscopic wormhole for the case (B) (see the text) when the wormhole keeps its connections to the original phantom universe, without resorting to connections to extra large universes, during the time period on which the wormhole throat is larger than the Universe and time follows closed curves.

and should be regarded as quantum spacetime constructs without classical analog. In fact, by itself, a phantom field can be considered as a quantum entity in the following sense. It is known [19] that in order to preserve weak energy condition, $\rho > 0$, the phantom scalar field should be Wick rotated so that, e.g., $\phi \to i\Phi$. Now, it can be seen that such a rotation is equivalent to Wick rotating the time t itself so that, e.g., $t \to -i\tau$, while preserving the field unchanged. For instance, for the dark energy model with scale factor (4), the scalar field ϕ can be expressed as [19] $\phi + \phi_0 = \frac{2}{3\sqrt{1+w}\ell_P} \ln T$, where ϕ_0 is the initial value of the scalar field and we have restored the Planck length ℓ_P. In the phantom case $w < -1$, this equation can be approximated to $\phi + \phi_0 \propto i(t - t_0)$, from which one can in fact deduce that Wick rotating ϕ while keeping t unchanged is equivalent to Wick rotating time t while keeping ϕ unchanged, always preserving the weak energy condition. Now, it is well known that a Euclideanized spacetime metric would describe a quantum system [20]. It is thus, in a way which parallels the procedure through which the quantum temperature and entropy of black holes can be derived [21], that the cosmic phantom fluid, and hence the wormhole, can be shown to be essential

quantum entities. In fact, it has recently been suggested [22] that the phantom stuff is characterized by a negative temperature and becomes therefore an essentially quantum fluid without classical analog.

An important caveat on the conclusions of this work is worth mentioning. Even though current data appear to support $w < -1$, they do not seem to favor a constant equation of state [23]. Therefore, it could well be that the equation of state would relax back into the stable region of $w > -1$ in the future, before the big rip singularity or even the wormhole reached an infinite size. However, even in that case, there would still be some tendency for the wormhole radius to grow, as we have found.

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